the general science journal

Full analysis & design solutions for EHD Thrusters at saturated corona current conditions
Category: Ionocrafts & Lifters

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Published on 1/1/2004 – Updated 1/1/2006

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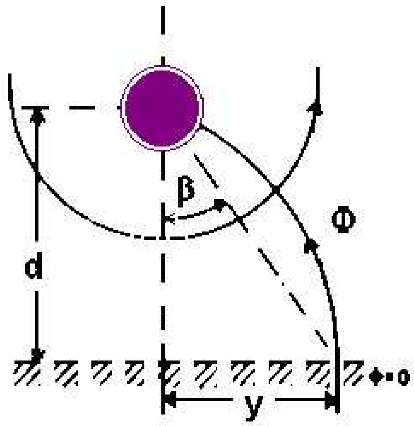
This paper deals with analysis & design solutions for single stage, multiple corona EHD Thrusters.

An EHD thruster is a propulsion device based on ionic fluid propulsion, that works without moving parts, using only electrical energy. The principle of ionic air propulsion with corona-generated charged particles has been known since the earliest days of the discovery of electricity, with references dating back to year 1709 in a book titled *Physico-Mechanical Experiments on Various Subjects* by Francis Hauksbee. The first publicly demonstrated tethered model was developed by Major De Seversky in the form of an Ionocraft, a single stage EHD thruster, in which the thruster lifts itself by propelling air downwards according to Newton's third law of motion. De Seversky contributed much to its basic physics and its construction variations during the year 1960 and has in fact patented his device US Patent 3,130,945, April 28, 1964, and has also demonstrated a working model capable of lifting up its own weight, excluding the power supply. During our research, and utilizing the calculations described in this paper, we have been able to design highly efficient single stage EHD thrusters, which not only generate enough thrust to lift their own weight, but to lift extra payloads in excess of their own weight. For example, the hexagonal multicorona thruster shown at http://www.blazelabs.com/e-exp14.asp has a structural weight of just 85 grammes, and generates a maximum of 200 gramme force.

Only electric fields are used in this propulsion method. In its basic form, it simply consists of two parallel conductive electrodes, one in the form of a fine wire usually referred to as the corona wire, and another which may be formed of either a wire grid, tubes or foil skirts with a smooth round surface, referred to as the collector. When such an arrangement is powered up by high voltage in the range of a few kilovolts, it produces thrust. The ionocraft forms part of the EHD thruster family, and is a special case in which the ionisation and accelerating stages are combined into a single stage. The aim of this paper is to provide the mathematical tools to design and predict the basic electrical and mechanical properties of this kind of thrusters. A mathematical solution for the best spacing between neighbouring thruster cells is also presented. The derived equations can also be used for the design of other EHD devices, such as calculating the air flow rate (CFM) in EHD coolers, which could be used as a silent alternative for cooling of electronic devices, or calculating the air pressure generated by EHD speakers.

Mathematical analysis



Wire to plane ionocraft
Corona current distribution
Blaze Labs 2004

The charge flow is simplified as multiple paths of unipolar ions drifting all together in the form of an ion cloud with mobility μ and negligible diffusion effects. A voltage source of voltage V is applied between the corona wire and collector grid. The corona is a discharge where ionisation is non thermal.

In the above diagram, the top conductor is the corona wire (not to scale), **d** is the vertical air gap distance from wire to plane grid, and **y** is the effective lateral distance over which the cloud spreads out during its journey to the collector. **β** is called the distribution angle and is a measure of spreading out of the ion cloud taking into account its interaction with neutral air molecules and ion space charge.

It can be assumed that, all charge transport through the gap is carried by charged particles having the same polarity as the corona as described in detail in http://blazelabs.com/l-intro.asp . The ion flow lines coincide with the electric field lines, but, the electric field distribution is strongly dependent on the ion space charge. At high currents or corona saturation currents, the current distribution j(β) is of a modified Laplacian form which was earlier found by Warburg in 1899, that it closely follows the so called Warburg $\cos^5$ distribution, given by:

$j(B) = j_0 \cos^m(\beta)$ …… m = 4.82 for positive corona and m = 4.65 for Negative Ionocrafts

If one plots the Warburg distribution it can be clearly seen that for angle range of 60° to 65°, the current density falls sharply from 4% towards 1%, indicating that the field lines further out than point y, at which β >= 65° are not acting upon the ion cloud. Due to the small difference in **m** for different polarities, the angle β for positive ion clouds at which this threshold occurs is slightly less, calculated to be just one degree less , that is 64°.

This first rule, clearly indicates that implementing collector grids, which laterally exceed 2d*tan(β) will not have any beneficial effect upon the resulting thrust, and result only in additional 'dead' weight. Knowing β, we can now conveniently express **y**, half the width of the ion cloud in terms of the height **d** of the wire above the plane, since

K Tan(β) = y/d (1)
Ktan(65°) = y/d
y = 2.1 Kd ….. half spread width for negative emitter corona wire (2)

for positive ions

Ktan(64°) = y/d
y = 2 Kd ….. half spread width for positive emitter corona wire (3)

K is a coefficient which depends on the actual geometry of the electrodes. See appendix.

Effective ion cloud cross sectional area

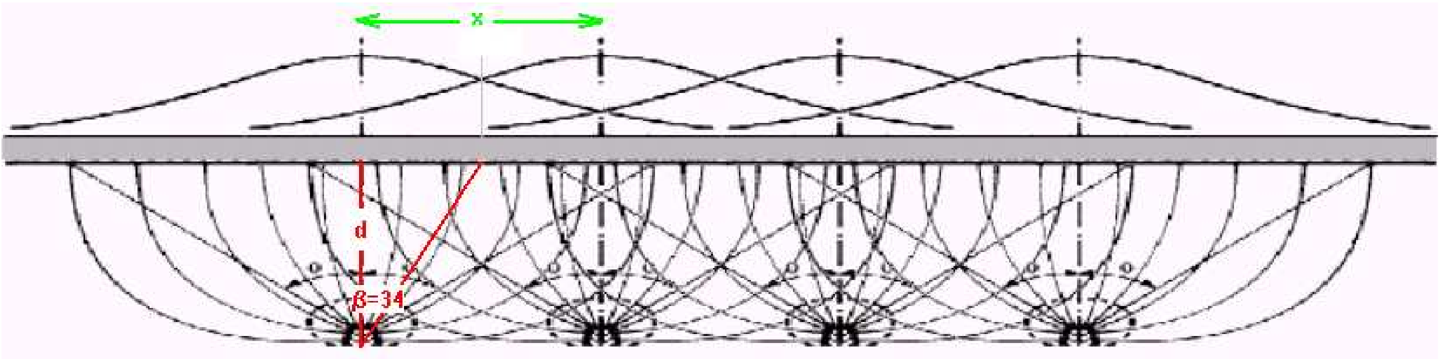
Thus, the effective area of the negative or positive ion clouds at the grid per unit length l of the grid becomes:

$A_n = 2\,y\,l = 4.2\,Kd\,l$ … for negative corona (4)
$A_p = 2\,y\,l = 4\,Kd\,l$ … for positive corona (5)

So, effective area is independent of actual grid lateral width (given that the width 2y >= 4.2Kd or 4Kd respectively)

Xavier's Law for multi-corona element spacing

There is a minimum distance 'x' between corona sources (point or wire), at which the effective force of the 2 sources will be less then twice of a single source. In fact for very close sources, the effective force will be the same as for one source and will virtually behave as one effective point or wire. In all theoretical work by other researchers in this field, this minimum distance is always derived from experimental work, but the situation can be easily analysed so that such distance can be accurately predicted for different voltage gradients and geometries. The cause of this current deficiency is due to the fact that the current density at any point in the cloud cannot exceed the saturated current limit $\mathbf{j_0}$. Ideally, with multiple parallel sources, assuming no ions are lost to the surroundings, the collector should see a constant current density of $\mathbf{j_0}$ all over its surface. In the below diagram, the current density at the collector from each corona wire is superimposed. The total current density at each position on the collector would be the sum of all the plots. As you can see, the region of least current density will always be at midpoint between the emitters, at which the collector should ideally receive ½ $\mathbf{j_0}$ from each source. Now solving for $\cos^{4.82}(\beta) = 0.5$, we get $\beta = 30^o$. So the optimal distance between 2 elements is x = 2y = 2Kd tan (30) = 1.15 d. This applies to all twin point to plane geometries, and also to twin wire to plane.



For 4 parallel elements, this becomes a little bit more complex, because the 3[rd] & 4[th] elements at the extreme ends will add another pair of components at the middle of the collector at an angle $(\tan^{-1}[3y]/\tan^{-1}[1y]) * \beta = 1.59\beta$

$j_{max} = j_0(2\cos^{4.82}(\beta) + 2\cos^{4.82}(1.59\beta)) <= j_0$ which gives $\beta \sim 34^o$, and a **spacing of x = 2Kd tan (34°) = 1.35d ……… (6)**

For six or more parallel elements, the β factors will result in angles greater than 65° and thus have virtually no effect, so x=1.35d can be taken as the maximum distance even for higher numbers of parallel elements. If the wires are set closer than this, the corona current density at the wire diminishes so that at no point does the current density ever exceeds the saturated value $\mathbf{j_0}$ for a single wire. In the limit where x=0, the saturation current will be equally divided in two, and they will therefore act as a single wire.

Lifter designed with parallel corona wires spaced at 1.3d

Derivations for maximum pressure, force, air flow, air velocity & corona saturation current

The ion flow is driven through the air gap by the electric field and braked by the collision and friction with the neutral gas molecules. Ion acceleration in the wire to plane geometry is negligible, and thus all the electric energy from the field is ultimately transferred to the neutral molecules. We can thus integrate the force qE along all field lines, and take the effective force projected over the plane grid. This has been worked out by Sigmond et all, and the total force exerted by the corona current i over a gap distance d is given by:

$F = i\, d / \mu$ ……. F is force in Newtons, I current in Amps, d air gap in me tres, μ = ion mobility in air

This shows that the vertical component of the force is independent of the actual ion path and electric field.
Again we have a small difference between negative and positive ions, due to a small variation in mobility for different polarities, μ_n= 2.7 E-4 m^2/Vs , μ_p= 2.0 E-4 m^2/Vs , thus we have:

Ion cloud velocity $\mathbf{v_{ion} = \mu E}$ **…. resulting in ion cloud velocities > 100m/s (7)**

F = i d / 2.7 E- 4 ….. for negative corona (8)

F = i d / 2.0 E- 4 ….. for positive corona (9)

This force, is equal to the momentum transfer rate between the fast ion cloud to the almost stationery neutral air molecules, and can be used to calculate gas flow and velocity at the lower side of the grid. If we assume a free flowing air stream through the collector grid effective area A, with average air flow S and velocity v (m/s), we have:

$S = v/A$
$F = i\, d / \mu = \rho S v = \rho S^2/A$, ………… ρ is the air density in kg/m^3 , S= flow rate in litres/sec

$\mathbf{S = (i\, A\, d / \mu\rho)^{1/2}}$ **(10a)**
$\mathbf{S\ (CFM) = 2.1186 * (i\, A\, d / \mu\rho)^{1/2}}$ **(10b)**
$\mathbf{v = S/A = (id / \mu\rho A)^{1/2}}$ **(11)**

Now, the total maximum pressure acting over the active grid area A (having dimensions: length l, width 2y), is equal to :

$P = F/A$
$P = id\ /(A\mu)$
$P = id\ /(2y\ l\ \mu)$ …. writing y in terms of d, y = 2.1Kd or y = 2Kd for the respective corona polarities we have:

$\mathbf{P_{max} = id / (4.2\ Kd\ l\ \mu) = i / (4.2\ K\mu\ l) = j / (4.2\ K\mu)}$ **… for negative coronas (12)**

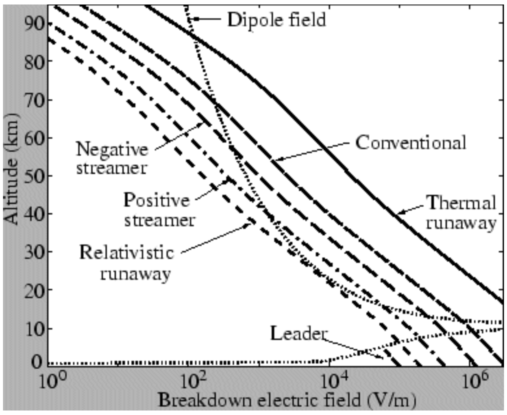
$\mathbf{P_{max} = id / (4\ Kd\ l\ \mu) = i / (4\ K\mu\ l) = j / (4\ K\mu)}$ **… for positive coronas (13)**

This shows that pressure generated does not depend on the length l ,but only on saturation current density j and the Borg-Sigmond electrode geometry coefficient K.

Now the pressure gradient is equal to coulombs force acting on each ion.

From Gauss law, we have $\delta p/\delta z = \varepsilon_o * E * (\delta E/\delta z)$ …. where E varies from zero to the streamer breakdown voltage

Integrating for total pressure increase $\mathbf{\Delta P_{max} = \frac{1}{2}\, \varepsilon_o\, E_{max}^2}$ **or** $\mathbf{\frac{1}{2}\, \varepsilon_o\, (V/d)^2}$ **(14)**



The breakdown in the ionised air gap occurs by means of a totally different mechanism than the conventional electron avalanche, namely by corona streamers. Streamers are ionisation waves which can propagate as narrow channels through regions where the electric field is less than the conventional breakdown voltage for air (30kV/cm). In the plot below, you can see the conventional Townsend avalanche breakdown voltage at sea level is approximately 3E6 V/m, but once we have corona streamers in action, this drops to either 1.1E6 V/m for negative streamers, or even worse, about 0.6E6 V/m for positive streamers. This self-propagation as we know, is due to highly nonuniform electric fields which result from significant gradient in current density, or space charge.

Note that again, we have another variation between streamer breakdown voltage (V/d) for negative and positive coronas, where E_n ~ -11kV/cm or -1.1E6 V/m and E_p ~ +6kV/cm or +0.6E6 V/m. However this average breakdown voltage varies with pressure, temperature, humidity, altitude and lateral wind speed. So, it is recommended to use actual streamer breakdown V/d values from experimental data.

We can now equate the above pressure equations:

$P_{max} = j_{max} / (4.2\ K\mu) = ½ * \varepsilon_o E_n^{\ 2}$ for negative coronas (15)

$P_{max} = j_{max} / (4\ K\mu) = ½ * \varepsilon_o E_p^{\ 2}$ for positive coronas (16)

Thus the saturation corona current per unit length (per metre) j_{max} in each case is given by:

$j_{\ max} = 2.1 * K\mu\varepsilon_o E_n^2$ for negative coronas E_n= -V/d (17)

$j_{\ max} = 2 * K\mu\varepsilon_o E_p^2$ for positive coronas E_p= +V/d (18)

This shows that an upper limit for current density exists which is independent of actual ionocraft size, given it's driven under saturated corona current conditions. The same applies for pressure generated.

Total current consumption $i_{max} = j_{\ max} * l$ (19)

Total power consumption $P = i_{max} * V$ (20a)
It is also given by $P = F_{max} * v_{ion}$ (20b)

Substituting for **$j_{\ max}$** in $F = id/\mu = j\ l\ \ d / \mu$, we get

$F_{max} = 2.1\ K\ \varepsilon_o\ V^{\ 2} * (l / d)$ for negative coronas (21)

$F_{max} = 2\ K\ \varepsilon_o\ V^{\ 2} * (l / d)$ for positive coronas (22)

Thrust in gF **T = (1000 / 9.8)* Fmax (23)**

Using equations (23) & (20a), the thrust to power ratio in g/Watt assuming $g=9.8m/s^2$:

T/P = (1000 / 9.8)* Fmax / (V * i_{max}) = 102 / (μE) where E= Ep or En depending upon polarity (24)

Machine efficiency = Mechanical energy output / Electrical energy input , assuming thruster reaches air velocity

Conversion efficiency % = 100 * F_{max} * v / (i_{max} * V) (25)

Performance F_{max} per unit length = $2.1\ K\ \varepsilon_o\ V^{\ 2}\ / d$ for negative coronas (26)

Performance F_{max} per unit length = $2\ K\ \varepsilon_o\ V^{\ 2} *\ / d$ for positive coronas (27)

Fan performance (CFM/Watt) = S(CFM) / P ….. (28)

Practical worked estimates for an Ionocraft
Operating voltage at +40kV, positive corona polarity, constructed of an array of 10 parallel elements 20cm each

From E_p ~ +0.6MV/m (see quick reference below), we know that for maximum thrust, the gap distance d at the given voltage, must be 40kV/(6kV/cm) = 6.7cm or 0.067m

Total element length = 10 elements * 0.2m = 2m

Eqn(3)…. Ion spread width y = 2 * K * d = 2 * 1 * d = 13.3cm or 0.133 m

Eqn(5) ….Ion cloud area reaching grid = $A_p = 2 * y * l$ = 2 * 0.133 * 2 = 0.53 m^2

Eqn(6) … Distance between parallel corona elements x= 1.3d = 1.3 * 0.067 = 0.087m or 8.7cm

Eqn(7) … Ion cloud velocity $v_{ion} = \mu E$ = 2E-4 * 0.6E6 = 120 m/s

Eqn(18)…. Saturated corona current per metre $j_{max} = 2\mu\varepsilon_o E_p^2$ = 2 * 2E-4 * 8.8542E-12 * $0.6E6^2$ = 1.28 mA/m or 12.8uA/cm

Eqn(19) … Current consumption for full length $i = j * l$ = 1.28 * 2 = 2.55mA

Eqn(10a) …. air flow rate $S = (i A d / \mu\rho)^{1/2}$ = (2.55E-3 * 0.53 * 0.067/ (2E-4 * 1.2)) $^{1/2}$ = 0.61 litres/sec
Eqn(10b) …. air flow rate CFM = 0.61 * 2.1186 = 1.29 CFM

Eqn(11) …..air velocity $v = S/A = (id / \mu\rho A)^{1/2}$ = (2.55E-3 * 0.067/ (2E-4* 1.2 * 0.53)) $^{1/2}$ = 1.15 m/s

Eqn(22)….. $F_{max} = 2\varepsilon_o V^2 * (l / d)$ = 2 * 8.8542E-12 * $40E3^2$ *(2/ 0.067) = 0.85 N/m = 87.18 gF

Eqn(16) ….. $P_{max} = j / (4 \mu)$ = 1.28E-3 / (4 * 2E-4) = 1.6 Pa (or $P_{max} = \frac{1}{2} \varepsilon_o E_{max}^2$ = 1.6 Pa) (or $P_{max} = F_{max} / A_p$ = 1.6 Pa)

Eqn(20a/b) …. Total power consumption = i * V = 2.55mA * 40kV = 102 W (or $F_{max} * v_{ion}$ = 0.85 * 120 = 102 W)

Eqn(24)…… T/P = 102 / (μE_p) = 102/ (2E-4 * 0.6E6) = 0.85 g / Watt

Eqn(25)……. Conversion efficiency = 100 * F_{max} * v / (i V) = 100* 0.85 * 1.15/ 102 = 0.96% (or 100* v/ v_{ion}= 100 * 1.15/120 = 0.96%)

Eqn(28) ….. Fan Performance = S (CFM) /P = 1.29/102 = 0.013 CFM/Watt.

So, this ionocraft will produce a 87.18gF thrust, and run at 2.55mA, 102 Watts.
A similar sized lifter will produce 43.6gF thrust, and run at 1.28mA, 51Watts.

This ionocraft can also be used as an EHD hovercraft or EHD speaker generating a total pressure of 1.59Pa.
As a fan, it is rated at 1.29 CFM performing at 0.013CFM/Watt.
As an EHD air pump it will move about 2212.69 litres of air per hour

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Quick reference:

μ_n = 2.7 E-4 m^2/Vs
μ_p = 2.0 E-4 m^2/Vs

E_n ~ -11kV/cm or -1.1E6 V/m
E_p ~ +6kV/cm or +0.6E6 V/m

Warburg current distribution $j(B) = j(0) \cos^m(\beta)$
m = 4.82 for positive corona
m = 4.65 for negative corona

Experimental distribution angle $\beta = 65^o$ for negative corona ionocrafts
Experimental distribution angle $\beta = 64^o$ for positive corona ionocrafts

K = Borg-Sigmond dynamic geometric coefficient (R.Sigmond uses a fixed constant K for the point to plane geometry)
(K=9/8 for planar geometry, K=1 for multiwire ionocrafts, K=0.5 for lifters)

ε_o = 8.8542E-12 F/m
air density ρ = 1.2kg/m^3
g= 9.8 m/s^2

Published on 1/1/2004 – Updated 1/1/2006