11. Simultaneity 2

Links to the book and to other chapters of the book.

11.1 Introduction

This short chapter contains a single argument that could have been included as a section in the previous chapter. But the argument is so simple and conclusive that I have found it convenient to devote a chapter of its own to it. Indeed, a flagrant contradiction derived directly from the phase difference in relativistic synchronization (lack of simultaneity) is demonstrated here.

The absence of a "*universal now*" is one of the most debated aspects of special relativity. The rotating bar argument developed in this chapter demonstrates in a very simple and conclusive way that, indeed, the absence of a "*universal now*" is inconsistent. Consequently, we could again consider the possibility of universally simultaneous events and simultaneous processes.

11.2 Demonstrations based on thought experiments

If a thought experiment is designed in strict accordance with the fundamental laws of physics, and the laws of logic, we have to admit its results, regardless of whether or not the experiment can be carried out in practice. If the inconsistency of a theory is formally demonstrated by one of these thought experiments, we will have to admit that inconsistency. Indeed, the formal consistency of a theory does not depend on the fact that this or that experiment can be carried out in practice, but depends on its fundamental principles, which are statements assumed without formal proofs, and from which all theoretical results are formally deduced, including those obtained by thought experiments under the conditions just indicated. In the experimental sciences their respective fundamental laws are usually established on the basis of inductive knowledge accumulated from observation and experimentation. In some cases, as in the special theory of relativity, these fundamental laws or principles are arbitrary, without a previous inductive basis (as is the case with a good part of mathematical theories). It is precisely this theory that will see its formal consistency compromised by the rotating bar argument included in this chapter.

11.3 The physical setting of the argument

The setting for this new argument about the relativity of simultaneity is very simple. Figure 11.1 fully illustrates it in its proper reference frame RF_o. In effect, the vertical support VS supports a motorized horizontal bar B that can rotate around its center of mass CM. Initially, the motor is off and B is in equilibrium in a horizontal position, parallel to the axis X_o of its own reference frame RF_o. Two electrons emitters E_1 and E_2 positioned as shown in the figure can fire electrons with adjustable uniform velocity. E_1 fires electrons e_1 to the sensor S_1 placed at the left end of B; and E_2 fires electrons e_2 to the sensor S_2 placed at the right end of B. Each sensor being placed at a distance $L_o - d_o$ from CM.

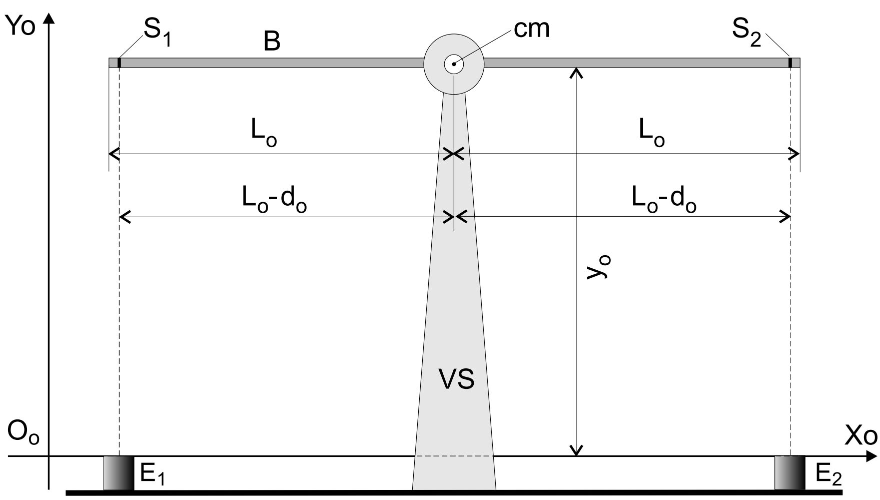

Figure 11.1 – The rotating bar scenario.

Both electrons e_1 and e_2 are always fired in the vertical direction (parallel to Y_o) with the same selected speed. The times at which both electrons are fired can be programmed independently and very precisely on each emitter, and so that in each experiment a unique couple of electrons $(e_1,\ e_2)$ is fired, one electron e_1 fired by the emitter E_1 towards the sensor S_1, and one electron e_2 fired by the emitter E_2 towards the sensor S_2, both electrons emitted at the same instant or at different instants of the RF_o-time.

Once activated by an electron, each sensor sends a signal to the bar's engine. If the engine simultaneously receives the signal from each emitter, it does not start. If it receives the signal from the left sensor S_1 first, it will start a clockwise rotation. If it receives the signal from the right sensor S_2 first, a counterclockwise rotation will be started. If, while already in rotation, the signal from the other sensor reaches the motor, the motor stops. In these conditions, with the two firings separated by any chosen time interval, there will be three possible proper types of shots in RF_o:

a) The electrons e_1 and e_2 are fired simultaneously and, consequently, they simultaneously reach their corresponding sensors S_1 and S_2. In this case B will not rotate.

b) The electrons are not fired simultaneously, although with a small time difference between the two shots, and so that each electron hits its corresponding sensor, but one before the other. In this case the first electron to hit B will cause it to start a spin, but the spin will be stopped by the second electron hitting.

c) The electrons are not fired simultaneously, and with such a time difference that the first electron hitting its sensor causes the bar to start spinning so that by the time the second electron reaches the height of its corresponding sensor, the bar has already rotated an angle sufficient to prevent the second electron from hitting the sensor. In this case B will rotate uniformly for a programmed large interval of time (see Figures 11.2 and 11.3).

11.4 The argument

The human and robotic observers in RF_o prepare the shots of electrons e_1 and e_2 according to the alternative c), so that only the electron e_2 hits the right sensor of the rigid bar B, taking into account all possible factors, including the time it takes for the information to propagate through the rigid bar B. The emitter E_2 to the right of VS fires the electron e_2 a time Δt_o before the emitter E_1 fires the electron e_1 (Figure 11.2). The time Δt_o has been calculated so that when the electron e_1

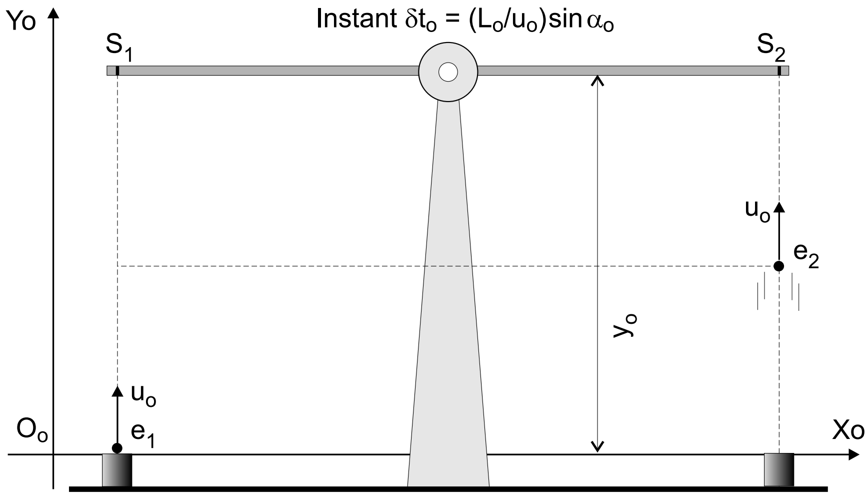

Figure 11.2 – The electron e_2 is fired by a time Δt_o before the electron e_1 is fired.

reaches the height of the left end of the bar B, it has already been hit by the electron e_2 and has rotated an angle α_o such that:

$$L_o \cos\alpha_o = L_o - 2d_o < L_o - d_o \tag{1}$$

and in these conditions e_1 cannot hit its corresponding sensor (Figure 11.3). In consequence, and assuming that both electrons were fired with the same velocity u_o and that it takes them a time t_o to reach the initial coordinate y_o of each sensor, we can write:

$$u_o t_o = y_o \tag{2}$$

$$u_o(t_o - \Delta t_o) = y_o - L_o \sin\alpha_o \tag{3}$$

$$\Delta t_o = \frac{L_o}{u_o}\sin\alpha_o \tag{4}$$

And taking into account (1):

$$\cos\alpha_o = 1 - \frac{2d_o}{L_o} \tag{5}$$

$$\sin\alpha_o = \sqrt{1 - \left(1 - \frac{2d_o}{L_o}\right)^2} \tag{6}$$

$$= \sqrt{\frac{L_o^2 - (L_o - 2d_o)^2}{L_o^2}} \tag{7}$$

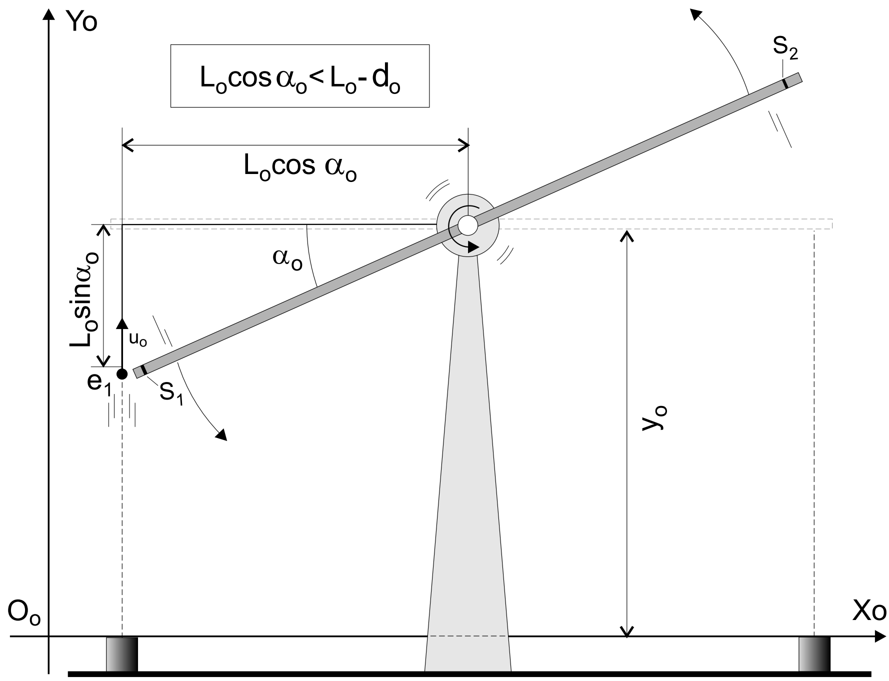

Figure 11.3 – Only the electron e_2 hits its corresponding sensor S_1, as a consequence of which the bar B will begin to rotate for a long period of time powered by the appropriate engine of the bar.

$$= \frac{\sqrt{4L_o d_o - 4d_o^2}}{L_o} \tag{8}$$

$$= \frac{2}{L_o}\sqrt{d_o(L_o - d_o)} \tag{9}$$

So, the interval Δt_o between the firings of e_2 and e_1 can be written:

$$\Delta t_o = \frac{L_o}{u_o}\sin\alpha_o \tag{10}$$

$$= \frac{2}{u_o}\sqrt{d_o(L_o - d_o)} \tag{11}$$

Figure 11.4 shows the variation of Δt_o in terms of the length of the bar (from 10 to 100 m) and the speed u_o of the electrons (from 1000 Km/s to 20000 Km/s). For instance, e_1 should be fired 1.999×10^{-5} seconds after e_2, for a bar of 50 meter and a speed u_o of 50 Km/s.

In RF_o there is, therefore, a lack of synchronism Δt_o between the

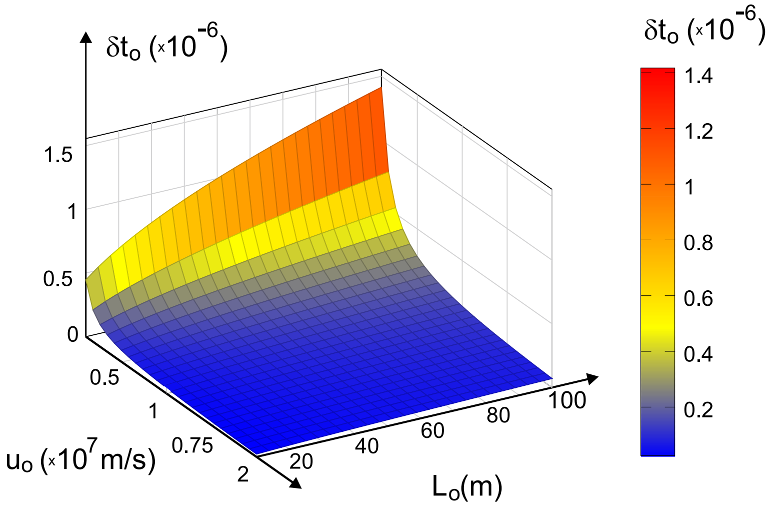

Figure 11.4 – The delay Δt_o in terms of the length (in meters) of the rigid bar L_o and the speed u_o (in m/s) of the electrons.

event: the electron e_2 reaches the coordinate y_o, and the event the electron e_1 reaches the coordinate y_o (corresponding to sensors S_1 and S_2). It is then immediate to ask about the existence of an inertial reference frame RF_v that coincide with RF_o at a certain instant and from whose perspective RF_o moves parallel to the axis X_v of RF_v in such a way that the corresponding phase difference in the synchronization δt_v (in RF_v-time) of two events separated in RF_o by a distance $2(L_o - d_o)$ in the direction of the relative motion is precisely Δt_o given by (11).

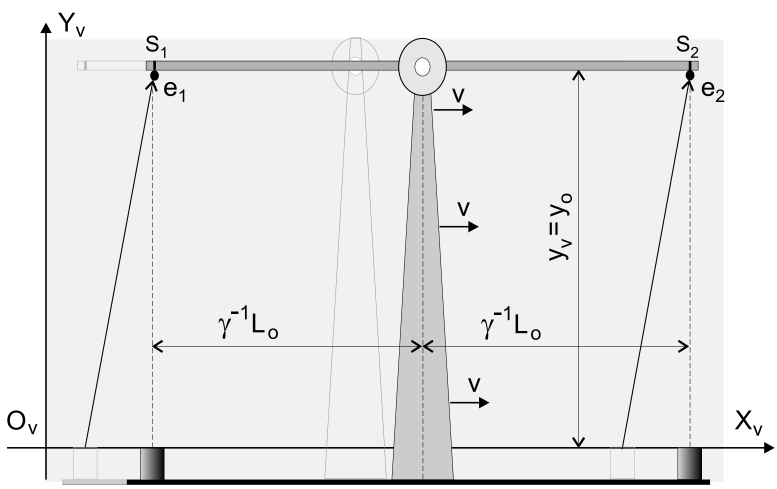

Figure 11.5 – According to the observers of RF_v the two photons simultaneously reach their respective sensors. The bar should not rotate. But they will see it rotates.

According to the Lorentz Transformation, the velocity $v = kc$, $(0 < k < 1)$, of RF_v should be such that:

$$\delta t_v = \Delta t_o \tag{12}$$

$$\frac{\gamma 2(L_o - d_o)k}{c} = \frac{2}{u_o}\sqrt{d_o(L_o - d_o)} \tag{13}$$

Therefore:

$$\frac{2(L_o - d_o)k}{\sqrt{1-k^2}\ c} = \frac{2}{u_o}\sqrt{d_o(L_o - d_o)} \tag{14}$$

$$\frac{k}{\sqrt{1-k^2}} = \frac{c\sqrt{d_o(L_o - d_o)}}{u_o(L_o - d_o)} \tag{15}$$

If we represent with the letter A the right term of the last equation (15), we can write:

$$\frac{k}{\sqrt{1-k^2}} = A \tag{16}$$

$$\frac{k^2}{1-k^2} = A^2 \tag{17}$$

$$k^2 = (1-k^2)A^2 \tag{18}$$

$$k^2(1+A^2) = A^2 \tag{19}$$

$$k^2 = \frac{A^2}{1+A^2} \tag{20}$$

and writing instead of A its definition as the right term of (15):

$$k^2 = \frac{\dfrac{c^2 d_o(L_o - d_o)}{u_o^2(L_o - d_o)^2}}{1 + \dfrac{c^2 d_o(L_o - d_o)}{u_o^2(L_o - d_o)^2}} \tag{21}$$

$$= \frac{\dfrac{c^2 d_o}{u_o^2(L_o - d_o)}}{1 + \dfrac{c^2 d_o}{u_o^2(L_o - d_o)}} \tag{22}$$

$$= \frac{\dfrac{c^2 d_o}{u_o^2(L_o - d_o)}}{\dfrac{u_o^2(L_o - d_o) + c^2 d_o}{u_o^2(L_o - d_o)}} \tag{23}$$

$$= \frac{c^2 d_o}{u_o^2(L_o - d_o) + c^2 d_o} \tag{24}$$

$$= \frac{d_o}{(u_o/c)^2(L_o - d_o) + d_o} \tag{25}$$

So that k is finally given by:

$$k = \sqrt{\frac{d_o}{(u_o/c)^2(L_o - d_o) + d_o}} \tag{26}$$

Figure 11.6 represent the values of the coefficient k of the relative velocity $v = kc$, $(0 < k < 1)$, in terms of te proper velocity u_o of the fired electrons and the proper length L_o of the bar B.

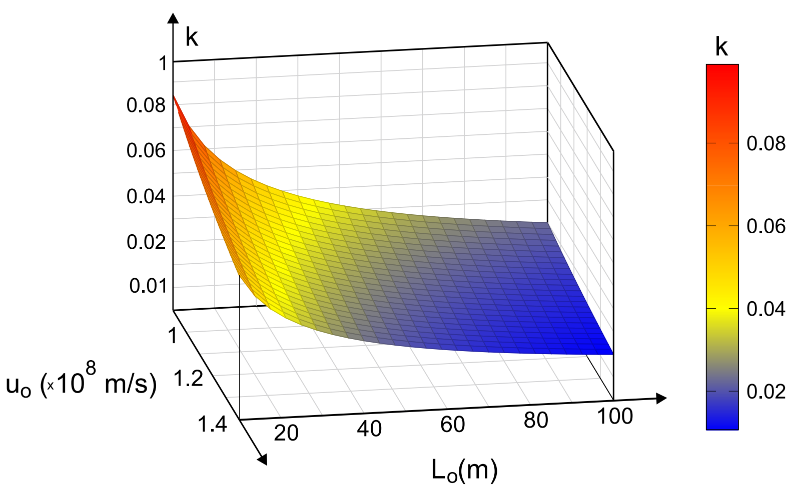

Figure 11.6 – Each point of the surface represents a contradiction deduced from the Lorentz Transformation: The bar B is and is not rotating.

We must, therefore, conclude that each point of the surface $k(u_o, L_o)$ depicted in Figure 11.6 represents a contradiction deduced from the Lorentz Transformation because in each reference frame RF_v from whose perspective RF_o moves in our standard conditions and the factor k of its relative velocity $v = kc$, $(0 < k < 1)$ satisfies (26), the events e_1 reaches the point of vertical coordinate y_v, and e_2 reaches the point of vertical coordinate y_v are simultaneous, while in RF_o are not. So, in each of these reference frames the rigid bar cannot be spinning. But all its observers will observe it is spinning. We have deduced a contradiction from the Lorentz Transformation, which in turns is formally deduced from the two Principles of Relativity. So one of them, or both, must be false. Both principle have in common the reference to reference frames:

> **Principle of Relativity**: The laws of physics have the same form in all inertial reference frames.
>
> **Principle of the Constancy of the Speed of Light**:
>
> *Statement 1*: The speed of light is always the same and does not depend on the speed of the measuring instrument relative to the light source.
>
> *Statement 2*: The speed of light is the same in all inertial reference frames.

Since reference frames are abstracts objects defined as continuums of points and instants, and these continuums are infinitist objects defined on the Hypothesis of the Actual Infinity, maybe the above inconsistency is a consequence of the formal inconsistency of that hypothesis.