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Infinity and special relativity

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Abstract.-The formal consistency of special relativity theory is inevitably tied to the formal consistency of the spacetime continuum, a densely ordered infinite set whose formal inconsistency is demonstrated in this article in a very brief and simple way.

1 Introduction: the actual infinity

Contemporary physicists work diligently on various theories to explain different aspects of the physical world, theories invariably constructed with the same infinitist mathematics, in spite of the enormous effort (renormalization process) they have to make every time infinity appears in some of their theories. And it is striking that none of them have considered the possibility that these mathematics may be inconsistent. Even more so when one of the axioms underlying them, the Axiom of Infinity, is anything but self-evident, and its statement has been debated for over twenty-five centuries. It is suspicious that, almost miraculously, these discussions abruptly ceased at the beginning of the 20th century, as if the actual infinity subsumed in the Axiom of Infinity had suddenly become the most obvious thing in the world, and its alternative, the potential infinity, the most absurd concept ever thought. This state has been reached by fervently adopting mathematical creeds instead of questioning them, as all scientists are obliged to do.

Remember, for example, that according to the hypothesis of actual infinity, the set of natural numbers in their natural order of precedence contains all natural numbers. All of them. Even though there is no last natural number to complete that ordering. As is well known, infinite sets (actual infinity) are defined and distinguished from finite sets by the fact that they can be put into one-to-one correspondence with (infinitely) many of their PROPER subsets. A true violation of the old Euclidean *Axiom of the Whole and the Part*, which instead of raising suspicion, provokes fascination. I have investigated the Axiom of Infinity for the last 30 years, and it is set theory itself that has provided me with the most tools to demonstrate its inconsistent nature [5]. The third section of this article contains the briefest of those proofs that I have been able to develop, and that I insistently include in my publications in the hope that someone will consider it (although my hopes are minimal). But before that, in the second section, I will introduce the most famous and ubiquitous infinite set in physics: H. Minkowski's spacetime continuum [6, 7, 8].

2 Space, time, and dense order

In Einstein's early 1905 paper [3], the concept of spacetime does not appear, at least not in the form in which it appears today in almost all physical theories, including the theories of relativity. Although he quickly accepted them, Minkowski's new ideas about space and time were not initially well received by Einstein [9, p. 102]:

> Since the mathematicians have invaded relativity theory, I do not understand it myself any more.

Finally, Einstein himself, and practically all physicists up to the present day have accepted it. According to the relativistic orthodoxy there is a complete symmetry between space and time, so that it is possible to inter-convert them, a very popular inter-conversion that appears almost always in the secondary literature on the subject, for example [4, p. 79]. An inter-conversion that seems questionable to me, at least because:

1.- Space is always different for each object in any inertial reference frame. Time is always the same for all objects in any inertial reference frame.

2.- Rest is possible in space, but impossible in time.

3.- Any direction of space can be traveled naturally in either of its two senses, but the direction of time can only be traveled naturally in one sense, which is always the same: towards the future.

4.- It is possible to come and go from the same point in space any number of times. It is not possible to come and go from the same instant of time.

5.- Space is a real physical object that can vibrate and be the transmitter of its own vibrations traveling at the speed of light. These vibrations are impossible in time because time can only travel in one direction.

6.- Space has well-defined physical properties, such as electrical permittivity ϵ_o, magnetic permeability μ_o or elasticity. Time has no such properties.

7.- The universe has a single temporal direction of evolution, but not a single spatial direction of evolution because it evolves in all directions.

As is well known, the spacetime of physics is modeled by the set $\mathbb{R}^4$ of quaternions (x, y, z, t), where the first three coordinates are spatial, and the fourth is temporal. What some physicists seem to overlook, including certain physics journal editors, is the dense order of this set and its formal consequences: between any two points in any spatial direction, there exists the same infinite number of different points: $2^{\aleph_0}$ different points; and between any two instants in the temporal direction, there also exists the same infinite number of different instants: $2^{\aleph_0}$ different instants. Therefore, the existence of contiguous points or instants is not possible; neither points nor instants touch. However, physicists commonly express themselves as if spatial and temporal contiguity were possible. This dense order of the spacetime of physics allows one to say, as I often provocatively repeat: light traverses the same number of points in space in a millionth of a second as it does in 14.8 billion years; or, put differently: light takes the same number of time instants to travel a millionth of a millimeter as it does to travel 90 billion light-years. This is the spacetime model that physicists never question as the central model of their theories.

The same physicists who talk and write about adjacent points or contiguous instants also speak of infinitesimal points and instants of infinitesimal duration. But neither points are infinitesimal, nor do instants have infinitesimal duration: points have exactly zero extension, and instants have exactly zero duration. And here arises a serious conflict with special relativity: the inertial contraction of space cannot occur by the contraction of its points because the points of space cannot contract; nor can the inertial contraction of space occur by reducing the number of its points because any space interval contains the same number of points before and after it is contracted ($2^{\aleph_0}$ points). Time cannot dilate by dilating its instants either because those dilated instants would cease to be instants; nor can time dilate by increasing the number of instants between two given instants because any interval of time has the same number of instants before and after being dilated ($2^{\aleph_0}$ instants).

Thus, neither the inertial contraction of space nor the inertial dilation of time can be explained in terms of the only basic components of spacetime: its points and its instants. Both relativistic deformations must then be explained through the measurement processes associated with the definitions of arbitrary metrics, but not through changes in the physical reality of space and time modeled by $\mathbb{R}^4$. And then one must ask about the physical usefulness of this mathematical model. Or about the real or apparent nature of these relativistic deformations.

3 Inconsistency of the dense order

In the proof of the following theorem, I will use some well-established results from classical set theory: the denumerable nature of the sets $\mathbb{N}$ of the natural numbers, and $\mathbb{Q}$ of the rational numbers [1], the latter being, moreover, densely ordered (between any two rationals other different rationals always exist). The same applies to the open rational interval $(0, 1)$ [2, §9]. The expression "complete totality" (used in the demonstration) refers to any set of a well-defined type of elements that contains ALL elements of that well-defined type, so that it is impossible to add to that set new elements of that well-defined type because it already contains them all.

Theorem 1 (of Actual Infinity) *The actual infinity subsumed in the set of all natural numbers and in the rational interval* $(0, 1)$ *is inconsistent.*

Proof.-The open interval of rational numbers $(0, 1)$ is densely ordered in the natural order of precedence (in symbol $<$) defined by the natural values of the rational numbers. It is also a denumerable set, so it can be put in one-to-one correspondence f with the set $\mathbb{N}$ of natural numbers in their natural order of precedence. Consequently, $(0, 1)$ can be rewritten as the set $\mathbb{Q}_{01} = \{q_1, q_2, q_3, \dots\}$, where $q_i = f(i), \forall i \in \mathbb{N}$, and the successive elements $q_1, q_2, q_3, \dots$ of $\mathbb{Q}_{01}$ are ordered by their respective subscripts, and not by their natural values as rational numbers. Obviously, these subscripts are the successive natural numbers of the domain $\mathbb{N}$ of the one-to-one correspondence f between $\mathbb{N}$ and $(0, 1)$. Let now x be a rational variable initially defined as q_1; and let (the current value of) x be $<$-compared (i.e. compared according to the natural values of rational numbers) with all the successive elements of the set $\{q_1, q_2, q_3, \dots\}$, so that x is redefined as q_n if, and only if (iff), $q_n < x$, i.e. iff q_n is LESS than the current value of x. Let us denote by $<$-comparison* this $<$-comparison and redefinition of x iff the element $<$-compared is less than the current value of x. Since, according to the Axiom of Infinity, all elements $q_1, q_2, q_3, \dots$ of $\mathbb{Q}_{01}$ are rational numbers that exist as a complete totality, x can be successively $<$-compared* with ALL OF THEM:

$$\forall n \in \mathbb{N} : \ x \text{ is } <\text{-compared* with } q_n \tag{1}$$

It is immediate to prove that for any natural number v it is possible to perform the first v $<$-comparisons* of x with the first successive v elements of $\mathbb{Q}_{01}$. Indeed, if it were not possible to do so, there would exist at least one natural number $n \leq v$ such that x could not be $<$-compared* with q_n, which is impossible because q_n is a rational number in $\mathbb{Q}_{01}$ that can be $<$-compared* with the current value of x, which is also a rational number. Once all possible $<$-comparisons* of x with the successive elements $q_1, q_2, q_3, \dots$ of $\mathbb{Q}_{01}$ have been carried out, the current value of x, whatever it is, will be the smallest rational number in that set. Indeed, if once all possible $<$-comparisons* of x with the successive elements of $\mathbb{Q}_{01}$ have been performed, the current value of x were not the smallest rational number in $\mathbb{Q}_{01}$, there would exist at least one element q_n in $\mathbb{Q}_{01}$ such that $q_n < x$. But this is impossible because n is a natural number, the first n $<$-comparisons* have been performed, and then x was $<$-compared* with q_n and redefined as q_n, and in all subsequent $<$-comparisons*, x could only be redefined with values less than q_n. So, it is impossible that $q_n < x$. But, on the other hand, it is also immediate to prove that once all possible $<$-comparisons* of x with the successive elements of $\mathbb{Q}_{01}$ have been performed, the current value of x is not the smallest rational in that set. In effect, each element of the infinite set $\{x/2, x/3, x/4 \dots\}$ is an element of $\mathbb{Q}_{01}$ less than x. This contradiction proves the assumed actual infinity of the denumerable sets $\mathbb{N}$ and $\mathbb{Q}_{01}$, is inconsistent. □

Corollary 1 *The spacetime continuum* $\mathbb{R}^4$ *is inconsistent.*

Proof.-All sets whose infinitude is the actual infinity, as is the case of the spacetime continuum $\mathbb{R}^4$, are inconsistent sets (Theorem 1). □

4 Special relativity is formally inconsistent

The spacetime continuum plays a major role in the theory of special relativity (and also in general relativity and many other physical theories). One may then ask: Can a theory built on a formally inconsistent concept be formally consistent? The obvious answer should be: no, because such a theory, and by means of such a concept, would allow to prove the two statements of any formal contradiction. Therefore, if the above Theorem 1 and Corollary 1 are correct, the theory of special relativity cannot be formally consistent, even if its predictions can be empirically verified. Predictions that, on the other hand, could be referring to apparent, not real, spacetime deformations. Or they could also be approximations made from the point of view of a continuous physical world that in reality is discontinuous, discrete, with indivisible minimal units of space (qusits) and time (qutits) and a maximum possible velocity of one qusit per qutit. In any case, what seems indisputable is that if the actual infinity is inconsistent, then the theory of special relativity is a formally inconsistent theory, as all the theories built on the basis of contemporary mathematical infinitism. A very serious issue that contemporary physicists insist on not addressing, probably as a defensive attitude in the face of the enormous impact that this inconsistency has on almost all their theories. But sooner or later they will have to face it. I can do nothing but write articles like this in the hope that someone will consider them.

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