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Einstein's False Interpretation Of Lorentz Transformations

By Harry H. Ricker III email:kc3mx@Yahoo.com

1.0 Introduction

This paper demonstrates that in special relativity the traditional interpretation of the Lorentz transformation equations, which is based on Einstein's fundamental papers, is incorrect. This error arises from Einstein's metaphysical interpretation of "the principle of relativity" which was used without rigor and care in the development of the theory. The uncritical acceptance of Einstein's metaphysical ideas led to them being copied by textbook writers without a critical examination of its mathematical method despite severe criticism and resistance to the acceptance of his ideas. In this paper, we will see that the failure to critically examine the mathematics of special relativity, despite the urging of critics to do so, has resulted in the institutionalization of a mathematically false interpretation of the Lorentz transformation equations.

2.0 The Danger In Metaphysical Speculation

This section demonstrates the elements of Einstein's metaphysical dream at the heart of the special theory of relativity as demonstrated in his fundamental papers. The strength and power of this dream has been hailed as one of the greatest intellectual achievements of the 20th century. However, this paper demonstrates that this metaphysical speculation was not backed up with rigorous mathematical proof and solid logical reasoning in physics. The insufficiently formed ideas of measurement of length and time in physics were exploited to create the false impression of an elegantly correct theory. But, a metaphysically elegant theory does not always translate into a correct physical theory as will be shown is this paper.

The fundamental flaw in Einstein's method of approach is that no where in the fundamental papers does he give a rigorous mathematical proof that proves his metaphysical speculation is consistent with the mathematical theory of relativity. Furthermore, there has never been an experiment performed that demonstrates that this speculation is physically correct. These two difficulties have generally been circumvented by avoiding the issue entirely; despite the arguments of critics demanding that this problem be addressed.

The metaphysical dream at the heart of Einstein's theory is his idea that the principle of relativity, i.e., covariance of reference frames, means that all relatively moving inertial frames are equivalent in the sense that the laws of physics apply equally for all of them. This is a beautiful and wonderful idea for a natural philosopher, although it may seem unnecessarily vague to an inductive experimenter in the Baconian tradition like Faraday.

An experimental as opposed to a theoretical physicist demands that the principle be derived from experiments and rendered general by induction as taught of Newton. But Einstein's physics is a new style of scientific philosophy, which insists that physical theory should be based on pure speculative thought and then rendered general by deduction.

In his 1905 paper, Einstein introduces the postulate of the principle of relativity as follows. "Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the "light medium," suggest that the phenomena of electrodynamics as well as mechanics posses no properties corresponding to the idea of absolute rest. They suggest rather…the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate…" Although this does fit the form that requires that principles be derived from experiments, then be rendered general by induction, Einstein gives no detailed analysis of experiments. It seems clear that the postulate is merely a speculative guess that is suggested by experiments. The real problem is that the principle is so vague, that there is no obvious way that it appears to apply to the physical theory that is developed based upon it.

The vagueness of the 1905 version of the Principle of Relativity is illustrated by the following definition of the principle given in Section 2, part I: "The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform translatory motion." The principle is then used to require that the length of a rod in a moving frame is the same as the length in a stationary frame: "In accordance with the principle of relativity the length to be discovered by the operation (a)---we will call it "the length of the rod in the moving system"-- must be equal to the length L of the stationary rod". This use of the principle leads to a famous contradictory conclusion. Having assumed the lengths are the same he demonstrates that they are different. (That has been definitively shown to be an incorrect conclusion in a previous paper.) Here we ask: What law of physics is involved in ordaining that the two lengths must be equal? We are not dealing in laws of physics, but simply in measurement of physical quantities. Hence, a different interpretation that leads to false conclusions has been introduced under the guise of the postulate of relativity. Einstein seems to think that length is a physical state, and that by a subtle change in the principle he claims that length is unaffected by motion. But, he then proceeds to contradict this by showing that the lengths are different. Which claim is the true one?

3.0 Mathematical Implications Of Principle Of Relativity

Much of the vagueness of the 1905 paper is corrected in 1907, where a more comprehensive introduction and explication of the principle is given. An important clarification of the mathematical application of the principle, is given in the first paragraph of section 3, Part I: Let S and S' be equivalent reference systems, i.e., these systems shall have unit measuring rods of the same length and clocks running at the same rate when these objects are compared with each other in a state of relative rest. It is then obvious that all physical laws that hold with respect to S will hold in exactly the same form for S' too, if S and S' are at rest relative to each other. The principle of relativity requires such total equivalence also if S' is in uniform translational motion with respect to S. Hence, specifically, the velocity of light in vacuum must have the same numerical value with respect to both systems." it is this statement which is really the foundation of the theory of relativity. It justifies that the relatively moving coordinate systems are physically equivalent, and that the velocity of light is the same in both of them. Unfortunately, both of these assumptions are contradicted by the Lorentz transformation equations, which are apparently derived from these two assumptions. Certainly this is a major contradiction fatal to the theory.

Using the principle of relativity the inverse Lorentz transformations are introduced as follows in the 1907 paper: "In general, according to the principle of relativity each correct relation between "primed" (defined with respect to S") quantities or between quantities of only one of these kinds yields again a correct relation if the unprimed symbols are replaced by the corresponding primed symbols, or vice versa, and if v is replaced by -v." In his 1910 paper this is interpreted in terms of a rotation of coordinate systems S and S', so that they can be viewed as inverses in terms of a rotation group. Einstein says: "By combining the transformation equations with the equations expressing the rotation of one system with respect to another one, we can obtain the most general transformations of coordinates." In this paper this idea, that the Lorentz and inverse Lorentz transformations are inverse elements of a rotation group, will be demonstrated as inconsistent with the solutions to the Lorentz transformation equations applied to time and space.

4.0 Later Textbook Interpretations

Here we will briefly examine the much simpler interpretations presented by textbook writers that have the advantage that they give more precise and succinct statements of the principles presented in Einstein's papers. In "Introduction To Theoretical Physics", third edition 1952, Leigh Page defines the principle of relativity this way: "the laws of physics are the same when determined relative to one inertial system as when determined relative to any other." He clarifies this by noting that the new or unique aspect of this is the application of it to the laws of electromagnetism and optics. He then shows that the speed of light "has the same constant value c in systems S and S'," by writing down the famous spherical light wave equations diverging from the origins of S and S'.

(1) $x^2+y^2+z^2- c^2t^2=0$

(2) $x'^2+y'^2+z'^2- c^2t'^2=0$

Here the principle of relativity establishes that the law of light propagation is of the same form in both S and S' and the constancy of light velocity postulate is used to justify that the constant of light velocity c has the same numerical value in the same units of measure in both frames. These equations form the basis of the theory of relativity and we will see that the interpretation of the Lorentz transform equations is inconsistent with their mathematics and correct physical interpretation. (This is not new, critics have been saying this for nearly 100 years.)

Page gives the following interpretation of the solution of the Lorentz transforms for length contraction as follows: "According to the relativity principle the dimensions in the direction of motion of material objects moving relative to the observer suffer… contraction ... The effect under discussion is reciprocal." A similar result is deduced for time: "therefore, the interval of time which has elapsed in S is greater than that which has elapsed in S'. Observers in S conclude, then, that the moving clock is running slow. Again the effect is reciprocal." Both of these conclusions result in famous paradoxes. For length there is the pole vaulter and barn paradox, while for time there is the twins paradox. These paradoxes arise from the requirement that the effect is reciprocal, which is demanded by the principle of relativity.

5.0 Evaluation Of Lorentz Equations For Space and Time

The mathematical method used here is to first solve the system of Lorentz and inverse Lorentz equations for space and time simultaneously using a specified condition of evaluation. Here the term evaluation is used in the same sense as it is used when a polynomial equation is solved for its roots by setting the equation to equal zero and solving for the indeterminates. The procedure used here is similar. A selected variable is set to zero, and the resulting solutions are obtained. Solutions are obtained by setting one of the following four variables equal to zero, and then solving for the remaining three. The following variables are set equal to zero and the resulting solutions obtained by evaluation: $x=x'=t=t'=0$, each taken in turn.

The Lorentz transformation equations in a simplified form are assumed as follows:
$x'=\beta(x-vt)$ $\quad t'=\beta(t-vx/c^2)$ $\quad x=\beta(x'+vt')$ $\quad t=\beta(t'+vx/c^2)$ $\quad \beta=(1-v^2/c^2)^{-1/2}$
Here there are four equations which express the simultaneous solutions for the transformation of coordinates. These equations are defined in the usual way in terms of two relatively moving reference frames S and S'. Where the origin of frame S' is in motion with velocity v in the positive x direction of S.

Notice that β is greater than unity when v is greater than zero, and that β^{-1} is less than unity when v is greater than zero. An equation of the form $t'=\beta t$ results in a dilation of the variable t' with respect to t because t' is greater than t The equation $t=\beta^{-1}t'$ results in a contraction of the variable t with respect to t' because t is less than t'. The definition of β implies that it is always equal to or greater than unity, and can never be less than unity.

The coordinate frames S and S' are assumed to be orthogonal coordinate systems with the requirement that time is defined such that $t=t'=0$ occurs when the origins coincide; i.e. $x=x'=y=y'=z=z'=0$ at $t=t'=0$. The axes for the x, y, and z directions are assumed to be parallel, and the y and z coordinates are assumed to be identical and coincide when the origins coincide at t=t'=0. The purpose of the solutions is to determine the relations governing the transformation of the x and t coordinates according to the Lorentz transform equations.

5.1 Results for x=0 (Specification of an evaluation at the same place in S)

To consider the role of evaluation in at the same place in S, we determine the simultaneous solution of the four equations when we specify the condition that x=0. The results are as follows:

(3) $x'=\beta(x-vt)=-\beta vt$

(4) $t'=\beta(t-vx/c^2)=\beta t$

(5) $x=\beta(x'+vt')=0$, Therefore $x'=-vt'$

(6) $t=\beta(t'+vx'/c^2)=\beta t'(1-v^2/c^2)=\beta^{-1}t'$

Notice that equation 6 is the inverse of equation 4, so they are the same solution. Equation 6 is solved by substitution with the result from equation 5. Therefore, from equations 4 and 6 we have the following solution for the condition x=0: $t'=\beta t$. The solutions for equations 3 and 5 give the results $x'=-\beta vt=-vt'$, from which we conclude that $t'=\beta t$. A result which is the same as obtained from equation 4. Henceforth, we will take the solution to be equation 4, when the evaluation condition x=0 is imposed. All of the results given here lead to the conclusion that time in frame S' is dilated relative to frame S. This result is confirmed by equations 3 and 5.

5.2 Results for x'=0 (Specification of an evaluation at the same place in S')

To consider the role of evaluation at the same place in S', we determine the simultaneous solution of the four equations when we specify the condition that x'=0. The results are obtained as follows:

(7) $x'=\beta(x-vt)=0$, Hence $x=vt$

(8) $t'=\beta(t-vx/c^2)=\beta t(1-v^2/c^2)=\beta^{-1}t$

(9) $x=\beta(x'+vt')=\beta vt'$

(10) $t=\beta(t'+vx'/c^2)=\beta t'$.

Notice that equation 8 is the inverse of equation 10, so they are the same solution. Equation 8 is solved by substitution with the result from equation 7. Therefore, from equations 8 and 10 we have the following solution for the condition x'=0: $t=\beta t'$. The solutions for equations 7 and 9 give the results $x=vt=\beta vt'$, from which we conclude that $t=\beta t'$. A result which is the same as obtained from equation 10 which is the primary result for the condition x'=0. This equation is the traditional result for time dilation, with 8 being the result of Einstein obtained in 1905. Notice that all these equations are symmetrical reciprocals, by exchange of primed and unprimed symbols, of the equations in section 5.1 and vice versa.

5.3 Results for t=0 (Specification of an evaluation at the same time in S)

To complete the analysis of evaluation, we now consider the role of evaluation at the same time in S. We determine the simultaneous solution of the four equations when we specify the condition that t=0. The results are as follows:

(11) $x'=\beta(x-vt)=\beta x$

(12) $t'=\beta(t-vx/c^2)= -\beta vx/c^2$

(13) $x=\beta(x'+vt')=\beta x'(1-v^{2/}c^2)=\beta^{-1}x'$

(14) $t=\beta(t'+vx'/c^2)=0$, therefore $t'=-vx'/c^{2.}$

Notice that equation 13 is the inverse of equation 11, so they are the same solution. Equation 15 is solved by substitution with the result from equation 14. From equations 11 and 13, we have the following solution for the condition that t=0: $x'=\beta x$. The solutions for equations 12 and 14 give the results $t'=-\beta vx/c^2=-vx'/c^2$ from which we conclude that $x'=\beta x$. A result which is the same as obtained from equation 11 which is the primary result for the condition t=0. Henceforth, we will take equation 11 as the solution instead of the usual interpretation which takes equation 13 as the solution showing that measurements obtained in S are contracted.

5.4 Results for t'=0 (Specification of an evaluation at the same time in S')

To consider the role of evaluation with the opposite condition, we determine the simultaneous solution of the four equations when we specify the condition that t'=0. The results are as follows:

(15) $x'=\beta(x-vt)=\beta x(1-v^{2/}c^2)=\beta^{-1}x$

(16) $t'=\beta(t-vx/c^2)=0$, therefore $t= vx/c^2$.

(17) $x=\beta(x'+vt')=\beta x'$

(18) $t=\beta(t'+vx'/c^2)= \beta vx'/c^2$.

Notice that equation 15 is the inverse of equation 17, so they are the same solution. Equation 15 is solved by substitution with the result from equation 16. From equations 15 and 17 we have the following solution for the condition that t'=0: $x=\beta x'$. The solutions for equations 16 and 18 give the results $t=\beta vx'/c^2=vx/c^2$ from which we conclude that $x=\beta x'$. A result which is the same as obtained from equation 17 which is the primary result for the condition t'=0. Henceforth, we will take this equation as the solution. It is a dilation of space, and not a contraction. It is clear that length in S is longer than in S' from the results of equations 16 and 18. Notice that all these equations are symmetrical reciprocals, by exchange of primed for unprimed symbols, of the equations in section 5.3 and vice versa.

6.0 Interpretation of Evaluation Solutions For Lorentz Transform Equations

In the traditional interpretation of relativity, equations 4 and 10 are considered to prove that moving clocks run slow and equations 13 and 15 that space or length contracts. This interpretation depends on the assumptions, justified by the principle of relativity, that equations 4 and 8 should be t'=t and t=t' instead of the actual results obtained. The principle also is used to justify the result that equations 11 and 15 should be x'=x and x=x' instead of the results actually obtained. Hence, the traditional interpretation which assumes, based on the principle of relativity, that a clock calibrated in S can be placed in frame S' without a change in its rate is false. Furthermore, the assumption that a rod whose length in S is one unit has when in motion at rest in S' a length of one unit is also false. This result is critically important. It is clear that the assumptions used to prove the results of special relativity is contradicted by the above solutions of the Lorentz transform equations. Hence, the theory based on the traditional interpretation is false, because it is based on an incorrect method of solution of the Lorentz transform equations. This result is proved by the correct solutions given in section 5.0.

6.1 First Contradiction Theorem

The following theorem shows that the traditional interpretation that there are no contradictions in the theory of relativity is false. The contradiction arises from the symmetry requirement derived from the principle of relativity.

Theorem: If the same standard of measure is valid in frames S and S', then either the solutions with evaluation in S are true, or the solutions with evaluation in S' are true, but solutions resulting from both evaluations; i.e. S and S' together, taken as simultaneously true, are contradictory.

Proof: Suppose both solution sets in section 4.0 are true and represent valid solutions. Then, for every true statement of the form A=L(B) obtained using an evaluation in S (S'), there exists a corresponding true statement of the form B=L(A) obtained using an evaluation in S' (S). Here the function L denotes a Lorentz transform, or an inverse transform, evaluated in S (S'). Both of these statements cannot be true simultaneously, so there is a contradiction.

In these statements, the objects A and B have the same meaning and are the same symbols in both statements. The functional relation can be expressed either as greater than or less than depending on the actual function. For $A=\beta B$, since $\beta>1$, we have $A>B$. For $A=\beta^{-1}B$, since $\beta^{-1}<1$, we have $A<B$. The procedure for forming statements with the opposite evaluation is Einstein's procedure for forming the inverse Lorentz transform. Given any valid statement, the true statement for the opposite evaluation is formed by exchanging symbols; i.e. replace A with B and B with A. The statements formed in this way are contradictory to the original statements. Hence the two statements taken together form a contradictory pair. Therefore, if we claim that only statements resulting from evaluation in S (S') are true, there is no contradiction because we no longer claim statements using the opposite evaluation S' (S) are true. QED

To make this clear consider the following example using equation 2 obtained by evaluation in S. We have for the condition x=0: $t'=\beta t$. Hence $t'>t$. By the procedure of exchanging symbols we have the result given in equation 8 obtained by evaluation in S', for the condition x'=0. Therefore, we have for the condition x'=0: $t=\beta t'$. Hence, $t>t'$or t’<t. (When we evaluate with x=0 and in the other case, when we evaluate with x’=0.) These results both taken as true are contradictory since they imply that t’ is both greater than and less than t simultaneously. So both statements can not both be true simultaneously, otherwise we have a contradiction.

Alternate Proof by Enumeration of Solutions: We enumerate the contradictory equations for transformation of time as equations 2 and 8. Section 4.5 indicates these are the primary solutions for time, the secondary solutions are equations 4 and 6. We show that equations 2 and 8 are contradictory as follows. Solving equation 2 we have $\beta=t'/t$ and solving equation 8 we have $\beta=t/t'$. Hence $t'/t=t/t'$ or $(t')^2=t^2$. Therefore t=t’, which contradicts both equations 2 and 8. By similar procedures contradictions are obtained for equations 4 and 6 the secondary solutions for time. For the primary solutions for transformation of space equations 9 and 15 there is a contradiction and for the secondary solutions equations 11 and 13 there is a contradiction. The reader can easily prove the contradiction using the method given above for equations 2 and 8. Hence, for every solution obtained with the condition x=0, there is a corresponding solution obtained with the condition x’=0 which contradicts it. Hence, both solutions can not be true at the same time. However, taken separately, either of the two sets of solutions is true, as long as we reject the second solution as false and do not use it. The same procedure when applied to the two sets of equations in paragraphs 4.3 and 4.4, lead to similar contradictions and to the conclusion that either the solutions of 4.3 are true or 4.4 are true but that for each solution in 4.3 there is a corresponding solution in 4.4 which contradicts it. Therefore, for every solution in paragraph 4.5 there is a solution obtained with the opposite evaluation, which contradicts it. QED

The meaning of this theorem is that the traditional reciprocal or inverse solutions can not both be simultaneously true solutions when the symbols used have the same meaning in the solutions of 5.1 and 5.2, or in the solutions of 5.3 and 5.4. However, we will see in the next section that by giving these symbols different physical interpretations the difficulty is eliminated. Hence in the following, the symbols used in 5.1 will not have the same meaning as those used in 5.2, and those used in 5.3 will not have the same meaning as those of 5.4. This change will be to interpret the equations as basis transformations in sections 5.1 and 5.3, and coordinate transformations in sections 5.2 and 5.4.

6.2 Explanation Of Correct Interpretation Of Transform Equations

Traditionally equations 4 and 10 have been interpreted as the reciprocal or inverse equations (in a rotation group) for time dilation of moving clocks as viewed from relatively moving reference frames. But using them results in the twins paradox, and violates the above theorem, which shows that the contradiction can not be successfully resolved within the traditional interpretation. Here it will be shown that these equations can be given a mathematically consistent interpretation by interpreting these equations as specifying two different types of transformation laws. We will interpret the first equation as a basis change law which defines how the unit of time measure in S changes when transformed into S’. The second equation will be interpreted as the coordinate transformation law which defines how the clock dial measurement of time changes when transformed from S’ into S.

We begin by noting that the laws for clock dial readings do not agree with the laws for the transformation of clock frequency, if we interpret the symbols as having the same meaning. Taking equation 10 and converting to frequency we obtain the following: $t=\beta t'$, and converting to frequency we have $1/f=\beta 1/f'$, so that $f'=\beta f$. This result indicates that the clock in S’ is running fast, because it has a higher frequency than the clock in S. However, if we take the law as given by equation 4, we obtain the correct relation as follows: $t'=\beta t$, $1/f'=\beta 1/f$, so we have $f=\beta f'$, hence $f'=\beta^{-1}f$. Therefore, the frequency of the moving clock in the S’ frame is slow as proven in experiments. We see from this that we can not interpret the two laws given by equations 4 and 10 in the same way.

Lets compare dial readings. The law in equation 10, the traditional result, demonstrates that when the dial on the moving clock in S’ reads one time unit, the dial on the clock in S reads β time units. Hence, the moving S’ clock is slow relative to the S clock. Now the law in equation 4 does not give the same result. It says that when the dial in S reads one time unit, the dial in S’ reads β time units, from which we conclude that the clock in S’ is fast. Therefore, when we interpret the equations as representing two different kinds of transformation laws, we can achieve a consistent interpretation of Lorentz transforms. But if we interpret them as having the same meaning in both equations, the result is a contradiction.

Traditionally, equations 13 and 15 have been interpreted as reciprocal or inverse transformations (in a rotation group) of length measurement. But this interpretation leads to a contradiction when we take the symbols used in both equations to have the same meaning. However, this contradiction can be avoided by taking 13 as the basis transformation law from S’ into S, and taking equation 15 as the coordinate transformation law going from S’ into S. Hence the symbols in these equations have different meanings and there is no contradiction. The correct interpretation is as follows. Equations 11 and 13 specify the basis transformation laws, and equations 15 and 17 specify the coordinate or measurement transformation laws. The basis change laws show that the unit of length measure in S’ is longer than the unit of length measure in S. Hence, when we make measurements of a rod defined at rest in S, using the units of S‘, i.e. the measuring scale defined in S’, we find that the rod measured in S’ units is shorter than the same length measured in S units. Also, a rod defined at rest in S’ units of measure, will have a longer length when measured in S units of measure.

The following results in a consistent interpretation of Lorentz transform equations. Equations 4 and 6 specify the transformation of time units of measure. Equation 4 shows that the time unit defined in S’ is β times longer than the time unit defined in S. Equation 6 shows that the time unit defined in S is β^{-1} times shorter than the time unit defined in S’. Equations 6 and 10 specify that time measurements defined as dial readings transform so that when the clock in S reads one unit of time the clock is S’ reads β^{-1} time units and is therefore, slow. Conversely, when the clock in S’ reads one time unit, the clock in S reads β units.

Equations 11 and 13 specify that the units of length measure, the distance scale, so that the length unit defined in S' is longer than the unit defined in S. Conversely, the length unit defined in S is shorter than the length unit defined is S'. Equations 15 and 16 specify that the reported measurement obtained in S' units is less than the same measurement reported in S units. Conversely, the measurement of a length reported in S units of measure is greater than the same measurement reported in S' units.

7.0 Summary of Results and Conclusions

The traditional derivation of the Lorentz transformations is based on the principle of relativity and the numerical equality of the light constant in two relatively moving reference frames. Notice the more precise statement of the second requirement, relative to the usual statement. The results presented here indicate that although the light constant is numerically the same in frames S and S', these frames do not have the same units of measure for time and distance in both frames, as is assumed in the traditional theory of relativity. Here it has definitely been shown that the assumption of identical units of measure in frames S and S' is inconsistent with the solutions of the Lorentz transform equations. The first contradictory theorem in section 6.1 proves this. Hence, the traditional interpretation which assumes the units are the same in S and S' is false.

The revised interpretation avoids the paradoxes of relativity by assigning different meanings to the symbols so that the equations do not contradict each other. The new interpretation, which is contradiction free, interprets the contradictory equations as basis transformation and coordinate transformation laws. These are reciprocally related laws of transformation the same as those used in the transformation of vector spaces.