13. Relativistic reflection of light

Links to the book and to other chapters of the book.

13.1 Introduction

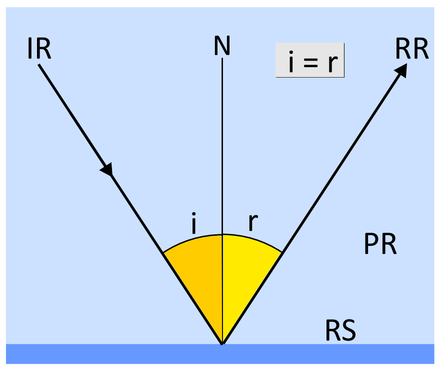

Figure 13.1 – The laws of the reflection of light. IR: incident ray; N: normal; RR: reflected ray; i: angle of incidence; r. angle of reflection;: RS reflecting surface; PR: Plane of reflection.

In spite of the fact that the reflection of light on a plain mirror appears in many relativistic discussions, the relativity of the reflection itself has not been, in my opinion, adequately addressed. In this chapter we will examine the violation by the Lorentz Transformation of the Second Law of the Reflection of Light. In fact we will see that (if being equal is not the same as being unequal) the Second Law of the reflection of light, which is satisfied in the proper reference frame of the medium on which light reflects, is not satisfied when te reflection is observed in relative motion and transformed according to the Lorentz Transformation.

Recall the First Law of the reflection of light states the incident ray, the normal (the perpendicular to the reflecting surface through the incident point on the surface) and the refracted ray lie in the same plane. And the Second Law of the reflection of light states the angle of incidence (the angle between the incident ray and the normal) is equal the angle of reflection (the angle between the normal and the reflected ray) (Figure 13.1)

13.2 A conflicting reflection

We will analyze the reflection of light on a mirror that, as Figure 13.2 shows, is attached to the inclined surface (hypotenuse AC) of a right angled structure ABC whose orthogonal sides have the same length L_o, being one of them parallel to the X_o axis and the other to the Y_o axis of its proper reference frame RF_o. In consequence, M and any perpendicular to M (as the normal N_o) are inclined at an angle of $\pi/4$ radians with respect to the Y_o axis, though in opposite senses.

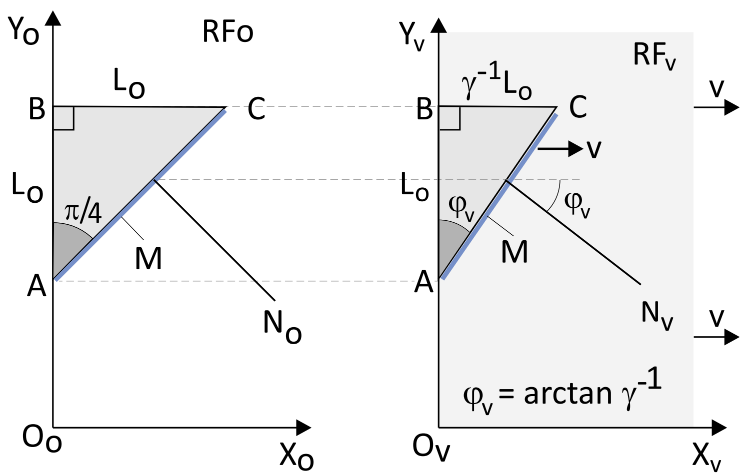

Figure 13.2 – Left: The mirror M and its normal N_o in RF_o. Right: The same mirror and its normal N_v as seen in RF_v.

RF_v is another reference frame that coincides with RF_o at a certain instant and from whose perspective RF_o moves according to our standard conditions at a uniform velocity $v = kc$, $0 < k < 1$ in the direction of the increasing x_v. Since all horizontal lines in the direction of the relative motion are contracted by the same factor $\gamma^{-1} = (1-k^2)^{1/2}$ and no contraction occurs in the direction orthogonal to the relative motion, the structure ABC in RF_v continues to be right angled, thought not isosceles, and its side AC (and then the mirror) is now inclined by an angle of $\arctan\gamma^{-1}$ relative to Y_v (Figure 13.2, right):

$$\tan\varphi_v = \frac{\gamma^{-1}L_o}{L_o} \tag{1}$$

$$\varphi_v = \arctan\gamma^{-1} \tag{2}$$

In these conditions, we will examine the reflection of a photon a^* fired vertically towards the mirror M in RF_o. The argument on the photon could also be applied to a rigid ball of mass $m > 0$ moving with a vertical uniform velocity towards a rigid surface inclined by an angle of $\pi/4$ radians to the vertical and on which the ball bounces off elastically.

Let us begin by analyzing the reflection of the photon a^* emitted at point P in the vertical direction towards the mirror M in the proper reference frame RF_o of the mirror. Let Q be the point where a^* hits the mirror and assume it is at a vertical distance y_o from the X_o axis. At point T, placed at the same vertical distance y_o from X_o and in the same plain of the photon incident-reflected trajectory, it is placed a target that fires a visible green flash when it is activated by a photon. (Figure 13.3, left).

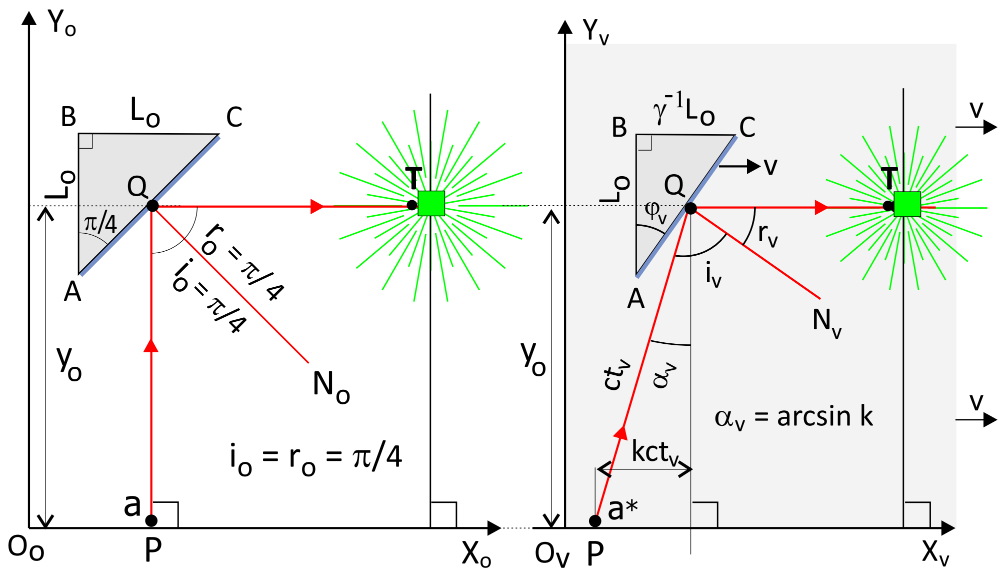

Figure 13.3 – Left: Reflection of the photon a^* by the mirror M as seen in the proper frame RF_o of the mirror M. Right: The same reflection as seen in RF_v.

From the perspective of RF_o the photon a^* is reflected according to the laws of the reflection of light so that the angle of incidence is equal to the angle of reflection. Since the incident trajectory is vertical and the normal N_o is tilted at an angle $\pi/4$ relative to the vertical direction, the angle of incidence and the angle of reflection will be both of $\pi/4$ radians. Therefore, the reflected trajectory QT will be horizontal. Consequently, the photon reaches its target and the green flash is fired. The green flash will be seen in all references frames.

Assume that RF_o moves parallel to X_v from left to right at a uniform velocity $v = kc$ $(0 < k < 1)$ with respect to the frame RF_v, and in such a way that its spacetime diagram coincides with that of RF_v at the precise instant at which the photon a^* is emitted by its source. In accordance with the Lorentz Transformation, the observers in RF_v will come to the following conclusions regarding the reflection of the photon a^* on the mirror M (Figure 13.3, right):

1. According to the Lorentz Transformation, the points Q (at which a^* hits M) and T (at which a^* ends its reflected trajectory) are at the same vertical distance ($y_v = y_o$) from the X_v axis.
2. Therefore, the reflected trajectory QT of the photon a^* will be horizontal.
3. The photon a^* takes a time t_v in going from P to Q.
4. During t_v the mirror M and its point Q moves a horizontal distance kct_v towards the right.
5. Therefore, the incident trajectory PQ of a^* will not be vertical but inclined at an angle:

$$\alpha_v = \arcsin\left(\frac{kct_v}{ct_v}\right) = \arcsin k \tag{3}$$

6. The horizontal side of the structure to which the mirror is fixed is contracted by a factor $\gamma^{-1} = (1-k^2)^{1/2}$. Therefore, the length of this side will be $(1-k^2)^{1/2}L_o$.
7. The vertical side of the right angled structure maintains its proper length L_o.
8. Therefore, the mirror is not inclined with respect to the vertical at an angle $\pi/4$ but at an angle:

$$\varphi_v = \arctan\left(\frac{\gamma^{-1}L_o}{L_o}\right) \tag{4}$$

$$= \arctan \gamma^{-1} \tag{5}$$

$$= \arctan\sqrt{1-k^2} \tag{6}$$

9. Consequently, the normal N_v to the mirror is not inclined at an angle $\pi/4$ relative to the vertical, as is the case of the normal N_o observed in RF_o.

As seen from RF_v, the reflected trajectory of the photon a^* is horizontal as in RF_o, but its incident trajectory is not vertical. Thanks to the relative velocity kc of RF_o, the incident trajectory is inclined at an angle $\alpha_v = \arcsin k$ relative to the vertical (parallel to the axis Y_v). Now then, since the normal N_v is not inclined at an angle $\pi/4$ relative to the vertical as it is in RF_o, it could be the case that both differences compensate each other so that the second law of reflection is also satisfied in RF_v, and $i_v = r_v$. As we will see next, elementary geometry suffices to prove that is not the case.

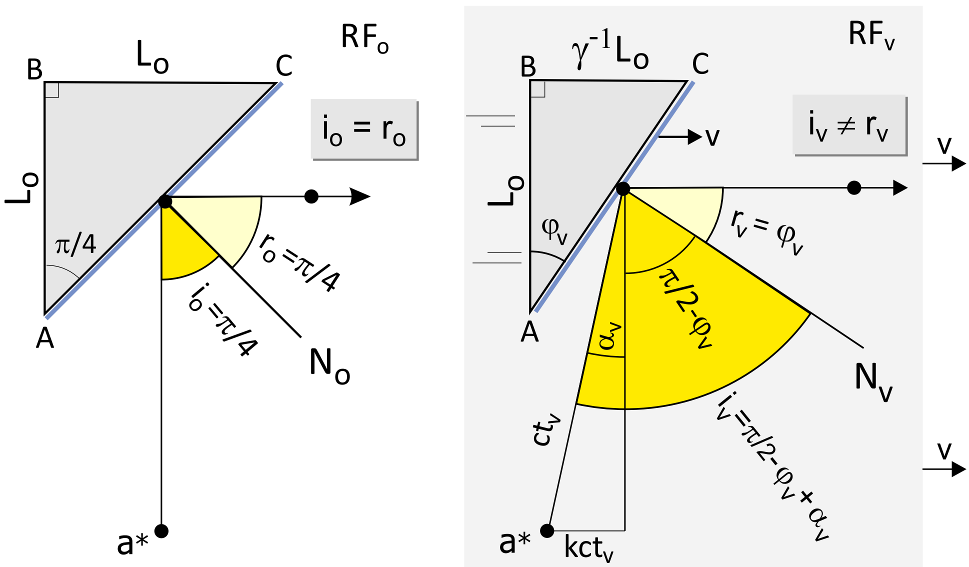

Figure 13.4 – Geometry of the reflection of the photon a^* by the mirror M in RF_o (left) and in RF_v (right).

It is immediate to prove the angle of incidence i_v is always greater than the angle of reflection r_v and that that difference increases as the relative velocity kc increases. In fact, basic geometry allows us to write (see Figure 13.4, right):

1. In the place of $\pi/4$, the mirror is now inclined at an angle φ_v given by:

$$\varphi_v = \arctan \gamma^{-1} \tag{7}$$

$$= \arctan \sqrt{1-k^2} \tag{8}$$

2. The angle α_v between the vertical and the incident trajectory in

RF_v is:

$$\alpha_v = \arcsin\left(\frac{kct_v}{ct_v}\right) \tag{9}$$

$$= \arcsin k \tag{10}$$

3. The angle of incidence i_v is:

$$i_v = \alpha_v + \frac{\pi}{2} - \varphi_v \tag{11}$$

$$= \frac{\pi}{2} + \alpha_v - \varphi_v \tag{12}$$

$$= \frac{\pi}{2} + \arcsin k - \arctan\sqrt{1-k^2} \tag{13}$$

4. The angle of reflection r_v is:

$$r_v = \arctan\sqrt{1-k^2} \tag{14}$$

Therefore, in this particular case, the Second Law of the Reflection of Light in the reference frame RF_v becomes:

$$i_v = \frac{\pi}{2} + \arcsin k - r_v \tag{15}$$

In accordance with (8)-(14), we can conclude that as the relative ve-

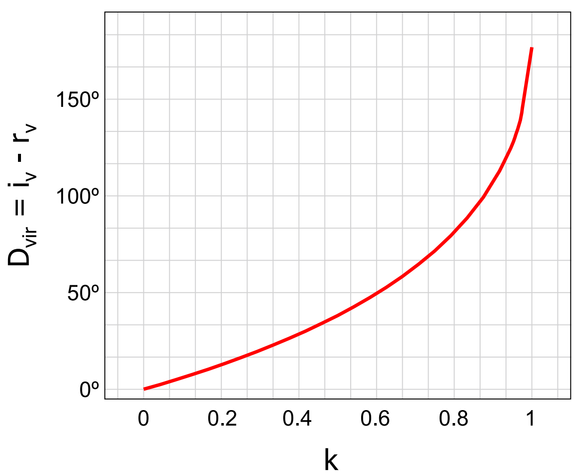

Figure 13.5 – In RF_v the difference $i_v - r_v$ between the angle of incidence and the angle of reflection increases as k increases within the interval $(0, 1)$.

locity $v = kc$ increases:

1. k increases within the real interval $(0, 1)$.
2. $\sqrt{1-k^2}$ and $\arctan\sqrt{1-k^2}$ decrease.
3. $\alpha_v = \arcsin k$ increases.
4. $\pi/2 - \arctan\sqrt{1-k^2} = \pi/2 - \varphi_v$ increases.

Therefore, as k increases

1. The angle of incidence $i_v = \dfrac{\pi}{2} + \alpha_v - \varphi_v$ increases.
2. The angle of reflection $r_v = \arctan\sqrt{1-k^2}$ decreases.

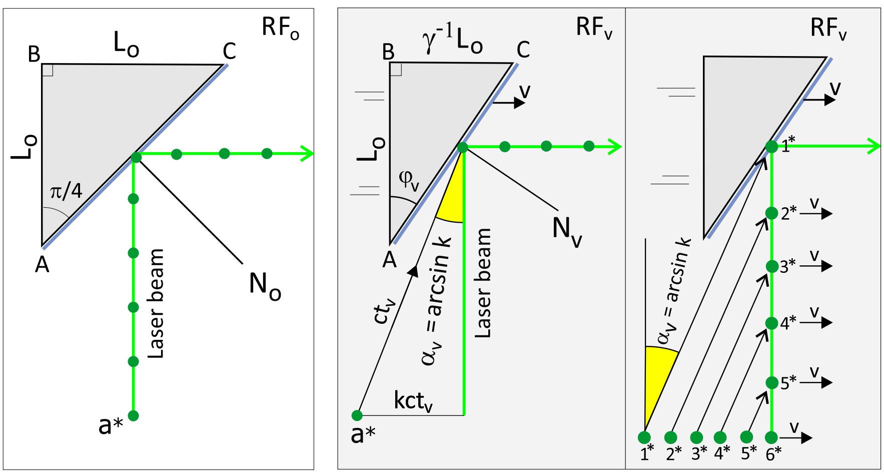

Figure 13.6 – Reflection of a laser beam in the proper reference frame RF_o of its source (left), and in the frame RF_v, from whose perspective RF_o moves from left to right at a uniform velocity v (center and right).

In consequence, from the perspective of RF_v the difference between the angle of incidence and the angle of reflection increases with relative velocity. Indeed, that difference can be expressed as a function of k:

$$D_{vir}(k) = i_v - r_v \tag{16}$$

$$= \frac{\pi}{2} + \arcsin k - 2\arctan\sqrt{1-k^2} \tag{17}$$

which is a strictly increasing function within the domain $(0, 1)$ of k (Figure 13.5).

In the place of the reflection of a photon, consider the reflection of a visible laser beam illustrated in Figure 13.6. The angles of incidence and reflection are equal only in RF_o. i.e. when the reflection is observed in the proper reference frame of the reflecting mirror.

In RF_o all photons follow the same vertical trajectory coincident with the visible vertical laser beam. In RF_v the laser beam is also seen as a vertical (moving) line (this will be formally proved in Chapter 31), but each of its photon follows a trajectory inclined by an angle $\arcsin k$ with respect to the visible vertical laser beam. This is a notable difference between RF_o and RF_v: while in RF_o all photons follow the same vertical trajectory coincident with the visible laser beam, in RF_v the trajectory of each photon is not vertical but inclined by an angle $\arcsin k$, so it does not coincide with the observed vertical laser beam.

13.3 General case

We will now extend the above discussion to the general case in which the incident trajectory of the photon makes any angle θ_o to the vertical. Assume the photon a^* follows a trajectory PQ that in RF_o makes an

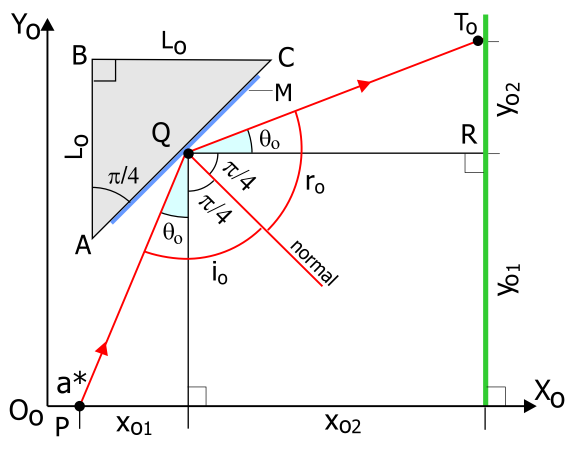

Figure 13.7 – The reflection of the photon a^*, that now follows an inclined trajectory, from the point of view of RF_o.

angle θ_o to the vertical, so that it hits the mirror at point Q, which is at a vertical distance y_{o1} from the X_0 axis. The photon is then reflected and, since the normal N_o is inclined by an angle of $\pi/4$ radians to the vertical, the reflected trajectory QT will make an angle θ_o to the horizontal QR. Finally, a^* reaches a target T placed at a vertical distance y_{o2} to the horizontal QR (Figure 13.7). In accordance with the Second Law of the reflection of light, and being i_o and r_o the angle of incidence and the angle of reflection respectively, we will have:

$$i_v = r_v = \pi/4 + \theta_o \tag{18}$$

RF_v is another inertial reference frame whose spacetime diagram coincides with that of RF_o at a certain instant, and from whose perspective RF_o moves parallel to X_v, from left to right and with a uniform velocity $v = kc$, $(0 < k < 1)$. From RF_v and according to the Lorentz Transformation, ABC continues to be a right angled triangle, though its horizontal side BC is now contracted by a factor γ^{-1}. Thus the mirror M is inclined at an angle $\varphi_v = \arctan\gamma^{-1}$ to the vertical (Figure 13.8).

According again to the Lorentz Transformation, a^* hits the mirror at the same vertical distance $y_{v1} = y_{o1}$ from X_v as in RF_o, and reaches the target T at the same vertical distance $y_{v2} = y_{o2}$ from the horizontal QR. In RF_v the angle of incidence i_v and the angle of reflection r_v are given by (see Figure 13.8):

$$i_v = \theta_{v1} + \pi/2 - \beta_v \tag{19}$$

$$r_v = \theta_{v2} + \beta_v \tag{20}$$

And then:

$$i_v - r_v = \theta_{v1} + \pi/2 - \theta_{v2} - 2\beta_v \tag{21}$$

We will now express the angles θ_{v1}, θ_{v2} and β_v in terms of θ_o and k. In

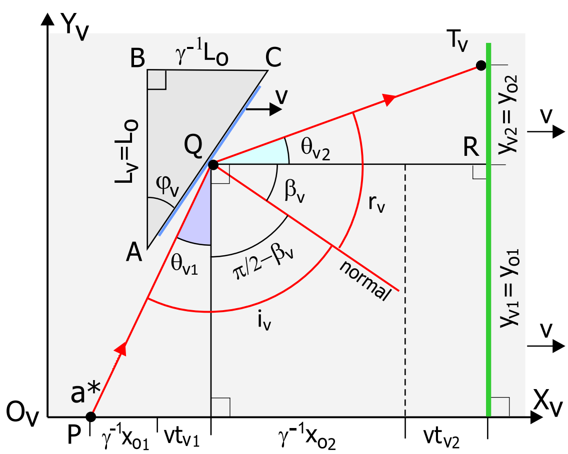

Figure 13.8 – The reflection of the photon a^* from the perspective of RF_v.

the first place, and taking into account that β_v and φ_v are both acute angles and that the sides of the one are perpendicular to the sides of the other, we will have:

$$\beta_v = \varphi_v = \arctan \gamma^{-1} \tag{22}$$

$$= \arctan \sqrt{1 - k^2} \tag{23}$$

Assume now that a^* lasts an RF_v-time $t_{v1} = \gamma(t_{o1} + x_{o1}k/c)$ to go from its current position P to the point Q on the mirror M. During that time, M moves a horizontal distance kct_{v1}. So, and taking into account that:

$$x_{o1} = ct_{o1} \sin \theta_o \tag{24}$$

$$x_{o1}/t_{o1} = c \sin \theta_o \tag{25}$$

we can write:

$$\tan \theta_{v1} = \frac{\gamma^{-1} x_{o1} + kct_{v1}}{y_{o1}} \tag{26}$$

$$= \frac{\gamma^{-1} x_{o1} + kc\gamma(t_{o1} + x_{o1}k/c)}{y_{o1}} \tag{27}$$

$$= \frac{\gamma^{-1} + kc\gamma(t_{o1}/x_{o1} + k/c)}{y_{o1}/x_{o1}} \tag{28}$$

$$= \frac{\gamma^{-1} + kc\gamma(1/(c \sin \theta_o) + k/c)}{\cot \theta_o} \tag{29}$$

$$= \frac{\gamma^{-1} + k\gamma(\csc \theta_o + k)}{\cot \theta_o} \tag{30}$$

$$= \frac{\gamma^{-2} + k(\csc \theta_o + k)}{\gamma^{-1} \cot \theta_o} \tag{31}$$

$$= \frac{1 - k^2 + k \csc \theta_o + k^2}{\sqrt{1 - k^2} \cot \theta_o} \tag{32}$$

$$= \frac{1 + k \csc \theta_o}{\sqrt{1 - k^2} \cot \theta_o} \tag{33}$$

$$\theta_{v1} = \arctan\left(\frac{1 + k\csc\theta_o}{\sqrt{1-k^2}\cot\theta_o}\right) \tag{34}$$

After its reflection, a^* lasts an RF_v-time $t_{v2} = \gamma(t_{o2} + x_{o2}k/c)$ to reach its final target T. During that time, T moves a horizontal distance kct_{v2}. So, and taking into account that:

$$x_{o2} = ct_{o2}\cos\theta_o \tag{35}$$

$$x_{o2}/t_{o2} = c\cos\theta_o \tag{36}$$

we can also write:

$$\tan\theta_{v2} = \frac{y_{o2}}{\gamma^{-1}x_{o2} + kct_{v2}} \tag{37}$$

$$= \frac{y_{o2}}{\gamma^{-1}x_{o2} + kc\gamma(t_{o2} + x_{o2}k/c)} \tag{38}$$

$$= \frac{y_{o2}/x_{o2}}{\gamma^{-1} + \gamma kc(t_{o2}/x_{o2} + k/c)} \tag{39}$$

$$= \frac{\tan\theta_o}{\gamma^{-1} + \gamma kc(1/(c\cos\theta_o) + k/c)} \tag{40}$$

$$= \frac{\tan\theta_o}{\gamma^{-1} + \gamma k(\sec\theta_o + k)} \tag{41}$$

$$= \frac{\gamma^{-1}\tan\theta_o}{\gamma^{-2} + k(\sec\theta_o + k)} \tag{42}$$

$$= \frac{\sqrt{(1-k^2)}\tan\theta_o}{1 - k^2 + k\sec\theta_o + k^2} \tag{43}$$

$$= \frac{\sqrt{1-k^2}\tan\theta_o}{1 + k\sec\theta_o} \tag{44}$$

$$\theta_{v2} = \arctan\left(\frac{\sqrt{1-k^2}\tan\theta_o}{1 + k\sec\theta_o}\right) \tag{45}$$

Equation (21) can now be rewritten as:

$$
\begin{aligned}
D_{vir}(\theta,k) &= i_v - r_v \\
&= \pi/2 + \arctan\left(\frac{1+k\csc\theta_o}{\sqrt{1-k^2}\cot\theta_o}\right) \\
&- \arctan\left(\frac{\sqrt{1-k^2}\tan\theta_o}{1+k\sec\theta_o}\right) \\
&- 2\arctan\sqrt{1-k^2}
\end{aligned}
\tag{46}
$$

Therefore, for $k=0$ we have:

$$
\begin{aligned}
D_{vir}(\theta,k) &= i_v - r_v \\
&= \pi/2 + \arctan\left(\frac{1}{\cot\theta_o}\right) \\
&- \arctan\left(\frac{\tan\theta_o}{1}\right) \\
&- 2\arctan 1 \\
&= 0
\end{aligned}
\tag{47}
$$

So that, as expected, $i_o - r_o$ is always equal to 0, independently of the angle θ_o. Therefore, (46) could be considered another expression,

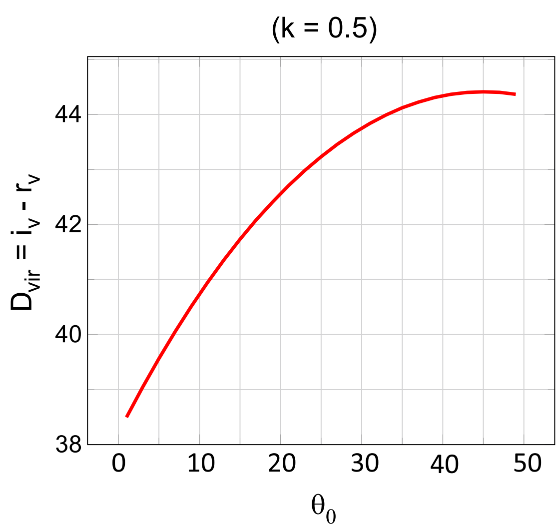

Figure 13.9 – D_{vir} for a relative velocity $k = 0.5c$.

though much more complex, of the Second Law of the Reflection of

Light. But, in any case, there is a significant difference between RF_o and RF_v: while in RF_o the difference $i_o - r_o$ does not depend on the angle of incidence, in RF_v it does, as Figure 13.9 illustrates for the case $k = 0.5$. Notice that, for this case, D_{vir} ranges from more than 38º to more than 44º. As Figure 13.10 (left) shows, each point of the surface:

$$z = D_{vir}(\theta_o, k) \tag{48}$$

represents a violation of the Second Law of the reflection of light because each of those points represents a difference between the angle of incidence i_v and the angle of reflection r_v greater than zero, a difference that increases with relative velocity (factor k). It is worth noting that for very small θ_o (incident trajectory very close to the vertical), the surface $D_{vir}(\theta_o, k)$ increases very faster (Figure 13.10, right).

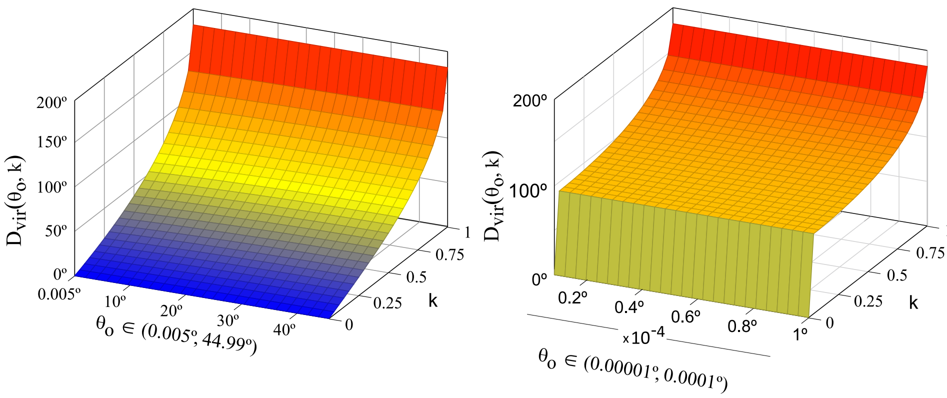

Figure 13.10 – Left: The surface $D_{vir}(\theta_o, k)$. Right: Surface $D_{vir}(\theta_o, k)$ for very small θ_o.

13.4 Conclusions

According to the above discussion, FitzGerald-Lorentz contraction can only be apparent. Otherwise it would be incompatible with the laws of optics, or the laws of the elastic collisions if in the place of the reflection of a photon we would have considered the case of the massive ball suggested above.

By symmetry, time dilation and phase difference in clocks synchronization with relative motion would also be apparent, at least until it be declared which relativist deformations are real, and which apparent. Simply because of all of them are direct consequences of the same Lorentz Transformation. The alternative would be that some physical laws that are satisfied in the proper frame of the events driven by those laws, are not satisfied when the same events are observed in relative motion.

P3 Thus, LT does not transform properly a *physical legality* into another physical legality. Or to put it another way, the physical laws are not always preserved when the events governed by those laws are observed in relative motion and the consequences on space and time of LT are interpreted as real physical phenomena.

The above conclusion P3 means that interpreting the consequences on space and time of LT as real physical phenomena implies a break of Lorentz symmetry. This could be a proof that the relativistic distortions of space and time are rather apparent, and that not always these distortions can be used to draw conclusions about what happens in the reference frame in which those distortions seem to take place.