P8. Times in Special Relativity, A Critical Analysis

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Abstract

The arguments behind adopting "different times" in different inertial frames, enunciated by Lorentz, Poincare, and Einstein, are investigated critically. It is shown that light's speed independence of the relative velocity between the light's source and the receiver does not follow from relativity principle. A simplest derivation of Lorentz transformations is given and a peculiar feature arising from relative simultaneity is presented. A comparison is made between the duration and length of a light trip as specified by relativity theory and the theory of universal space and time.

Key Words: synchronization paradox, relative time, global time.

1.Introduction:

In previous works [1-6] we argued that, in order a tick's period of a clock, say a second, bear a meaning as far as motion is concerned, it should be correlated to what can happen during "a second" in the world outside the clock. More precisely, it should quantify the amount of the spatial displacement intrinsic to some reference physical phenomenon, such as the propagation of light from an arbitrary point in an inertial frame S. A "second" must thus be quantifiable (and actually measured) by a certain distance traveled by light inside S during a corresponding period. Time measurements therefore must be reducible to specific types of spatial displacement's measurements.

2. Measuring Time by Spatial Displacements

Global timing in an inertial frame S is set up by synchronization with an arbitrary observer $O \in S$ employing light's signals. *The concept of quantitative global time emerges through envisaging a "linear" correspondence between each instant of time T read by the timer O and the compound event:* (*the wave front of the pulse that was emitted from O at* $T = T_0$ *occupies, at T, points at* *equal distances R from O*). Through this correspondence, time duration ΔT is essentially measured by distance R, i.e.

$$\Delta T \equiv (T - T_0) = a\,R. \qquad (2.1)$$

The proportionality constant defines a constant velocity c of light by $c = 1/a = R/\Delta T$. The homogeneity of time follows from the homogeneity of space and the constancy of the light's velocity. The above correspondence is only one step short of *synchronization, which is accomplished by each S observer at* $(R,\emptyset,\theta)$ *taking note of his radial coordinate R and the instant of time* T_0 *at which the pulse emanated from O, and thus sets his timer at* $T = T_0 + aR$ *when the pulse is received.*

The rest of this section won't be needed till section 7 and the reader may move over to section 3. The theory of universal space and time (TUST) [5-7] determines the duration t_{bO} of a light trip from a *moving light source b* to an observer $O \in S$ by

$$\frac{ct_{bO}}{\sqrt{1-\beta^2}} = E(\beta, \pi - \theta)L \qquad (2.2)$$

where:- $L(= cT)$ is the geometric length of light trip in S, which is the distance of source b from O when it emits the pulse, say at $t = 0$,

- $E(\beta,\theta)$ is the Euclidean factor:

$$E(\beta,\theta) = \frac{-\beta\cos\theta + \sqrt{1-\beta^2 sin^2\theta}}{1-\beta^2}, \qquad (2.3)$$

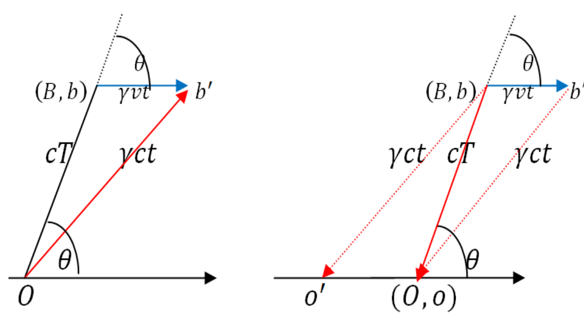


Fig. 2

in which, $\beta = v/c$ is the velocity of the source, and $\theta = \measuredangle(\boldsymbol{OB}, \boldsymbol{v})$ is the angle between the velocity vector and the initial radius vector (Fig.2). The duration of the latter trip, namely that which starts at $t = 0$, when b is at $B \in S$ and ends at $O \in S$ is the same as the duration of the light trip which starts from $O \in S$ at $t = 0$ (i.e., when

$(b\ is\ at\ B)$ and intercepts b, say at $b' \in S$. More about global time and synchronization can be found in [1-7].

General Remarks: 1. An arbitrary observer $O \in S$ may associate the time unit "a second" read by his clock with an evolution of a light wave-front from a sphere of radius $R[meter]$ to a sphere of radius $(R+c)[meter]$. The common perception of a "second" envisaged as the duration associated with the ticks of a clock becomes identical to our previous conception only if we demand that light travels rectilinearly c meters inside S during a second. The latter definition conditions the clocks to conform to the reference criterion, which is light propagation. We have thus on one hand the measuring instruments, "the clocks" that can be synchronized and distributed (hypothetically) everywhere in the space, and on the other hand the happenings in the outside world, namely the distance that light travels during the period read by the clocks. Note that the clocks which are indeed indispensable for measuring time *are no more than instruments that can be manipulated to conform to the reference criterion defining time, and their performance should certainly be rejected when they do not.* A clock measuring time at *one point* in S is in accord with the set criteria if the periods it reads, agree with distance covered by a return light's trip from the clock's location to a reflector at rest in S and back to the clock location.

2. Because the space is geometrically homogeneous *time durations* defined by equation (1) is independent of the master timer's position O. I.e. if another timer (not necessarily identical, ticking different units of time say $seccond'$) placed at $O' \in S$ records in a similar process a duration $\Delta T'\,[seccond'] = a\,R'[m]$, then

$$\frac{\Delta T'}{\Delta T}\frac{seccond'}{second} = \frac{R'}{R},$$

and in particular, if $R' = R$ then $\Delta T' seccond' = \Delta T\ second$. If the two timers use the same unit then $\Delta T' = \Delta T$.

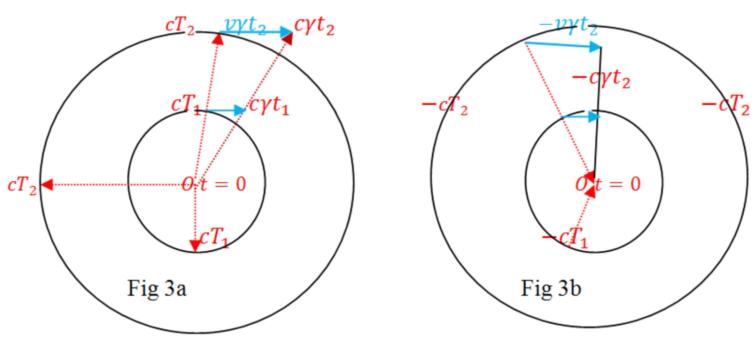


Fig 3a Fig 3b

Fig 3a. The spherical outgoing wave-fronts $R = cT$ in S correspond to the future time $(T \geq 0)$. Displacement of bodies $b_1, b_2, \ldots$ moving at velocity $\boldsymbol{v}$ till intercepted by the wave that emanated from O are shown in blue. The durations of the latter trips are denoted by $t_1, t_2, \ldots$(equation (2.2), and their lengths are $c\gamma t_1, c\gamma t_2 \ldots.$

Fig 3b. Light received at O at $t = 0$, from sources at rest in S, comprises emissions at instants T in past from all those in spherical surfaces $R = c(-T)$. The instant of light's emission t from a source that is moving at velocity $\boldsymbol{v}$ when was at a geometric distance $c(-T)$ from O depends on T, v, and the angle between $\boldsymbol{v}$ and the line of sight to b (equation (2.2)); the corresponding length to O is the product of the duration $(-t)$ by the effective light's velocity $c\gamma$.

Time in Special Relativity

3. Simultaneity According to Relativity

In section VIII, On the Idea of Time in Physics, of his book "Relativity: The Special and General Theory"[8], Einstein introduced through an example a criterion by which simultaneity of two events is judged. Quotations from this book appear in blue, whereas the arguments of the theory of universal space and time (TUST) is shown in black.

"Lightning has struck the rails on our railway embankment at two places *A* and *B* far distant from each other. I make the additional assertion that these two lightning flashes occurred simultaneously. If now I ask you whether there is sense in this statement, you will answer my question with a decided "Yes." But if I now approach you with the request to explain to me the sense of the statement more precisely, you find after some consideration that the answer to this question is not so easy as it appears at first sight. ……….After thinking the matter over for some time you then offer the following suggestion with which to test simultaneity. By measuring along the rails, the connecting line *AB* should be measured up and an observer placed at the mid-point *M* of the distance *AB*. … …If the observer perceives the two flashes of lightning at the same time, then they are simultaneous.

I am very pleased with this suggestion, but for all that I cannot regard the matter as quite settled, because I feel constrained to raise the following objection: Your definition would certainly be right, if I only knew that the light by means of which the observer at *M* perceives the lightning flashes travels along the length $A \to M$ with the same velocity as along the length $B \to M$. But an examination of this supposition would only be possible if we already had at our disposal the means of measuring time. It would thus appear as though we were moving here in a logical circle."

Authors' comment: *The means of measuring time are available by TUST in which the duration required by light to cross a space interval is proportional to its length. This yields:* $t_{AM}/|AM| = t_{BM}/|BM| = a\ (constant)$, *which gives* $t_{AM} = t_{BM}$.

Quotation continued ……..That light requires the same time to traverse the path $A \rightarrow M$ as for the path $B \rightarrow M$ is in reality neither a *supposition nor a hypothesis* about the physical nature of light, but a *stipulation* which I can make of my own freewill in order to arrive at a definition of simultaneity."

Authors' comment: Thinking of a general case of synchronization, this stipulation is equivalent to state that light travels in vacuum 'in straight lines' with a constant velocity $1/a \equiv c$. Moreover, the geometry of the 3-dimensional space is Euclidean, which implies homogeneity and isotropy. Einstein's stipulation therefore involves assumptions about the geometry of space and on light propagation. [More accurately, to guarantee homogeneity and the isotropy the space has to be a Riemannian space of constant curvature, positive, zero, or negative. A vanishing curvature corresponds to a Euclidean space].

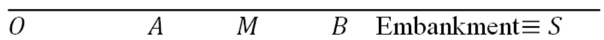
Quotation continued "It is clear that this definition can be used to give an exact meaning not only to *two* events, but to as many events as we care to choose, and independently of the positions of the scenes of the events with respect to the body of reference[1] [footnote[1]….] …We are thus led also to a definition of "time" in physics. For this purpose we suppose that clocks of identical construction are placed at the points *A*, *B* and *C* of the railway line (co-ordinate system), and that they are set in such a manner that the positions of their pointers are simultaneously (in the above sense) the same. Under these conditions we understand by the "time" of an event the reading (position of the hands) of that one of these clocks which is in the immediate vicinity (in space) of the event. In this manner a time-value is associated with every event which is essentially capable of observation.

Authors' comment: This corresponds to what we have called in TUST "global time" which is set up in an inertial frame S through synchronization with respect to a standard clock at an arbitrary point $O \in S$. As a result, the simultaneity of two events at A and B in S means the fulfillment of either of the following conditions:

-Both events took place at the same instant t_0.

-The timers at A and B read the same time when the two events took place.

- An observer at *O*, which is situated to the left of *AB*, sees the event at *A* advancing the event at *B* by $|AB|/c$.



- An observer *O*, wherever was his location relative to *AB*, sees a difference

$$t_B - t_A = (|\boldsymbol{OB}| - |\boldsymbol{OA}|)/c$$

between the two events.

Also in special relativity each of the above conditions is sufficient for the simultaneity of events *A* and *B* inside one chosen inertial frame *S*.

4. Simultaneity is Relative

In section IX, *The Relativity of Simultaneity*, Einstein advocates that simultaneity can be attached only to the set of bodies that are at rest with respect to each other, or as to say, each inertial frame has its own time.

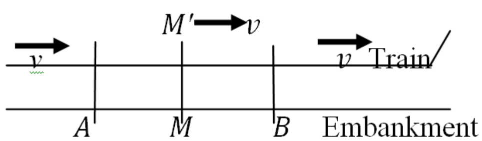


Figure 1

Quotation continued: We suppose a very long train travelling along the rails with a constant velocity ***v*** and in the direction indicated in Fig. 1. People travelling in this train will with advantage use the train as a rigid reference-body (coordinate system); …. the following question arises: Are two events (*e.g.* the two strokes of lightning *A* and *B*) which are simultaneous *with reference to the railway embankment* also simultaneous *relatively to the train?* We shall show directly that the answer must be in the negative. ……. But the events *A* and *B* also correspond to positions *A* and *B* on the train. Let *M'* be the mid-point of the distance *A* → *B* on the travelling train. Just when the flashes (As judged from the embankment) of lightning occur, this point *M'* naturally coincides with the point *M*, but it moves towards the right in the diagram with the velocity *v* of the train. If an observer sitting in the position *M'* in the train ……Hence the observer will see the beam of light emitted from *B* earlier than he will see that emitted from *A*. Observers who take the railway train as their reference-body must therefore come to the conclusion that the lightning flash *B* took place earlier than the lightning flash *A*. We thus arrive at the important result: Events which are simultaneous with reference to the embankment are not simultaneous with respect to the train, and *vice versa* (relativity of simultaneity). Every reference-body (co-ordinate system) has its own particular time; unless we are told the reference-body to which the statement of time refers, there is no meaning in a statement of the time of an event.

Authors' comment: By the TUST, if the inertial frame S is claimed universal (stationary and endowed with a global time) then the time of the embankment S is imposed in the train's frame S' ($\equiv s$ in our old notations). From this follows that the simultaneous events A and B in S are simultaneous in S' in spite of the observer M' sees event B taking place before event A. According to TUST, seeing the two events at the same time by a midway observer is not a sound criterion for simultaneity; the criterion here is that: the clocks at A and B in the universal frame read the same instant of time for the two events. Moreover, the passages of two objects a and b at the same instant t_0 by the points $A \in S$ and $B \in S$ respectively constitute two simultaneous evens in S and in any other frame regardless of the objects' velocities. Obviously it would make no difference if each object, a and b, emit a pulse of light at that instant.

Quotation continued. Now before the advent of the theory of relativity it had always tacitly been assumed in physics that the statement of time had an absolute significance, *i.e.* that it is independent of the state of motion of the body of reference. But we have just seen that this assumption is incompatible with the most natural definition of simultaneity; if we discard this assumption, then the conflict between the law of the propagation of light *in vacuo* and the principle of relativity (developed in Section VII) disappears.

Summary of chapter VII: The apparent incompatibility of the of propagation of light with the principle of relativity: Here is hardly a simpler law in physics than that according to which light is propagated in empty space. Every child at school knows, or believes he knows, that *this propagation takes place in straight lines with a velocity c* = 300,000 km./sec…..

A ray of light sent along the embankment frame S propagates in S with velocity c. By the Galilean law of velocity addition, the velocity of the light's ray relative to the train S' is

$$W = c - v.$$

But this result comes into conflict with the principle of relativity set forth in Section V {≡ If, relative to S, S' is a uniformly moving co-ordinate system devoid of rotation, then natural phenomena run their course with respect to S' according to exactly the same general laws as with respect to S.} by which light should propagates in vacuum at the same velocity c whether relative to the embankment or the train. In view of this dilemma there appears to be nothing else for it than to abandon either the principle of relativity or the simple law of the propagation of light *in vacuo*.

Two comments:

1.. Second Postulate of Special Relativity

The first thing to be said about the proclaimed law: "light propagates rectilinearly in vacuum with a constant speed c" is that it does not follow from relativity principle! Perhaps that is why Einstein stated it as a separate postulate. The law's statement could refer to either of the following cases: (i) Light's source and receiver are at rest in the given inertial frame, or (ii) Light's source and receiver have arbitrary relative velocity. Only in case (i) relativity principle, *as stated in relativity theory*, does imply the validity of the said law in every inertial frame. This is because experiments done under identical conditions (source and receiver are at rest in the given frame) in different inertial frames should, by relativity principle, produce the same result. The same conclusion is valid in TUST, but in non-intervening, or independent, inertial frames. We should elaborate on the latter statement in a separate part, but at present, we remain in the domain of special theory of relativity (SR for short).

In an inertial frame S, case (ii) comprises three distinct cases (a) a stationary source and a moving receiver (b) a moving source and a stationary receiver or (c) both source and receiver are moving in S. The relativity principle should imply the same light velocity c' under the same conditions (equal velocities of the source and the receiver) in S and S'. However, is $c' = c$? If $c' \neq c$ then how does c' depend on the velocities of the source and the observer? The answer to these questions should be settled by experiments, but certainly not by a second postulate of SR which states that: the speed of light in vacuum is independent of the motion of the source and of the inertial frame of reference of the observer. Moreover, in order to trivially fulfill the second postulate of SR, a transformation between S and S' that preserve the speed of light, namely Lorentz transformation, is constructed and claimed as the true physical transformation between inertial frames.

2. The Lorentz Transformations

Reverting to the above quoted materials, we note that the quest to preserve the velocity of light in vacuum regardless of the relative velocity between the source and observer was already done by Hendrik Lorentz (1904). Lorentz transformation (LT), in essence, modifies Galilean transformation (GT) between the inertial frames S and S' and sets up one that preserves the velocity of light. I.e., if a light pulse emanates from the stationary source $b \in S'$ and propagates at velocity c in S', then the new transformation will result in the pulse propagating also at velocity c in S. Henri Poincare [9-11] derived LT in a different method and studied its group structure; he also discussed clock synchronization and expounded the importance of form invariance of the laws of physics. For the sake of smoothness in presentation we give in the next section a very simple and short derivation of LT [12].

5. A Simplest Method to Derive LT

We assume that the inertial frames S and $S' \equiv s$ are in standard configuration and $S'$is translating relative to S at a uniform velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$. The Lorentz Transformations (LT) between S and S',

$$x = \gamma(x' + vt'), y = y', z = z', t = \gamma(t' + vx'/c^2), \qquad (5.1)$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$, relate the coordinates (time and spatial coordinates) of the same event, (t, x, y, z) in S and (t', x', y', z') in S'. We shall suppress the coordinates (y, z) and (y', z') which are equal, and focus on the LT between (t, x) in S and (t', x') in S'. The inverse of (5.1) is obtained by swapping primed and unprimed coordinates and replacing the velocity of S' relative to S, namely v, by $(-v)$ which is the velocity of S relative to S':

$$x' = \gamma(x - vt),\ t' = \gamma(t - vx/c^2). \qquad (5.2)$$

The Lorentz Transformations were tailed specifically to fulfill the requirements:

(i) The velocity of light emanating from a source of light b is the same in S and S'. I.e., the LT preserve the velocity of light regardless of the relative velocity of its source and the observer.

(ii) The LT grant the inertial frames S and S' a symmetric (or equivalent) status,

(a) regarding time and distance measurements.

(b) regarding being stationary or moving relative to the other frame.

To derive LT we start from [12] the Galilean transformations GT

$$x = x' + vt' \,, \; t = t' \tag{5.3}$$

in which we suppressed the coordinates $(y = y', z = z')$. From (5.3) follows the law of velocity addition: $W = u + v$, where u and W are the velocities of the same particle in S' and S respectively. This however violates requirement (i) because if $u = c$ (photon) then $W = c + v$. Or, if $W = c$ then $u = c - v$.

Having failed to fulfill requirement (i) by $t = t'$ we assume that every frame has its own time and seek a transformation between (x, t) and (x', t'). We check first the simple transformations

$$x = x' + vt', \quad t = t' + kx'. \tag{5.4}$$

We take the zero of time in both frames $(t = t' = 0)$ the instant at which the origins $O \in S$ and $O' \in S'$ coincide and assume that at this instant a ray emanates from O'. At an instant t' the ray arrives at the point $x' = ct'$. Substituting for x' in (5.4) we get

$$x = (c + v)t', \quad t = (1 + kc)t'. \tag{5.5}$$

To fulfill requirement (i) we should have $x = ct$, or

$$c + v = c(1 + kc). \tag{5.6}$$

This yields $c^2 k = v$, and hence the transformations we seek take the tentative form

$$x = x' + vt', \quad t = t' + vx'/c^2, \tag{5.7}$$

or

$$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} 1 & v \\ v/c^2 & 1 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}, \tag{5.8}$$

which preserve light's velocity. By requirement (*iia*,*b*), the inverse transformations should result from (5.8) by changing the sign of *v*:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} 1 & -v \\ -v/c^2 & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}. \tag{5.9}$$

But this is not the inverse transformation because the product of the above matrices (which corresponds to the composition of the transformation and its inverse) is not the identity matrix I_2:

$$\begin{pmatrix} 1 & v \\ v/c^2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -v \\ -v/c^2 & 1 \end{pmatrix} = \begin{pmatrix} 1 - v^2/c^2 & 0 \\ 0 & 1 - v^2/c^2 \end{pmatrix} =$$

$$(1 - v^2/c^2) I_2 = \gamma^{-2} I_2. \tag{5.10}$$

To fulfill requirement (*iia*) we should multiply each matrix by γ, which is equivalent to multiply the right hand-sides in (5.7) by the same factor, γ, to obtain LT (5.2).

If $x = x_0$ and $x' = x'_0$ coincide at t_0 in S and t'_0in S'then the LT take the form

$$\Delta x = \gamma(\Delta x' + v\Delta t'),\ \ \Delta t = \gamma(\Delta t' + v\Delta x'/c^2), \qquad (5.11)$$

where

$$\Delta x = x - x_0,\ \Delta t = t - t_0, \qquad (5.12a)$$

$$\Delta x' = x' - x'_0,\ \Delta t' = t' - t'_0. \qquad (5.12b)$$

Velocity Addition: Suppose a body is moving in S' at velocity $\boldsymbol{u} = u\hat{\boldsymbol{x}}$ where S' itself is moving in S at velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$. We pose the question: what is the velocity of the body, W, relative to *S*? From LT (5.2) we have:

$$\frac{dx}{dt} = \frac{d(x'+vt')}{d(t'+vx'/c^2)} = \frac{u+v}{1+uv/c^2} \equiv W. \qquad (5.13)$$

For a photon, $u = c$, and hence $W = c$, which means that light propagates in vacuum at a constant velocity *c* in all inertial frames.

6. Relative Simultaneity

By LT there corresponds to an instant of time t_0 in *S* different instants t' in S'that depend on the position x' of the event in S',

$$t' = \sqrt{1 - v^2/c^2}\, t_0 - vx'/c^2. \qquad (6.1)$$

The latter relation shows that clocks at distinct locations $x' \in S'$ read distinct times t' in spite that all clocks in *S* reads the same instant t_0. In other words, *there corresponds to the set of simultaneous events* $\{(t_0, x): -\infty < x < +\infty\}$ *in S, the set of non-simultaneous events*

$$\{(t', x') \equiv \gamma(t_0 - vx/c^2, x - vt_0): -\infty < x < +\infty\ \} \qquad (6.2)$$

in S'. In different warding, events with different abscissas (i.e. in different planes perpendicular to relative velocity) can be simultaneous only in one inertial frame of reference. Moreover, events (t, x_0, y, z) with the same abscissa x_0 in *S* are simultaneous in S' if and only if simultaneous in *S*. The latter statement is also evident on the account of the inverse of (5.11),

$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2) = 0, \qquad (6.3)$$

in which we set $\Delta x = 0$.

By (6.1) the instants of time t'descends linearly with $x(or\ x')$ from $t' = +\infty\ at\ x' = -\infty$ to $t' = -\infty\ at\ x' = +\infty$ passing through $(t' = 0\ at\ x' = c^2 t_0/v\gamma)$. In particular, for $t_0 = 0$,

$$(t', x') = \gamma\ (-vx/c^2, x), \qquad (6.4)$$

and the descending line passes through ($t' = 0, x' = 0$).

A peculiar feature arises from relative simultaneity which we illustrate in this paragraph. Suppose that a rod $A'B'$which is stationary in the train frame S' (one may take the full train to be $A'B'$) is observed from the embankment frame *S* to coincide at an instant t_0 with the rod *AB* which is at rest in *S*. In the frame S' the end B' is observed to touch *B* before A' touches *A* (by $\Delta t' = \gamma v|AB|/c^2$). If the two rods were arranged so that the ends of a rod can touch the other rod's ends and form a circuit, then this circuit is closed in *S* at t_0 but never closed in S'. If the circuit contains a battery and a bulb, the bulb flashes in *S* but never in S'!!

7. Durations of Light Trips as Determined by SR and TUST

We examine here the views of SR and TUST of time and simultaneity applied to the example of the train (frame S') and embankment (frame *S*) discussed by Einstein. It is noted from start that the peculiar features of length contraction, time dilatation, and relative simultaneity in SR are absent altogether in TUST. In the latter theory, timing in the stationary frame *S* prevails in all other frames, and the length of a solid bar (geometric length) retains its value when the bar is moving.

Consider two simultaneous events $A(t = 0, x = -l)$ and $B(t = 0, x = l)$ in *S* which represent two pulses of light emerging at the same instant of time from the points *A* and *B*. Suppose that the two pulses head towards the mid-point $M \equiv O$, where *O* is the origin of *S*.

The coordinates of the latter two events in S' according to LT are:

$$t'_A = \gamma\, vl/c^2, \quad x'_A = -\gamma l, \tag{7.1a}$$

$$t'_B = -\gamma\, vl/c^2, \quad x'_B = \gamma l. \tag{7.1b}$$

At $t = 0$ the points *A* and *B* occupy positions in S' that are symmetric with respect to its origin O'. The coordinate of mid-point of the latter events in $S'$is $\frac{1}{2}(x'_A + x'_B) = 0$, and hence $M' \equiv O'$. Or by LT the event $(x = 0, t = 0)$ in *S* is the same as the event $(x' = 0, t' = 0)$ in S'.

In *S*, the two pulses originating from the points *A* and *B* at $t = 0$ arrive at the mid-point *O*, at $t_A^O = t_B^O = l/c$.

In S', and using (7.1a), the pulse emanating from *A* takes $|x'_A|/c = \gamma l/c$ to arrive at O'at

$$t_A'^{O'} \equiv t'_A + \frac{\gamma l}{c} = \frac{v}{c}\frac{\gamma l}{c} + \frac{\gamma l}{c} = \frac{l}{c}\frac{(1+v/c)}{\sqrt{1-v^2/c^2}} = \frac{l}{c}\sqrt{\frac{1+v/c}{1-v/c}}\,, \tag{7.2a}$$

whereas the pulse emanating from *B* arrives at O' at

$$t_B'^{O'} \equiv +\frac{\gamma l}{c} = -\frac{v}{c}\frac{\gamma l}{c} + \frac{\gamma l}{c} = \frac{l}{c}\frac{(1-v/c)}{\sqrt{1-v^2/c^2}} = \frac{l}{c}\sqrt{\frac{1-v/c}{1+v/c}}\,, \qquad (7.2b)$$

advancing that from A by

$$\Delta' = \frac{2v}{c}\gamma\frac{l}{c} = 2\frac{v}{c}\frac{l/c}{\sqrt{1-v^2/c^2}}. \qquad (7.3)$$

The corresponding readings in S to the instant of light arrival at O' are

$$t_A^{O'} \equiv \gamma t_A'^{O'} = \frac{l}{c}\frac{1}{1-v/c}, \quad t_B^{O'} \equiv \gamma t_B'^{O'} = \frac{l}{c}\frac{1}{1+v/c}, \qquad (7.4)$$

These value are also the durations of the trips $(A \rightarrow O')$ and $(B \rightarrow O')$ in S. Thus, as seen in S, the pulse from B arrives at O' before the pulse from A by a period

$$\Delta = \gamma\Delta' = 2\frac{v}{c}\frac{l/c}{1-v^2/c^2}\,. \qquad (7.5)$$

In TUST the universal time prevails everywhere in the sense that even moving timers should read the same time read by the stationary timer in its vicinity. The rays emanating from A and B at $t = 0$ arrives at M at l/c. The observer $M' \equiv O' \in S'$ which is contiguous to $M \equiv O \in S$ at $t = 0$ is moving in S at velocity v and it is at equal distances l from A and B. The scaling transformations determine the time at which the ray emanating from A intercepts the moving bodyO' by

$$t(A \rightarrow O')_U = \frac{l}{c}\Gamma(\beta, 0) = \frac{l}{c}\sqrt{\frac{1+u/c}{1-u/c}} \equiv t_U^A, \qquad (7.6a)$$

where we briefed the duration of the trip by t_U^A. Similarly,

$$t(B \rightarrow O')_U = \frac{l}{c}\Gamma(\beta, \pi) = \frac{l}{c}\sqrt{\frac{1-u/c}{1+u/c}} \equiv t_U^B. \qquad (7.6b)$$

The identical values (7.2a,b) and (7.6a,b) should not convey an impression that SR and TUST produce the same results, for, these values refers to different entities. The values (7.6a,b) represent the durations of the light trips (in all frames) $A \rightarrow M'$ and $B \rightarrow M'$ that both started at $t = 0$, whereas those in (7.2a,b) are the arrival times in S$'$ of the two trips which started at two different instants t_A' and t_B' respectively.

We distinguish the predictions of the SR and TUST by subscripts SR and U respectively.

(1) The durations of the trips $A \rightarrow O'$ and $B \rightarrow O'$ are given by (7.6a,b) in TUST, and in SR by

$$t'^A_{SR} = t'^B_{SR} = \gamma\frac{l}{c} = \frac{l/c}{\sqrt{1-v^2/c^2}} \ \ (\text{in } S') \qquad (7.7a)$$

$$t_{SR}^{A} = \frac{l}{c}\frac{1}{1-v/c}, \quad t_{SR}^{B} = \frac{l}{c}\frac{1}{1+v/c}, \quad \text{(in } S\text{)} \qquad (7.7b)$$

(2). The geometric lengths of each trip $A \to O'$ and $B \to O'$ in SR is the product of its duration by light's velocity:

$$L_{SR}^{\prime A} \equiv L_{SR}^{\prime B} = \frac{l}{\sqrt{1-v^2/c^2}}. \qquad \text{(in } S'\text{)} \qquad (7.8)$$

$$L_{SR}^{A} = \frac{l}{1-v/c}, \quad L_{SR}^{B} = \frac{l}{1+v/c}, \quad (in\ S) \qquad (7.9)$$

(3) The geometric length of each trip $A \to O'$ and $B \to O'$ in TUST is the product of its duration by the effective light velocity γc:

$$L_U^A = c\gamma.\, t_U^A = \frac{l}{1-v/c}, \qquad (7.10)$$

$$L_U^B = c\gamma.\, t_U^B = \frac{l}{1+v/c}. \qquad (7.11)$$

The distance travelled by O' till intercepted by the light ray sent from A, denoted by l_U^A, is the product of the duration of this trip by the moving body's effective velocity $v\gamma$:

$$l_U^A = v\gamma.\, t_U^A = \frac{vl/c}{1-v/c} \qquad (7.12)$$

The difference between the geometric lengths of the light trip $A \to O'$ and the distance travelled by O'during its duration must be l:

$$L_U^A - l_U^A = \frac{l}{1-v/c} - \frac{vl/c}{1-v/c} = l. \qquad (7.13)$$

Similarly the distance travelled by O' till intercepted by the light ray from B is

$$l_U^B = \frac{vl/c}{1+v/c}. \qquad (7.14)$$

The sum of this distance and the length of the light trip $B \to O'$ must be l:

$$L_U^B + l_U^B = \frac{l}{1+v/c} + \frac{vl/c}{1+v/c} = l. \qquad (7.15)$$

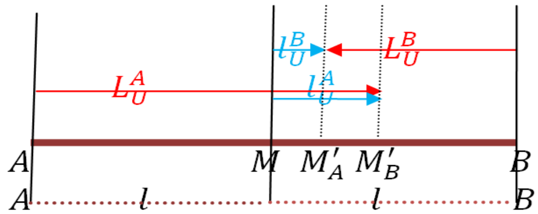


Fig. 4.

Similar results hold in the frame S employing SR

$$L_{SR}^{A} - l_{SR}^{A} = l, \qquad L_{SR}^{B} + l_{SR}^{B} = l.$$

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