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Lorentz Factor in Discrete Physics

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Abstract.-In addition to Lorentz's theory of relativity and Einstein's theory of special relativity, the relativistic Lorentz factor also appears in non-relativistic mechanics as H.E. Ives' theory, in the discrete sum of velocities, and in the conversion between the continuous and discrete versions of Pythagoras' Theorem. It could also be the conversion factor between the current infinitist mechanics based on the (inconsistent) spacetime continuum and a new mechanics based on a finite and discrete space and time, which would explain the experimental success of physical theories based on the inconsistent continuum.

Keywords: Relativistic Lorentz factor, spacetime continuum, inconsistent infinity, discrete sum of velocities, discrete version of Pythagoras' theorem.

1 Introduction: Lorentz factor

The relativistic Lorentz factor, $\gamma = (1-v^2/c^2)^{-1/2}$, needs no presentation. Although it may be necessary to recall that this factor already appears in Lorentz's theory of relativity [8], which was much better known than Einstein's until 1916. Of course, it also appears in Einstein's special theory of relativity [1]. And it also appears in the absolute space-time theory of H.E. Ives [2, 3, 4, 9, 5], the first scientist to prove experimentally the dilation of time (compatible with his own absolutist theory). Although different, especially that of Ives, the three theories have in common that they are infinitist theories based on the continuum of the real numbers. Theories that therefore have the Axiom of Infinity as one of their foundations.

2 Is the continuum consistent?

Scientists often repeat with some pride that their theories must be, and are, falsifiable. The problem is that falsification is not widely practiced; scientists do not always practice what they preach. The case of infinity is a real scandal. It has been more than a century since the concept of infinite set was defined and its hypothetical existence accepted by the Axiom of Infinity. Then came the boom of infinitist mathematics, the same with which physical theories are constructed. And where are the falsifications of this mathematics? Well, in the work of some dissidents who, like the author of this article, have been ostracized, with some added punishment in the form of scorns and insults.

I have devoted many years to the Hypothesis of the Actual Infinity, subsumed in the Axiom of Infinity, and it is ironic that it was infinitist mathematics that provided the first formal tools to prove that this hypothesis, and then this axiom, are inconsistent, something that some have suspected for more than twenty-seven centuries. Here I reproduce a demonstration of this inconsistency that will take less than two minutes to read. Maybe it is worth it[1]. And if it is worth it, here [7] you will find about forty more proofs.

Theorem 1 *The Axiom of Infinity is inconsistent.*

Proof.-The interval of rational numbers $\mathbb{Q}_{01} = (0,1)$ is denumerable and densely ordered. So, it can be put in one-to-one correspondence f with the set $\mathbb{N}$ of natural numbers in their natural order of precedence; and $\mathbb{Q}_{01}$ can be rewritten as the set $\{f(1), f(2), f(3), \dots\}$. Let now x be a rational variable initially defined as $f(1)$; and let (the current value of) x be compared with the successive elements $f(1)$, $f(2)$, $f(3)\dots$ so that x is redefined as $f(i)$ if, and only if, $f(i)$ is LESS THAN the current value of x. Since all elements $f(1), f(2), f(3), \dots$ of $\mathbb{Q}_{01}$ are rational numbers which exist as a COMPLETE TOTALITY, x can be successively compared with ALL of them:

$$\begin{gathered}\forall n \in N: \ x \text{ is compared with } f(n) \\ \text{and redefined as } f(n) \text{ iff } f(n) < x\end{gathered} \tag{1}$$

Once compared with all elements[2] of $\mathbb{Q}_{01}$, the current value of x is the smallest rational of that set. Indeed, if once compared with all elements of $\mathbb{Q}_{01}$, the current value of x were not the least rational of $\mathbb{Q}_{01}$, there would exist at least one element $f(n)$ in $\mathbb{Q}_{01}$ such that $f(n) < x$. But this is impossible according to (1). Therefore, it was compared with $f(n)$ and redefined as $f(n)$. So, it is impossible that $f(n) < x$. But it is also immediate to prove that: Once compared with all elements of $\mathbb{Q}_{01}$, the current value of x is not the smallest rational of that set. In effect, once compared with all elements of $\mathbb{Q}_{01}$, and whatsoever be the current value of x, each element of the infinite set $\{x/2, x/3, x/4 \dots\}$ is an element of $\mathbb{Q}_{01}$ less than x. This contradiction proves the Axiom of Infinity legitimizing the existence of $\mathbb{Q}_{01}$ as an actual (not potential) infinite totality is inconsistent. Or in other words: a COMPLETE and ordered list, such as the rational interval $(0,1)$, without a first element that starts the list is inconsistent. □

This inconsistency makes all infinite sets inconsistent, infinity in the sense of actual (not potential) infinity, which is the only infinity of contemporary infinitist mathematics. This includes the spacetime continuum. Thus, all physical theories based on this continuum, such as Einstein's two theories of relativity, are formally inconsistent, which does not mean that they cannot be operationally efficient. It may be that all infinitist physical theories are operationally adequate, but they cannot explain the world in a fundamental and consistent way if the world is consistent. And it is, as can be deduced from the Principle of Directional Evolution of the Universe (directional in the direction of its increasing entropy) [6, p.

[1] I do not think so, I have included it in my recent works without known consequences.
[2] Though it is not necessary, this is formally proved by induction in [7], and can also be proved by Modus Tollens and by supertask theory.

8-9].

3 Space and time can only be discrete

The inconsistency of the actual infinity has very broad and serious consequences in mathematics and in a large part of most theoretical physics. The rest of the sciences are ignorant of the actual infinity. For example, it can be proved almost immediately that nothing can be divided into an (actual) infinite number of parts, so all physical entities must be discrete: space, time, matter, mass, energy, electric, magnetic, and color charges must be discrete, which is already accepted for all but the first two and some types of energy. So once the inconsistency of the actual infinity is accepted, if that ever happens (I do not think it will because of the enormous amount of academic and authorial interest), space and time will also have to be considered discrete entities.

4 Sum of discrete velocities

In a discrete space-time, nothing can last less than an indivisible unit of time (qbeat), nor move less than an indivisible unit of space (qseat). The ratio between one qseat and one qbeat would then be the maximum natural speed c of this discrete space-time, that could be, or not, the speed of light (here we will assume it is). Being a constant maximum velocity, it could not be altered by adding any other velocity v:

$$c + v = c \tag{2}$$

Therefore, the algebraic addition of velocities in this discrete space-time must be multiplied by a factor Γ in order to obtain the discrete addition of velocities:

$$\Gamma(c + v) = c \tag{3}$$

$$\Gamma = \frac{1}{1 + v/c} \tag{4}$$

To find the algebraic relation α between Γ and the Lorentz factor γ, we write:

$$\Gamma = \alpha\gamma \tag{5}$$

$$\alpha^2 = \frac{\Gamma^2}{\gamma^2} \tag{6}$$

$$= \frac{(1 + v/c)^{-2}}{(1 - (v/c)^2)^{-1}} \tag{7}$$

$$= \frac{1 - (v/c)^2}{(1 + v/c)^2} \tag{8}$$

$$= \frac{(1 + v/c)(1 - v/c)}{(1 + v/c)^2} \tag{9}$$

$$= \frac{1 - v/c}{1 + v/c} \tag{10}$$

$$= \frac{c - v}{c + v} \tag{11}$$

Therefore:

$$\Gamma = \gamma\sqrt{\frac{c - v}{c + v}} \tag{12}$$

The same factor Γ, and for the same reason (3), is obtained for the relativistic sum of the velocities c and v.

5 Pythagoras discrete theorem

(Partially taken from [6])

The classical Pythagorean theorem is fundamental for calculating distances; and distances are also fundamental magnitudes in mechanics. Therefore, it makes sense to analyze the Pythagorean Theorem from the perspective of discrete geometry. A simple piece of graph paper will help the reader to demonstrate that the number of qseats of the hypotenuses of a right triangle is equal to the number of qseats of the larger of its legs.

The factor for converting between continuous and discrete hypotenuses would have the algebraic form of the relativistic factor γ. In fact, let h, x, and y be the number of qseats of the hypotenuse and legs of a right triangle in discrete spacetime, and let λ be the length of a qseat in both the discrete and the continuum geometries. Assume $x < y$. In the discrete geometry we have $h = y$. In the continuum geometry, the length of the hypotenuse would no longer be $h\lambda$, but $h'\lambda$, being $h' > h$, since it is greater than the length $y\lambda$ of the largest leg (note that while h, x, and y are natural numbers, λ and h' are real numbers). According to the classical Pythagorean theorem, it can be written:

$$\text{Hypotenuse: } h'\lambda = \sqrt{(y\lambda)^2 + (x\lambda)^2} \tag{13}$$

$$\text{leg: } y\lambda = \sqrt{(h'\lambda)^2 - (x\lambda)^2} \tag{14}$$

$$y = \sqrt{h'^2 - x^2} \tag{15}$$

The ratio between the continuous and the discrete hypotenuse is given by:

$$\frac{h'\lambda}{h\lambda} = \frac{h'}{h} = \frac{h'}{y} = \frac{h'}{\sqrt{h'^2 - x^2}} = \frac{1}{\sqrt{1 - (x/h')^2}}$$

where the last term on the right side of (5) as the algebraic form of the relativistic Lorentz factor γ. It can ve rewritten as:

$$\frac{h'\lambda}{h\lambda} = \frac{1}{\sqrt{1 - (x\lambda/h'\lambda)^2}} \tag{16}$$

Let a^* be a photon that moves through a vertical distance $y\lambda$ in the rest frame RF_o of its source. Assume a^* moves the same vertical distance $y\lambda$ from the perspective of another inertial frame RF_v while RF_o moves with respect to RF_v the horizontal distance $x\lambda$ at a uniform velocity v parallel to X_v for a time t_v. So, a^* moves with respect to RF_v along the hypotenuse of a right triangle whose legs are $y\lambda$ and $x\lambda = vt_v$, i.e. along $h'\lambda$ (13). And it will hold $h'\lambda = ct_v$. Therefore, (16) can be rewritten:

$$\frac{h'\lambda}{h\lambda} = \frac{1}{\sqrt{1 - (vt_v/ct_v)^2}} = \frac{1}{\sqrt{1 - (v/c)^2}} = \gamma$$

which proves the ratio between the continuous hypotenuse and its corresponding discrete alternative is the relativistic Lorentz factor γ.

6 Lorentz factor as a converting factor

According to what has been recalled and proved in the previous sections, it can be stated that

1. The Lorentz factor has been used in non-relativistic theories, such as the theory of H.E. Ives [2, 3, 4, 9], and in relativistic theories such as Lorentz's [8] and Einstein's [1].
2. The Lorentz factor is involved in the same way in both the discrete addition of velocities and the relativistic addition of velocities.
3. The Lorentz factor converts between the continuous and discrete versions of the Pythagorean Theorem.

It makes sense then to ask whether the Lorentz factor could also be a conversion factor between the current spacetime continuum-based mechanics and a new discrete mechanics based on discrete space and time. If this were the case, and taking into account the inconsistency of the infinitist spacetime continuum, the experimental success of the continuum-based mechanics could be explained because the Lorentz transformation would not only convert between observations in different reference frames, but would also convert between the discrete mechanics based on the discrete space and time, and the continuum-based mechanics.

From the Principle of Directional Evolution of the Universe it can be proved that the laws of physics must be the same in all regions of the Universe and at all times in its history. Something that the naturalists of the 19th century had already established in the form of the Principle of Actualism-Uniformism, with the advantage that it does not need to refer to reference frames, as the Principle of Relativity does. The principle of the constancy of the speed of light would also not have to refer to reference frames, it would just have to assume that the speed of light is a universal constant c, which is also unnecessary, since this speed is defined in terms of two universal constants: $c = (\epsilon_0 \times \mu_0)^{-1/2}$, where ϵ_0 is the electric permittivity and μ_0 is the magnetic permeability.

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