the general science journal

November 26, 2012

Algebraic Graviton Quantizing

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Abstract
The Unification of General Relativity and Quantum Mechanics has difficulties because of the renormalization problem. The graviton is a Spin-2 particle that would cause UV divergences in quantum field theory. This paper shows a Quantum Gravity model that make it possible to treat gravitons as a system of Spin-1 particles. By using algebraic techniques the Spin-1 particles are connected so that the Spin-2 characteristic of the graviton is conserved.

Introduction

In General Relativity, Gravity is treated by the curvature of spacetime . For each point in spacetime $\vec{x}$, there exists a metric tensor $g_{\mu\nu}$ that describes the geometry of the spacetime. May be $\vec{g}_\mu$ and g_ν two tangent vectors. Then, the metric tensor can be expressed as

$$g_{\mu\nu} = \vec{g}_\mu \vec{g}_\nu = |\vec{g}_\mu||\vec{g}_\nu| cos(\vec{g}_\mu, \vec{g}_\nu). \tag{1}$$

Hence, the metric tensor depends on the absolute value of tangent vectors $|\vec{g}_\mu|$ and the angle between $\vec{g}_\mu$ and g_ν. For the energy-momentum-tensor $T_{\mu\nu}$, the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R Einstein's field equations can be written as [1]

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}. \tag{2}$$

If the energy-momentum-tensor is non-symmetric, the spacetime has torsion and must be treated with Einstein-Cartan-Theory [2]. The source of the spacetime torsion is the Spin tensor $S^\lambda_{\mu\nu}$. May be $C^\lambda_{\mu\nu}$ the torsion tensor defined by

$$C^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}, \tag{3}$$

the connection between torsion tensor and Spin tensor is given by

$$C^\lambda_{\mu\nu} + \delta^\lambda_\mu C^\kappa_{\nu\kappa} - \delta^\lambda_\nu C^\kappa_{\mu\kappa} = 8\pi S^\lambda_{\mu\nu}. \tag{4}$$

Considering the Uncertainty principle of Heisenberg [3]

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{5}$$

with the position uncertainty Δx and the momentum uncertainty Δp. If the position uncertainty tends to zero, the momentum uncertainty tends to infinity. In a theory of Quantum Gravity, such infinities must be avoided. The divergence $\Delta p \to \infty$ can be avoided if $\Delta x \neq 0$ at every point in spacetime. The metric tensor must be modified so that the metric include the position uncertainty. In Loop Quantum Gravity the metric is quantized by the introduction of Ashtekar variables [4]. Length elements and time increments in Quantum Gravity are expressed in Planck units. In this paper, the graviton field, which is a Spin-2 field, can be expressed as Spin-1 fields n_μ, where algebraic connection lead to the Sin-2 characteristic of the graviton. Since other fundamental forces are described by a vector field in quantum field theory, gravity can be assumed as a vector field by using Algebraic Graviton Quantizing. The tensor field characteristic of gravity can be induced by using object-morphism transformations. Since n_μ quantizes the classical field $|\vec{g}_\mu|$

in equation (1), the angle between two tangent vectors can be quantized by introducing morphisms between n_μ and n_ν. By the structure of morphisms, the torsion of the spacetime can be quantized by the additional vector field m_ν. Both fields n_μ and m_μ describe the number of algebraic steps, which characterize the graviton in terms of two connected Spin-1 particles.

Theory

May be V and V' topological spaces of Spin-1 fields, where $n_\mu \in V$ and $n_\nu \in V'$. Consider the topological mapping

$$q : V \times V' \mapsto Q. \tag{6}$$

The morphism (6) with $q(n_\mu, n_\nu) = q_{\mu\nu} \in Q$ represents the mapping of tangent vectors to the angle between tangent vectors. Defining

$$|n_\mu - n_\nu| = q_{\mu\nu} \mod m_{\mu\nu} \tag{7}$$

with the constant quantum period $m_{\mu\nu}$, the function $q_{\mu\nu}$ is the measure of the quantum angle difference between $\vec{g}_\mu$ and $\vec{g}_\nu$. By the introduction of adjoint spaces $Adj(V)$ and $Adj(V')$ that describe an Space of Spin-1 fields isomorphic to V, V', the Spin-1 spaces V and V' are interlinked to a Spin-2 characteristic with the commutative diagram:

$$\begin{array}{ccc} V & \xrightarrow{a_*} & Adj(V) \\ \downarrow{\scriptstyle a_* \circ c} & & \downarrow{\scriptstyle c \circ a} \\ Adj(V') & \xrightarrow{a} & V' \end{array} \tag{8}$$

. The diagram (8) implies the relationship $(c \circ a) \circ a_* = a \circ (a_* \circ c)$ with the space changing $c : V \mapsto V'$, the adjunction a_* and its inverse a. If G is the topological space of Spin-2 fields, it can be written

$$G = V \otimes V' \otimes Q. \tag{9}$$

Adjunction is defined so, that for $q' : Adj(V) \times Adj(V') \mapsto Adj(Q)$ holds the identity:

$$Adj(Q) = Q. \tag{10}$$

Since adjunctions maps a Spin-1 contribution of the graviton to another possible Spin-1 contribution, there are existing multiple ways for adjuncting V or V'. May be n_μ the number of adjunctions. Then, the exact sequence

$$0 \longrightarrow Adj_0(V) \xrightarrow{i_0} Adj_1(V) \xrightarrow{i_1} ... \xrightarrow{i_{n_\mu}} Adj_{n_\mu}(V) \longrightarrow 0 \tag{11}$$

for n_μ adjunction types and interchanging mappings i holds, because adjunctions $i_r : Adj_r(V) \mapsto Adj_{r+1}(V)$ are isomorphic mappings, if all other adjoint spaces $Adj_{r+s}(V)$ with $s \notin \{0, 1\}$ vanish. If the interchanging mappings are nonisomorphic, the spacetime geometry is deformed. May be β_r a projection mapping defined by

$$\beta_r i_{r+1} \beta_r^{-1} = i_r. \tag{12}$$

The definition (12) is a transformation of the group of mappings I_{r+1} with $i_{r+1} \in I_{r+1}$ to the subgroup of mappings I_r with $i_r \in I_r$. For $I_{r+1} \cong I_r$, β_r contains only one element, i.e. $\#\beta_r = 1$. In a deformed spacetime, $\#\beta_r > 1$, because of nonisomorphic interchanging mappings. Since whole spacetime deformations are given by the mapping

$$a_* : V \equiv Adj_0(V) \mapsto Adj_{n_\mu}(V) \equiv Adj(V) \tag{13}$$

the absolute value of a tangent vector in Quantum Gravity can be written as a the complete projection product

$$|\vec{g}_\mu| = \prod_{i=0}^{n_\mu} \#\beta_i := F(n_\mu). \tag{14}$$

The functions $\#\beta_i$ are called *Graviton projection functions* that are similar to Spin network functions in Loop Quantum Gravity [5]. The exact sequence (11) can be extended with the substitution $V \mapsto Q$ and the relations (6) and (7) to the similar exact sequence

$$0 \longrightarrow Adj_0(Q) \xrightarrow{j_0} Adj_1(Q) \xrightarrow{j_1} ... \xrightarrow{j_{q_{\mu\nu}}} Adj_{n_\mu}(Q) \longrightarrow 0 \tag{15}$$

with further adjunction mappings j_r. By setting $\gamma_r j_{r+1} \gamma_r^{-1} = j_r$ with the projection mapping γ_r. In analogy to (14), the cosine between two tangent vectors is given by:

$$cos(\vec{g}_\mu, \vec{g}_\nu) = \prod_{i=0}^{q_{\mu\nu}} \#\gamma_i := R(n_\mu, n_\nu, m_{\mu\nu}). \tag{16}$$

Hence, the metric tensor (1) in Algebraic Graviton Quantizing has the form:

$$g_{\mu\nu} = F(n_\mu) R(n_\mu, n_\nu, m_{\mu\nu}) F(n_\nu). \tag{17}$$

In analogy to (17), the inverse metric tensor can be written as:

$$g^{\mu\nu} = F(n^\mu) R(n^\mu, n^\nu, m^{\mu\nu}) F(n^\nu). \tag{18}$$

The Christoffel symbols in General Relativity are defined as:

$$\Gamma^\lambda_{\mu\nu} = \vec{g}^\lambda \partial_\mu \vec{g}_\nu = |\vec{g}^\lambda| \vec{e}^\lambda \partial_\mu (|\vec{g}_\nu| \vec{e}_\nu). \tag{19}$$

The angle between the unit vectors $\vec{e}^\lambda$ and $\vec{e}_\nu$ and the period m^λ_ν varies with the point in spacetime. By setting $\vec{e}^\lambda \vec{e}_\nu = R(n^\lambda, n_\nu, m^\lambda_\nu)$ and $\vec{e}^\lambda \partial_\mu \vec{e}_\nu = \partial_\mu R(n^\lambda, n_\nu, m^\lambda_\nu)$ it follows:

$$\Gamma^\lambda_{\mu\nu} = F(n^\lambda) \partial_\mu (R(n^\lambda, n_\nu, m^\lambda_\nu) F(n_\nu). \tag{20}$$

Here, the angles between $\vec{e}^\lambda$ and $\partial_\mu \vec{e}_\nu$ can be obtained by setting

$$m^\lambda_\nu = |m_\nu - m^\lambda| \tag{21}$$

with the Spin-1 fields m_ν depending on spacetime. This additional Spin-1 field quantizes the torsion. Since $|n_\nu - n^\lambda| = q_\nu^\lambda \mod |m_\nu - m^\lambda|$, where q_ν^λ is the number of adjunction mappings respective to Q, the exact sequence (15) has more possible adjunction mappings respective to Q in the case of torsion. Hence, by substituting the quantities (17), (18) and (20) into the Einstein-Hilbert action [6]:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{|g|} R_{\mu\nu} g^{\mu\nu}, \tag{22}$$

Feynman's path integral for the Algebraic Graviton Quantizing has the form:

$$Z = \int \int d[n_\mu] d[m_\mu] e^{iS}. \tag{23}$$

Conclusions

It is possible to quantize the graviton with vector fields. The graviton remains a Spin-2 particle, but the position uncertainty in Quantum mechanics allows to distribute a Spin-2 field into two Spin-1 fields by using algebraic connections. Since Spin-1 fields are renormalizable, the UV divergences are avoided in the graviton field description.

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