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The reciprocal Lorentz Factor (LF): $\beta=1/(1-v^2/c^2)^{1/2}$.

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Abstract.

Einstein's 1905 paper[1] is of singular importance because he here tries to derive the relativistic effect, the reciprocal Lorentz Factor (LF), from the pathlength equations of Newtonian mechanics. This procedure makes the relation between SR error(s) and the LF transparent. Although he investigates rays in different directions[2], the τ equation in the full set is that for rays along the X-axes only (y,y',z,z'=0).
Nevertheless, even for the simple case of the ray in the positive direction of the X-axes, any attempt to refute Einstein's proof of the LF seems to involve one in wearisome algebra. I show here how the "algebra" can be simplified to a surprising extent, thus revealing in a few simple steps how the factor arises, and how the SR "proof" (in the identity transformation) appears to succeed: the twofold transformation applies the LF twice, and then, in consequence of the mistake $vt=v\tau$, cancels it again, so that $x=\beta^2x/\beta^2$.

Einstein's first τ equation, instead of the LF, includes its square:

$$\tau=a\beta^2(t- vx/c^2).$$

As the β^2 is not what he wants, he presents this equation in an over-elaborate form, then eliminates the surplus β by an algebraic substition: ϕ for $a\beta$.

Einstein had obtained this β^2 by the assumption that

$$1/2[x'(c-v)+x'/(c+v)]=x'/(c-v)=\tau.$$

This is the form of the equation $1/2[L/(c-v) + L/(c+v)]=\beta^2L/c$ for the case of the interferometer experiment, not applicable in Einstein's case: his pathlengths are not fixed lengths because, for the ray to the right $x'=(c-v)t$, and to the left $x'=-(c+v)t$.

The question is: why should there be any need for a LF? From the diagrams we have:
For the ray in the positive direction of the X-axes

(In the geometry of moving points it is customary to place an arrow to indicate direction over expressions like OP. I can't find a way to do this: direction is therefore assumed to be, in OP, "from O to P".)

O....O'............P(P' in S')

where $OP=x=ct$, $O'P'=x'=ct'=(c-v)t$.
The SR assumption that $OO'=vt=vt'$ is obviously false: if we use v', then

$$OO':OP'=v't':ct'=vt:(c-v)t, \text{ and } v'=vc/(c-v).$$

We have $x'=(c-v)t=x-vt$ and $x=x'+vt=x'+v't'$.

Simplication of the "algebra":
Since $x=ct$ and $t=x/c$, $x'=ct'$ and $t'=x'/c$, we can rewrite all expressions such that they contain

either x, x' or t, t' only:

for instance,

$$x'=(c-v)t=x(1-v/c) \text{ or } t'=t(1-v/c),$$
$$x=x'+v't'=x'(1+v'/c) \text{ or } t=t'(1+v'/c).$$

I opt for using x,x' only. First, this keeps the geometry at the front of our minds. Secondly, there is evidence that imagery, unlike the algebra-like symbol t, stimulates the mind (creativity, imagination, model building).

There is here no need for a LF: if x'=x-vt,

$$x=x'+vt=x'+v't'=x'(1+v'/c)=x(1-v/c)[(1+vc/c(c-v)]=x(1-v/c)/(1-v/c)=x.$$

If, on the other hand, we assume that x=x'+vt', we find

$$x=x'[(1+v/c]= x(1-v/c)(1+v/c)=x(1-v^2/c^2)=x/\beta^2.$$

SR proceeds to remedy this (see below) by assuming that x'=β(x-vt) as well as x=β(x'+vt'), because only then $x=\beta^2x/\beta^2=x$.

What happens in the case of the ray returning from P,P' to "the origin"? As the time t is the same as for the ray to the right, we have x=-ct: "the origin" is O. Here

O........O'........P...P'

During the time t O' and P' have moved through vt to the right. We are interested in ratios, by comparing absolute values we can avoid the acrobatics of negative signs. And so

|OP|=|PO|=|x|=|ct|,
|OP'|=|P'O|=|x'|=|ct'|=|x|+|vt|=|(c+v)t|,
and with the negative signs x'=-ct'=-(c+v)t.

Here |vt|=|PP'|=|'PP|=|v't'|. We have |PP'|:|OP'|=v't':ct'=vt:(c+v)t, and v'=vc/(c+v).

Since |x'|=|x|+|vt|=|x|+|v't'|, where |t'|=|x'|/c, we have

$$|x|=|x'|-|v't'|=|x'|(1-v'/c)=|x|(1+v/c)(1-v'/c)=|x|(1+v/c)[1-vc/(c+v)]=|x|(1+v/c)/(1+v/c)=x.$$

If we falsely assume that PP'=vt', we find, that

$$|x|=|x'|(1-v/c)=|x|(1+v/c)(1-v/c)=|x|(1-v^2/c^2)=|x|/\beta^2,$$

again with the SR remedy of assuming that x'=β(x-vt) and x=β(x'+vt'), for then $x=\beta^2x/\beta^2=x$.

This is what happens in the identity transformation before Einstein's final set of the SR equations. For this purpose he introduces a third system of coordinates S'(x',y',z',t'), defined as moving relatively to the second system with the velocity v in the negative direction of the X-axes, its origin coinciding with the origins of the two other systems at the time t',t,τ=0, with the false assumption that vt=vτ. This third system is found to be at rest with the first, and x'=x. The maths is as in the cases above where we had seen the effect of that error: Einstein's $x'=\beta^2x/\beta^2=x$.

Conclusion.
Not all critics of SR are interested in such mathematical technicalities. In view of so many obvious errors in Einstein's paper, many will see no reason whatsoever to dig further. Those who try, however, will find the "algebra" excessive if not intimidating. I write in the hope that my simplifying tricks might be useful for anybody who, now or in future, is prepared to look more closely at Einstein's mathematical proof that lengths in relative motion are reciprocally "contracted" in the measure of the Lorentz Factor.

Reference:

1. "On the electrodynamics of moving bodies" [Annalen der Physik 17 (1905):891-921].
2. He finds that the t for rays along the y'- and z'-axes, where x'=0, differs from the t for x=+/-ct; the complete set of equation has only the latter (y,y',z,z'=0).