9. Simultaneity 1

9.1 Introduction

One of the best known and celebrated results of the theory of special relativity is the relativity of simultaneity: two events that are simultaneous in an inertial reference frame may not be simultaneous in another inertial reference frame that moves relative to the first one. They are not simultaneous if the events occur at points separated by a distance greater than zero in the direction of the relative motion. In those conditions, the time difference between two unsynchronized clocks will be called *phase difference in synchronization*.

As we will see in this chapter, an immediate consequence of the apparent nature of the relativistic length contraction and time dilation discussed in the previous chapters is that the relativistic phase difference in synchronization (lack of simultaneity) must also be apparent. This chapter proves that is the case. It also proves that thanks to the relativistic phase difference in synchronization optically isotropic materials can never be observed in relative motion, while an unknown bipolar anisotropy will always be observed. These facts point in the same direction of the apparent nature of the relativistic phase difference in synchronization, otherwise the Principle of Relativity would be violated.

This chapter has a second part in Chapters 10, 29 and 37 (in my opinion the most important and conclusive of the book) in which several conclusions related to phase differences in synchronization are demonstrated. Those conclusions are devastating for the Lorentz Transformation as an operator for converting between measurements performed in different inertial reference frames, and consequently for the realistic interpretation of special relativity.

9.2 Inertial phase difference in synchronization

Let E_1 and E_2 be two simultaneous events, in their proper reference frame RF_o, whose spacetime coordinates in that frame are respectively (x_{o1}, t_{o1}) and (x_{o2}, t_{o2}) (Figure 9.1). Obviously we will have $t_{o1} = t_{o2}$. Let (x_{v1}, t_{v1}) and (x_{v2}, t_{v2}) be the spacetime coordinates of E_1 and E_2 in the frame RF_v from which RF_o moves with a constant velocity $0 < v < c$ in the direction from x_{o1} to x_{o2}. According to the

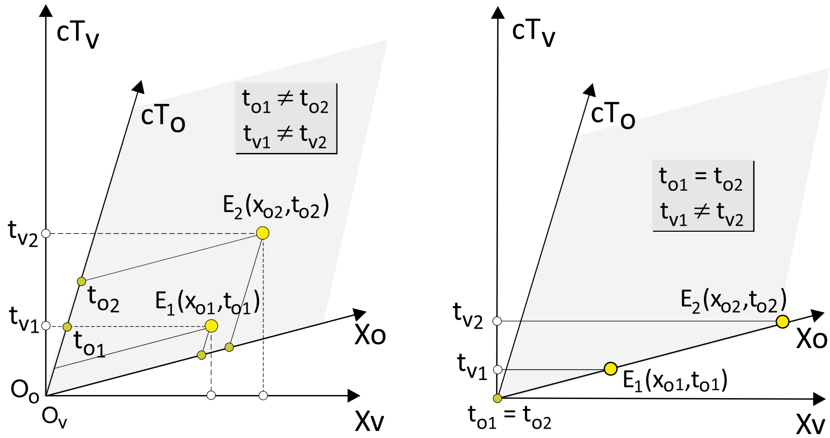

Figure 9.1 – Left: The events E_1 and E_2 are not simultaneous neither in RF_o nor in RF_v. Right: The events E_1 and E_2 are simultaneous in their proper frame RF_o but not in RF_v.

Lorentz Transformation we can write:

$$t_{v1} = \left(t_{o1} + \frac{vx_{o1}}{c^2}\right)\gamma \tag{1}$$

$$t_{v2} = \left(t_{o2} + \frac{vx_{o2}}{c^2}\right)\gamma \tag{2}$$

By subtracting the first equation from the second one and taking into account that $t_{o1} = t_{o2}$ we obtain:

$$t_{v2} - t_{v1} = \left(t_{o2} + \frac{vx_{o2}}{c^2}\right)\gamma - \left(t_{o1} + \frac{vx_{o1}}{c^2}\right)\gamma \tag{3}$$

$$= \left(t_{o2} - t_{o1} + \frac{vx_{o2}}{c^2} - \frac{vx_{o1}}{c^2}\right)\gamma \tag{4}$$

$$= \frac{(x_{o2} - x_{o1})v\gamma}{c^2} \tag{5}$$

$$= \frac{\gamma L_o v}{c^2} \tag{6}$$

where $L_o = x_{o2} - x_{o1}$ is the proper length of the segment $x_{o1}x_{o2}$. As noted above, the time $\gamma L_o v/c^2$ corresponds to the phase difference in synchronization of RF_o clocks as seen from RF_v and expressed in terms of RF_v time. In short, if E_1 and E_2 are two simultaneous events in RF_o separated in this frame by a distance L_o, in other frame RF_v from which RF_o moves in the direction from E_1 to E_2 with a velocity v, the event E_1 happens before the event E_2, being the interval of time elapsed between E_1 and E_2 equal to $\gamma L_o v/c^2$ in terms of RF_v time. An usual

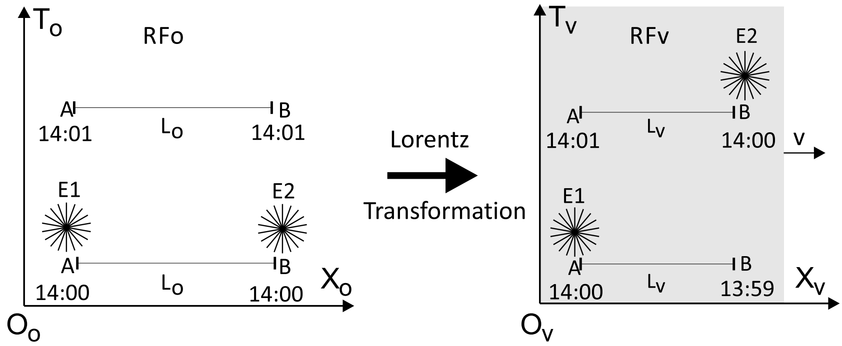

Figure 9.2 – Two simultaneous events $E1$ and $E2$ in RF_o are not simultaneous in RF_v. Notice that in RF_v the 'chasing' events $E1$ happens before the 'chased' event $E2$. And noting also the 'chasing' clock A is ahead of the 'chased' clock B.

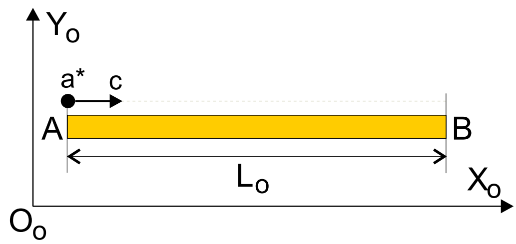

Figure 9.3 – Photon moving along the rod AB at rest in its proper reference frame RF_o.

reminder for this rule is: the 'chasing' event happens before (Figure 9.2).

9.3 Relativistic phase difference in synchronization

Let AB be a rod of proper length L_o at rest in its proper reference frame RF_o and placed parallel to the X_o axis (Figure 9.3). Assume a photon a^* moves from A to B. In RF_o the photon a^* takes a time $t_o = L_o/c$ to go from A to B. According to the Lorentz Transformation, from the perspective of the reference frame RF_v, the time t_v it takes the photon to complete its travel from A to B is given by:

$$t_v = \gamma t_o + \gamma \frac{v\, L_o}{c^2} \tag{7}$$

where the first term γt_o denotes the relativistic dilation of time and the second term $\gamma v L_o/c^2$ denotes the relativistic phase difference in synchronization (lack of simultaneity) between a clock placed at A and a clock placed at B. Now then, if according to the arguments given in the previous chapters length contraction and time dilation were only apparent, we would have:

$$\left.\begin{aligned} L_v = \gamma^{-1} L_o = L_o \\ t_v = \gamma t_o = t_o \end{aligned}\right\} \Rightarrow (\gamma = \gamma^{-1} = 1) \Rightarrow v = 0 \tag{8}$$

so that, once corrected the appearances, the measurements of length and time carried out in RF_v coincides with the measurements carried out in RF_o, as if RF_v were at rest with respect to RF_o. Obviously, in these conditions we would have:

$$\gamma \frac{v\, L_o}{c^2} = 0 \tag{9}$$

which means that, if length contraction and time dilation are only apparent, then the relativistic phase difference in synchronization would also be apparent. As we will see in the next sections, there are other independent arguments pointing in the same direction.

9.4 Isotropic media

The main objective of optical crystallography is the study of the propagation of light through different media. Fluids, amorphous and cubic minerals exhibit isotropy: light propagates through them with the same speed in all directions. In contrast, light propagates in different directions with different velocities through other solid materials, which for this reason are called anisotropic, and of which several types exist: uniaxial positive and negative (tetragonal, trigonal and hexagonal minerals); and biaxial positive and negative (orthorhombic, monoclinic and triclinic minerals).

A significant consequence of SR is that isotropic materials cannot be observed in relative motion. Optical isotropy can only be observed in media which are at rest in its proper reference frame. In effect, let m be any isotropic medium at rest in its proper reference frame RF_o and a^* a photon moving through m with a velocity c_o along a trajectory AB that makes an angle α_o with the axis X_o, and whose horizontal and vertical components are respectively AC and BC, (Figure 9.4). Being m optically isotropic, the photon a^* will move with the same speed c_o in any direction through m.

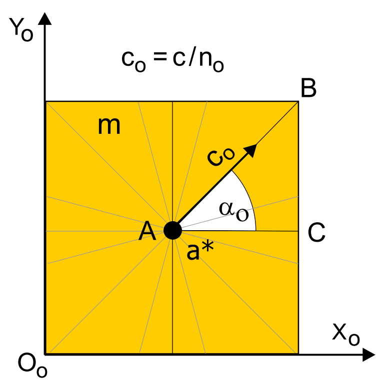

Figure 9.4 – Optically isotropic material at rest in its proper reference frame RF_o.

From the perspective of RF_v things are quite different. According to LT, and being $v = kc$; $0 < k < 1$ and c_v the speed of a^* from A to B through m, the photon a^* moves through m along the trajectory AB whose horizontal and vertical components are respectively $\gamma^{-1}AC$ and BC. The speed c_v of a^* through

m (which is different from the speed c of light with respect to RF_v) is then given by:

$$c_v = \frac{\sqrt{BC^2 + \gamma^{-2}AC^2}}{\gamma t_o + \gamma kcAC/c^2} \tag{10}$$

$$= \frac{c\sqrt{BC^2 + \gamma^{-2}AC^2}}{\gamma ct_o + \gamma kAC} \tag{11}$$

$$= \frac{c\sqrt{BC^2 + \gamma^{-2}AC^2}}{\gamma n_o c_o t_o + \gamma kAC} \tag{12}$$

$$= \frac{c\sqrt{BC^2 + \gamma^{-2}AC^2}}{\gamma n_o AB + \gamma kAC} \tag{13}$$

$$= \frac{c\sqrt{\sin^2\alpha_o + \gamma^{-2}\cos^2\alpha_o}}{\gamma n_o + \gamma k\cos\alpha_o} \tag{14}$$

$$= \frac{c\sqrt{\sin^2\alpha_o + (1-k^2)\cos^2\alpha_o}}{\gamma n_o + \gamma k\cos\alpha_o} \tag{15}$$

$$= \frac{c\sqrt{1 - k^2\cos^2\alpha_o}}{\gamma n_o + \gamma k\cos\alpha_o} \tag{16}$$

$$= \frac{c\gamma^{-1}\sqrt{1 - k^2\cos^2\alpha_o}}{n_o + k\cos\alpha_o} \tag{17}$$

$$= \frac{c\sqrt{\left(1-k^2\right)\left(1 - k^2\cos^2\alpha_o\right)}}{n_o + k\cos\alpha_o} \tag{18}$$

which means that for each different direction at an angle α_o with the axis X_o and each relative velocity $v = kc$, from the perspective of RF_v light travels through m with a different speed, as Figure 9.5 illustrates.

9.5 Bipolar anisotropy

Not only it is impossible to observe isotropic materials in relative motion, the phase difference in synchronization is responsible for an exotic anisotropy which is unknown in optical crystallography, and that we will call here *bipolar anisotropy* (Figure 9.6). Indeed, although in anisotropic materials the speed of light changes with direction, for a given direction the speed of light is the same in both senses

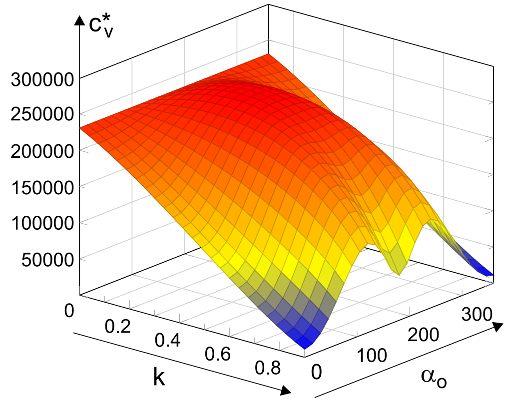

Figure 9.5 – Relativistic anisotropy of a material that is isotropic in its proper reference frame.

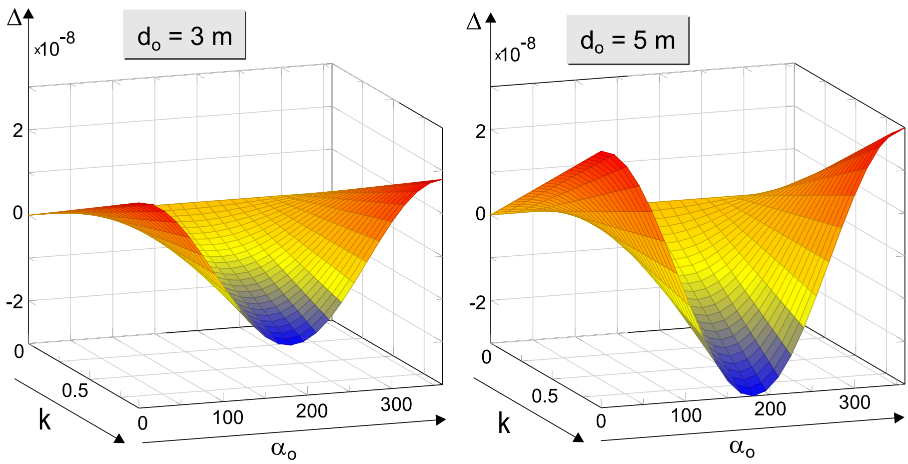

Figure 9.6 – Bipolar anisotropy in terms of k and α_o for two d_o: 3 m and 5 m.

of the given direction, which is plenty of physical meaning because the type and number of light interactions with a transparent medium is the same in both senses of the same direction. In contrast, when observed in relative motion, and due to the phase difference in synchronization, light travels with different speeds in each sense of the same direction through the transparent medium m, a difference Δ given by:

$$\Delta = \left(\gamma t_o + \gamma \frac{kcd_o \cos\alpha_o}{c^2}\right) - \left(\gamma t_o - \gamma \frac{kcd_o \cos\alpha_o}{c^2}\right) \quad (19)$$

$$= \frac{2\gamma k d_o \cos\alpha_o}{c} \quad (20)$$

$$= \frac{2k d_o \cos\alpha_o}{c\sqrt{1-k^2}} \quad (21)$$

where α_o is the angle that the trajectory of light in RF_o makes with the axis

X_o, and d_o is the proper length of the trajectory of light through the transparent medium. This bipolar anisotropy goes against all we know on the propagation of light through transparent media. In accord with the argument of this and the previous section, we conclude:

(a) Optical isotropy can only be observed and measured in the proper reference frame of the corresponding media.

(b) Bipolar anisotropy can only be observed and measured in media in relative motion.

Therefore, we must face the the following alternative:

Either:

> The relativistic phase difference in synchronization deduced from LT is only apparent.

Or:

> Not all reference frames are equivalent, according to (a) and (b).

9.6 Digital clocks and simultaneity

This last argument on phase difference in synchronization makes use of digital clocks whose screen display the actual time in the numerical terms of the decimal numbering systems. Consider a grid of such digital clocks which are synchronized and work correctly in their proper reference frame where they remain at rest. Assume the rows of the grid are parallel to the X_o axis and it columns parallels to the axis Y_o, as Figure 9.7 shows. All human and robotic observers in RF_o agree in that all clocks work correctly and are synchronized: all of them display the expected numerical digits in accordance with their correct operation.

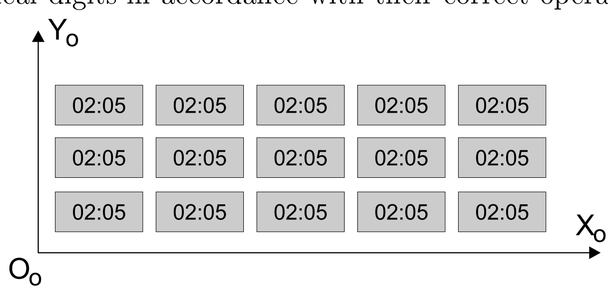

Figure 9.7 – Grid of digital synchronized clocks at rest in their proper reference frame RF_o.

In the reference frame RF_v all its human and robotic observers placed along the X_v axis will simultaneously (RF_v simultaneity) read the same figures in the screen of the clocks, but according to their relativistic LT-measurements they will appreciate all clocks malfunction and all clocks of each row are unsynchronized, but not at random but subject to strict rule: the lack of synchronization δt_v depends on their relative velocity kc an on their proper separation x_o in the direction of their relative motion according to:

$$\delta t_v = \frac{2kx_o \cos\alpha_o}{c\sqrt{1-k^2}} \tag{22}$$

On the contrary, all clocks placed in the same column are perfectly synchronized. We must again face two alternatives:

a) All clocks synchronized in their proper reference frame malfunction when observed in relative motion, though not randomly but in strict accordance with the law (22).

b) The observed phase difference in synchronization is not real but apparent, as apparent of the deformation of a rod partially submerged in water.