the general science journal

EINSTEIN'S FALSE DERIVATION OF TIME DILATION

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1.0 Introduction

This paper demonstrates that the traditional derivation of the time dilation formula, which is based on Einstein's method, is incorrect. This error arises from Einstein's first paper of 1905, which has been copied by textbook writers without a critical examination of its mathematical method and physical interpretation.

The following shows that the traditional formulas are incorrect. In 1905 Einstein obtained the result:

(1) $t' = \beta^{-1} t$

Where t is a time interval defined in the stationary frame, and t' is a time interval defined in the moving frame. We make the assumption that these time intervals physically represent oscillation periods of a harmonic oscillation. These harmonic oscillations being the basis for the measurement of time by clocks in the rest and moving frames, S and S' respectively. Hence, when time is converted to frequency, equation (1) transforms into the law for transformation of frequency of the clock oscillators used to measure time. Transforming equation (1) to frequency gives:

(2) $1/f' = \beta^{-1} 1/f$ or $f = \beta^{-1} f'$. Therefore, $f' = \beta f$

This formula shows that the frequency of the clock oscillator in the stationary frame, f is less than the frequency of the clock oscillator in the moving frame, or that the clock in the moving frame is fast. This result contradicts the traditional interpretation of relativity and is inconsistent with experiment which shows that the correct formula is: $f' = \beta^{-1} f$, which means that the clock oscillator of the moving clock is slow relative to the stationary clock.

In 1907 Einstein obtained a revised time dilation law which is the inverse of equation (1):

(3) $t = \beta t'$

Converting to frequency we obtain the same result as in equation (2).

(4) $1/f=\beta 1/f'$ or $f'=\beta f$

This formula shows that the clock oscillator frequency in the moving frame f' is greater than clock oscillator frequency in the stationary frame. This result contradicts the traditional interpretation of relativity and is inconsistent with experiment. This result shows that the moving clock runs fast contrary to what is accepted physics taught in our universities.

One of the objectives of this paper will be to explain how this contradictory state of affairs became accepted physics.

2.0 Einstein's Method

Einstein gives two distinctly different methods. One based on clock dial comparison and another based on clock frequency comparison. The first is the basis of the 1905 paper and the 1912 manuscript and the second is the basis of his 1907 and 1910 papers. It is curious to notice that the unpublished manuscript review of relativity written in 1912 (later published in 1995 and in a revised edition 2003) is a regression to the 1905 version of relativity.

2.1 The 1905/1912 Version Of the Method

This section examines Einstein's 1905 method of proof of the time dilation. The proof consists of the following steps. The first step which appears in section 1 of part I, is the definition of simultaneity. A procedure is defined to establish a synchronization of clocks. Then in step 2, which appears in section 2 of part I, he proves the relativity of simultaneity and declares: "So we see that we cannot attach any absolute signification to the concept of simultaneity, but that two events which, viewed from a system of co-ordinates, are simultaneous, can no longer be looked upon as simultaneous events when envisaged from a system which is in motion relatively to that system." This has traditionally been interpreted to mean that the passage of time, i.e., its rate, is not absolute or uniform for all observers, and suggests that the time dilation effect is related to the relativity of simultaneity.

The third step involved the derivation of the Lorentz transformation equations in section 3 of part I. This was then followed by the mathematical proof in section 4 that "the time marked by the clock (viewed in the stationary system) is slow."

The mathematical derivation of the time dilation proceeds as follows. Einstein says "We imagine one of the clocks which are qualified to mark the time t when at rest relatively to the stationary system, and the time τ when at rest relatively to the moving system, to be located at the origin of co-ordinates of k, and so adjusted that it marks the time τ". Here it is assumed that a clock defined at rest in k is the same physically as if the same clock were at rest in the stationary system K. The time coordinates defined in the system k are equated to those in system K using the Lorentz transformation equations of section

3, part I:

(5) $\tau=\beta(t-vx/c^2)$

He then performs the following substitution, which is a very important step, saying "we have, evidently, x=vt." However, this substitution is not very evident, nor is the assumption that the clock in K runs at the same rate as the clock in k, because he will contradict this assumption by concluding that the clocks run at different rates. The substitution into (5) gives the famous result given above in equation (1), where t' replaces τ. Although this proof has been accepted for over 100 years, it is not entirely convincing. The assumption that a clock calibrated in the stationary frame can be placed in motion so that it runs at the same rate as before has no justification and is contradicted by the conclusion. Furthermore, the substitution x=vt has no mathematical justification either. As we will see both of these assumptions are the cause of the false result.

Consider the 1912 method. This method is similar to the 1905 method in that clock dial readings are used to compare time. Einstein asks to consider, what is the dial reading on t the clock at rest in S (K), when the clock in S'(k) advances one unit of time? He refers to the Lorentz transformation for time and writes the transformation for time as:

(6) $\Delta t=\Delta t'/(1-v^2/c^2)^{½}$

Einstein says "Thus, Δt is always greater than Δt'; i.e., measured with the system of clocks of S, the clock moving with the velocity v runs slower than it would run if it were at rest." The reader should notice that (6) is the same as equations (3) and (5), which are inverses of equation (1). Unfortunately, Einstein does not give a detailed derivation of equation (6) in the 1912 manuscript. To determine the derivation of equation (6) we have to examine the 1907 and 1910 papers.

2.2 Einstein's 1907/1910 Version

The 1907 method of proof uses a comparison of clock frequency by showing that moving clocks run slow just as concluded in 1905, but based on a calculation of observed or apparent frequency. This method of proof was motivated by new experimental results obtained by J. Stark. This suggested to Einstein that moving atomic clocks run slow because "...the oscillation process that corresponds to a spectral line is to be considered an intra-atomic process whose frequency is determined by the ion alone, we may consider such an ion as a clock...the effect of motion on the light frequency...reduces the (apparent) proper frequency of the emitting ions..." The revised theory presented in 1907 demonstrated the mathematical proof of this supposition by showing that the Lorentz transformation of the frequency of a moving clock observed from the stationary frame was less than the observed frequency of the same ions at rest. Hence the clocks of the moving ions appeared to run slow. This seemingly brought the theory and experiment into agreement, and therefore, served as an experimental proof of the theory. However, the method was flawed as we will see later.

Another revision was that the interpretation made in 1905, that moving clocks lost time due to motion, was revised. In 1907 the interpretation became that the rate of a moving clock only appears slow when viewed from the rest frame. Hence in the 1907 revision, the interpretation that the slowing of moving clocks is a real change in clock rate gives way to the interpretation that the change is in the apparent rate of the clock observed from the moving frame. In the following, this altogether different interpretation will become apparent.

In the 1907/1910 method Einstein uses the equation

(7) $t=\beta t'$

The method presented in the 1910 paper derived this equation from the inverse Lorentz transform with the condition x'=0. Converting (7) into frequency he obtains the same result as given above in equation (2) but in the following form:

(8) $\nu=\beta^{-1}\nu_0$,

where ν_0 is the frequency of a clock calibrated at rest in S and then given velocity v so that it appears to an observer at rest in S to have the frequency ν. Einstein gives the conclusion that "a clock moving uniformly with velocity v with respect to a reference system runs, as observed in this system 1: $(1-v^2/c^2)^{1/2}$ times slower than an identical clock that is at rest with respect to this system". Notice that now the meaning of the symbols has changed from the meaning assigned in section 1.0. There the symbols f and f' referred to the rest rates of the clocks in their respective frames. Now the symbols f and ν refer to the frequency of the clock in the moving frame, not the clock in the stationary or rest frame S (or K). The trick in this change in meaning is obscured by the new meaning, which refers to the rate of the clock in the moving frame as seen from the rest frame. Now for the complete system of relativity where frames S and S' are in relative motion, we can view S' from S and S from S'. In this case there are now four different symbols that need to be used in order to make clear the difference between the rate of a clock in its rest frame versus the rate as observed from a relatively moving reference frame. However, this distinction does not appear in the theory of relativity or in textbooks which discuss the theory. This lack of rigor and clarity is just another source of argument, confusion, and acrimonious debate.

It is clear that by 1907 Einstein believed his derivation of time dilation in the 1905 paper was unsound. The new interpretation required a different approach. For the new interpretation, it was necessary to show that the time marked by the clock in the moving frame was shifted in frequency. This led to an apparently more elegant and convincing method of proof than that used in 1905. Hence, in 1907, the derivation of the transformation of time intervals was dramatically revised and appeared in a completely different form. The new form posed a completely different problem. It addressed the transformation of clock rate, as opposed to the previous approach based only on the transformation of time interval. But, while in 1905, the idea was that the rate of the clock was slow due to its actual motion through space, the 1907 revision resulted in the interpretation that the clock in the moving frame only appeared to be slow when viewed by an observer in relative motion. As Einstein puts it "The motion...reduces the (apparent) proper frequency of the emitting ions..." The confusion created by these two different versions of the theory has engendered considerable confusion in physics textbooks and fueled criticism and controversy.

3.0 Three Experimental Demonstrations of Time Dilation

There are three different types of experiments which experimentally demonstrate the time dilation equations of special relativity. They are: moving canal ray experiments, particle decay time experiments, and traveling or moving clock experiments. Each of them requires a different interpretation of the Lorentz transformation laws.

3.1 The Canal Ray Experiments

The moving canal ray experiments are based on the measurement of the spectral lines of fast moving canal rays, which are ionized hydrogen atoms accelerated to high velocities inside vacuum tubes. The earliest of these experiments was performed shortly after the publication of Einstein's 1905 paper. In 1907 Einstein published a second paper on relativity in which he demonstrated that the theory predicted the red shift of high velocity canal rays in accordance with the reported experimental results. Einstein's proof used the transformation given in equation 3 above, i.e., $t=\beta t'$. as shown above in the introduction, this law predicts that the moving canal rays should be blue shifted, and not red shifted as is observed. However as discussed above, by a clever argument which is incorrect, Einstein obtained that the special theory predicts that the spectral lines are red shifted.

The following shows that the correct transformation law is, $t'=\beta t$. Converting to frequency we have: $1/f'=\beta 1/f$, which becomes $f'=\beta^{-1}f$, showing that the oscillation frequency of the spectral lines of the moving canal rays are red shifted, or have a frequency less than the frequency at rest.

3.2 Particle Decay Time experiments

These experiments are based on the comparison of the low velocity decay times of elementary particles with the decay times of high velocity particles. The supposition is that the clocks of the particles run slow in the fast moving case, which explains the longer decay times. To apply the Lorentz time dilation laws, the low velocity decay times are taken as measures of rest frame time in frame S, and the high velocity results are taken as measures of time in a relatively moving inertial reference frame S'. Suppose we assign the symbol t' to indicate the rest frame S decay time and the symbol t' to represent the moving frame S' decay time. This seems a very natural assignment of symbols. However, the resulting transformation law based on experiment is $t'=\beta t$, which is not the traditional law derived by Einstein for time dilation.

Evidently there is a fundamental difficulty with the traditional time dilation law. This is clear from the following considerations. The traditional claim is that time is stretched or dilated in the moving frame S’ relative to the rest frame S. However, the law which is given in equation (3) states that a time in the rest frame is longer than time in the moving frame. The answer to this contradiction is as follows. The precise meaning of the time dilation law in equation (3) is this. When a clock dial in the moving frame S’ reads one unit of time in terms of S’ measure, the clock in the rest frame S, reads β units of S frame time measure. The difficulty in applying this meaning to the canal ray and particle decay experiments is now apparent. In both cases, we do not have clock dials which provide time measures. Hence, neither of these experiments employs a clock, although traditional relativity textbooks like to use them as clocks, and erroneously imply that they can be used as if they were clocks. Unfortunately, this attempt at analogy is faulty as has been shown here.

3.3 Traveling Or Moving Clock Experiments

The deficiency of the two previous types of experiments is avoided by using high precision clocks and placing them is relative motion, instead of using clock proxies. The Haefle-Keating experiment used clocks traveling around the world on commercial jet aircraft and compared them to laboratory standard clocks at rest on the ground. In this case, and only in this case, can the law given in equation (3) be used. It shows that the elapsed time recorded on the moving clocks was less than the time recorded on the rest clocks. Surprise again, the law apparently fails. But in this case the demonstrated result is the same as the law derived by Einstein in 1905, i.e., $t'=\beta^{-1}t$.

3.4 What Is The Correct Law Of Time Dilation?

In sections 3.1 and 3.2 we obtained the time dilation law based upon experiment as: $t'=\beta t$. In this equation the symbols t and t’ refer to units of time defined in frames S and S’ respectively. In section 3.3, the applicable law is $t=\beta t'$. The traditional interpretation claims that $t'=\beta t$ defines the transformation which shows that the clock in S is slow relative to S’, but the above experiments contradict this interpretation. Here is another contradiction. Does the law represent the rate of a moving clock observed from S’ or the rate of the clock in S’ relative to the rate of the clock in S?

If you are not confused by these results, then you are not paying attention. The confusing state of affairs with regard to the meaning of the time transformation laws in special relativity is the result of an incorrect interpretation of the Lorentz transformation equations. The author of this paper is not confused, nor is he incorrect, the problem is real, created by a lack of rigor in Einstein’s derivations and perpetuated by 100 years of confusion and misunderstanding. Critics such as Herbert Dingle focused on this problem starting in the 1950’s but were ignored by the physics establishment.

4.0 Einstein’s Big Blunder

Einstein's blunder is as follows. He says that when we have a Lorentz transformation between two time intervals, representing time marked by clocks, when we set the time indicated on the reference clock to 1 unit and calculate time on the opposite clock, if the result is less than 1 unit, the subject clock is slow. In 1905 he used this interpretation, based on the operational definition of the symbols in the Lorentz transforms as clock dial readings. He then obtained the result $t'=\beta^{-1}t$ which confirmed that moving clocks run slow based on this interpretation. He failed to obtain the correct result $t'=\beta t$, because in his interpretation of symbols, this result showed that the moving clock is fast. But we saw above that experiments confirm this law.

Einstein's blunder is confusing dial readings with time intervals. He assumes a direct 1 to 1 relation such that clock dial or display readings represent time. Dial readings are counts of clock beats. Beats are oscillations with a fundamental cyclic frequency of occurrence. They are usually assumed uniform of fixed period. Clocks keep time by counting these periods of oscillation. Oscillations with shorter periods, i.e., contracted time intervals, mean the clock runs fast because more beats are counted during a given reference period as measured by the reference clock. Now in 1907, Einstein used frequency as the measure of the clock rate. But, while frequency is inversely related to dial readings or counts, converting the law $t'=\beta^{-1}t$, into its inverse law $t=\beta t'$, did not correct the 1905 error. So the law used in 1905 for dial readings failed to give the correct result in 1907 based on frequency.

There is a simple and clear mathematical explanation of all of these difficulties. This will be stated here and explained in detail mathematically in the following section. The answer is that there are two ways to transform time. We can transform the time scale or standard of time measure, the time unit, or we can transform the measured units of time as dial readings. These two transformations are commonly known as basis and coordinate transformations. The basis transform changes the units of time and the coordinate transforms change the clock dial readings. Both types of transformation can be derived from the Lorentz transformation equations, all that is required is that we clearly specify which equations are used to transform the time units and which are used to transform dial readings.

5.0 Evaluation Of Lorentz Equations For Time

The mathematical method used here is to first solve the system of Lorentz and inverse Lorentz equations for time simultaneously using a specified condition of evaluation. Here the term evaluation is used in the same sense as it is used when a polynomial equation is solved for its roots by setting the equation to equal zero and solving for the indeterminates. The procedure used here is similar. A selected variable is set to zero, and the resulting solutions are obtained. Solutions are obtained by setting one of the following four variables equal to zero, and then solving for the remaining three. The following variables are set equal to zero and the resulting solutions obtained by evaluation: $x=x'=0$, each taken in turn.

The Lorentz transformation equations in a simplified form are assumed as follows:
$x'=\beta(x-vt)$ $t'=\beta(t-vx/c^2)$ $x=\beta(x'+vt')$ $t=\beta(t'+vx/c^2)$ $\beta=(1-v^2/c^2)^{-1/2}$
Here there are four equations which express the simultaneous solutions for the transformation of coordinates. These equations are defined in the usual way in terms of two relatively moving reference frames S and S'. Where the origin of frame S' is in motion with velocity v in the positive x direction of S.

Notice that β is greater than unity when v is greater than zero, and that β^{-1} is less than unity when v is greater than zero. An equation of the form $t'=\beta t$ results in a dilation of the variable t' with respect to t because t' is greater than t The equation $t=\beta^{-1}t'$ results in a contraction of the variable t with respect to t' because t is less than t'. The definition of β implies that it is always equal to or greater than unity, and can never be less than unity.

The coordinate frames S and S' are assumed to be orthogonal coordinate systems with the requirement that time is defined such that t=t'=0 occurs when the origins coincide; i.e. x=x'=y=y'=z=z'=0 at t=t'=0. The axes for the x, y, and z directions are assumed to be parallel, and the y and z coordinates are assumed to be identical and coincide when the origins coincide at t=t'=0. The purpose of the solutions is to determine the relations governing the transformation of the x and t coordinates according to the Lorentz transform equations.

5.1 Results for x=0 (Specification of an evaluation at the same place in S)

To consider the role of evaluation in at the same place in S, we determine the simultaneous solution of the four equations when we specify the condition that x=0. The results are as follows:

(9) $x'=\beta(x-vt)=-\beta vt$
(10) $t'=\beta(t-vx/c^2)=\beta t$
(11) $x=\beta(x'+vt')=0$, Therefore $x'=-vt'$
(12) $t=\beta(t'+vx'/c^2)=\beta t'(1-v^2/c^2)=\beta^{-1}t'$

Notice that equation 12 is the inverse of equation 10, so they are the same solution. Equation 12 is solved by substitution with the result from equation 11. Therefore, from equations 10 and 12 we have the following solution for the condition x=0: $t'=\beta t$. The solutions for equations 9 and 11 give the results $x'=-\beta vt=-vt'$, from which we conclude that $t'=\beta t$. A result which is the same as obtained from equation 10. Henceforth, we will take the solution to be equation 10, when the evaluation condition x=0 is imposed. All of the results given here lead to the conclusion that time in frame S' is dilated relative to frame S.

5.2 Results for x'=0 (Specification of an evaluation at the same place in S')

To consider the role of evaluation at the same place in S', we determine the simultaneous solution of the four equations when we specify the condition that x'=0. The results are obtained as follows:

(13) $x'=\beta(x-vt)=0$, Hence $x=vt$

(14) $t'=\beta(t-vx/c^2)=\beta t(1-v^2/c^2)=\beta^{-1}t$

(15) $x=\beta(x'+vt')=\beta vt'$

(16) $t=\beta(t'+vx'/c^2)=\beta t'$.

Notice that equation 14 is the inverse of equation 16, so they are the same solution. Equation 14 is solved by substitution with the result from equation 13. Therefore, from equations 14 and 16 we have the following solution for the condition $x'=0$: $t=\beta t'$. The solutions for equations 13 and 15 give the results $x=vt=\beta vt'$, from which we conclude that $t=\beta t'$. A result which is the same as obtained from equation 16 which is the primary result for the condition $x'=0$. This equation is the traditional result for time dilation, with 14 being the result of Einstein obtained in 1905. Notice that all these equations are symmetrical reciprocals, by exchange of primed and unprimed symbols, of the equations in section 5.1 and vice versa.

5.3 Interpretation Of The Results

The traditional interpretation is that equations 16 and 10 are the transformation of time in special relativity. Some authors, however, think that 14 and 12 are the solutions. The first derives from Einstein's 1907/1910 papers and the second derives from the 1905 paper. The current consensus of textbooks appears to support the 1907/1910 version which proclaims equations 16 and 10 as the time dilation laws. Unfortunately, equation 16 is consistent with experiment in terms of the dial transformation interpretation, while 10 is consistent with the time unit transformation interpretation. But this contradicts the idea that these equations are inverse transformations of dial readings.

A further problem is that there are four transformation equations and the traditional theory only has room for two. The four transformation equations are 10, 12, 16 and 14. (The traditional theory gives only 10 and 16.) As noted before, these form bijective pairs and are inversely related. Hence, we have more solutions than the traditional theory which insists there are only two. How can we resolve these problems. The answer is simple, instead of insisting that we have two solutions lets accept that there are four and then interpret them as basis and coordinate transformations of time. Now we have equations 10 and 12 as the basis transforms and 16 and 14 as the coordinate transforms. All the difficulties now disappear because we can now assign the correct law to the correct type of experimental situation. Equations 10 and 12 apply in the experiments of sections 3.1 and 3.2, while we apply equations 16 and 14 to the experiments of section 3.3.

6.0 Summary and Conclusions

The purpose of this paper was to show that Einstein's false conclusions obtained in his fundamental papers resulted in over 100 years of confusion in the interpretation of the Lorentz transforms for time. This paper has shown that Einstein's failure to recognize the difference between basis and coordinate transforms is the main error. Unfortunately, textbook writers for the last 100 years have reproduced Einstein's mistakes. One aim was to show that the side by side use of both the basis transformation and coordinate transformation laws should have been seen as a contradiction. But the lack of rigor in relativity obscured this difficulty. It is clear that the same transformation law can not be correct when used to transform frequency and dial readings as Einstein did. The failure to closely examine his false derivation in 1907 is clearly a failure of physics to examine its fundamental ideas with mathematical rigor.