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Einstein's False Derivation Of Lorentz-Fitzgerald Contraction

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1.0 Introduction

This paper demonstrates that the traditional derivation of the Lorentz-FitzGerald contraction formula, which is based on Einstein's method, is incorrect. This error arises from Einstein's first paper of 1905, which has been copied by textbook writers without a critical examination of its mathematical method.

2.0 Einstein's Method

This section examines Einstein's 1905 method of proof of the Lorentz contraction. The proof consists of the following steps. The first step which appears in section 1 of part I, is the definition of simultaneity. A procedure is defined to establish a synchronization of clocks. Then in step 2, which appears in section 2 of part I, two definitions of length are given. One, in (a), defines "the length of the rod in the moving system", while the other in (b), defines "the length of the (moving) rod in the stationary system". He then declares that "we shall determine on the basis of our two principles" that the length defined in (a) is different from that defined in (b). Or that the length of a stationary rod is different from that of a moving rod.

The crucial part of the procedure is contained in step 3 which involves an assumption justified as follows. "In accordance with the principle of relativity the length to be discovered by the operation (a)…must be equal to the length of the stationary rod." The meaning of this does not become clear until we arrive at the actual procedure which shows that the two lengths defined in (a) and (b) are different. However, there does not appear to be any reason to accept this claim. It asserts the following. When we define the length of a rod at rest in a stationary system, we can assume by the principle of relativity that this length is the same when the rod is placed in motion and hence can be mathematically described as having the same length at rest as in motion. This leads to an immediate absurdity. Einstein says the lengths are the same then says that they are different.

This contradiction deserves amplification. The assumption is this. The length measure of the rod defined at rest in the stationary system can be transferred into the length measure at rest in the moving system. Then it is claimed that when the rod is measured in motion from the stationary system while in motion, the measured moving length is different from the measured length obtained by an observer co-moving with the rod. The acceptance of this contradiction was partly justified by the Michelson-Morley experiment which could be explained by a Lorentz-FitzGerald contraction of length in the direction of motion. So the idea that a moving rod contracted was not a physically absurd idea.

The fourth step involved the derivation of the Lorentz transformation equations in section 3 of part I. This was then followed by the mathematical proof in section 4 that lengths defined at rest in the moving system "appear modified by the motion" when measured in the stationary system.

The mathematical derivation of the Lorentz-FizGerald contraction proceeds as follows. By invoking the principle of relativity, as given in step 3, Einstein says "We envisage a rigid sphere of radius R, at rest relatively to the moving system". Here it is assumed that a rigid sphere defined at rest in k is the same physically as if the same sphere were at rest in the stationary system K. The coordinates defined in the system k are equated to those in system K using the Lorentz transformation equations of section 3, part I. Hence, we have:

(1) $\xi^2=\beta^2(x-vt)^2$ where $\beta=1/(1-v^2/c^2)^{-½}$

The assumption that the length of the moving rod is measured in the stationary frame by measuring the ends of the rod simultaneously at t=0 in terms of stationary frame time, i.e., time is simultaneous in the stationary frame, is invoked to obtain the result:

(2) $\xi^2=\beta^2x^2$

Now solving this equation for the measure of length in frame K in terms of the length defined in frame k, gives the result:

(3) $\xi=\beta x$

Which is solved for x by inversion (going backwards) to obtain:

(4) $x=\beta^{-1}\xi$

This is interpreted to mean that the measured length in the stationary frame is contracted relative to the rest length defined in the moving frame. In the 1907 and 1910 papers the method of proof was essentially the same with a simplification of the mathematical method. In the 1910 paper he added the following: "One recognizes at once in these equations the hypothesis of Messrs. Lorentz and FitzGerald. This is the hypothesis that looked so strange to us and that had to be introduced to explain the negative results of the experiment of Michelson and Morley. Hence this hypothesis appears naturally as an immediate consequence of the principles assumed."

3.0 Later Relativity Textbook Interpretations

Textbook writers followed Einstein's method without questioning its rigor and mathematical logic. Their contribution was to simplify the procedure without essentially changing its basic method. However, the clear statement of the use of the principle of relativity to justify the assumption that the length of a rod defined at rest in a non-moving frame was the same as one defined by a co-moving observer in a moving frame was usually absent and the transformation became mysterious. An example is the following taken from Rindler's "Essential Relativity," second edition page 40.

"Consider two inertial frames S and S' in standard configuration. In S' let a rigid rod of length $\Delta x'$ be placed at rest along the x' axis. We wish to find its length in S, relative to which it moves longitudinally, its end points must be observed simultaneously. Consider, therefore, two events occurring simultaneously at the extremes of the rod in S, and use (2.8)(i) [the Lorentz transform]. Since $\Delta t=0$, we have $\Delta x'=\gamma\Delta x$, or, writing for Δx, $\Delta x'$ the more specific symbols L, L_0, respectively,

(5) $L=L_0/\gamma=(1-v^2/c^2)^{1/2}L_0$

This shows, quite generally, that *the length of a body in the direction of its motion with uniform velocity v is reduced by a factor $(1-v^2/c^2)^{1/2}$."*

The questionable mathematical step in this proof is the inversion of the equation $\Delta x'=\gamma\Delta x$, to obtain $\Delta x=\gamma^{-1}\Delta x'$. This means we go backwards from S' into S, after exchanging frames S and S' in the first part. The substitution of L_0 for $\Delta x'$ is also a problem but is justified if we accept the claim that this is justified by the principle of relativity. However, Rindler neglects to mention this fact. Essentially there are two difficulties in accepting this proof, however the last step of inversion is the one which lacks justification and does not seem a valid procedure. In a following section we will see that this is the case, after we examine the correct method to be used.

4.0 Correct Mathematical Derivation

Here we do not make the assumption based on the principle of relativity that the standard of length measure is the same at rest in S' as it is in S. The mathematical method used here is to first solve the system of Lorentz and inverse Lorentz equations for space and time simultaneously using a specified condition of evaluation. Here the term evaluation is used in the same sense as it is used when a polynomial equation is solved for its roots by setting the equation to equal zero and solving for the indeterminates. The procedure used here is similar. A selected variable is set to zero, and the resulting solutions are obtained. Solutions are obtained by setting one of the following four variables equal to zero, and then solving for the remaining three. The following variables are set equal to zero and the resulting solutions obtained by evaluation: $x=x'=t=t'=0$, each taken in turn.

The Lorentz transformation equations in a simplified form are assumed as follows:

(6) $x'=\beta(x-vt)$ (7) $t'=\beta(t-vx/c^2)$ (8) $x=\beta(x'+vt')$ (9) $t=\beta(t'+vx/c^2)$
(10) $\beta=(1-v^2/c^2)^{-1/2}$

Here there are four equations which express the simultaneous solutions for the transformation of coordinates. These equations are defined in the usual way in terms of two relatively moving reference frames S and S'. Where the origin of frame S' is in motion with velocity v in the positive x direction of S.

Notice that β is greater than unity when v is greater than zero, and that β^{-1} is less than unity when v is greater than zero. An equation of the form $t'=\beta t$ results in a dilation of the variable t' with respect to t because t' is greater than t The equation $t=\beta^{-1}t'$ results in a contraction of the variable t with respect to t' because t is less than t'. The definition of β implies that it is always equal to or greater than unity, and can never be less than unity.

The coordinate frames S and S' are assumed to be orthogonal coordinate systems with the requirement that time is defined such that $t=t'=0$ occurs when the origins coincide; i.e. $x=x'=y=y'=z=z'=0$ at $t=t'=0$. The axes for the x, y, and z directions are assumed to be parallel, and the y and z coordinates are assumed to be identical and coincide when the origins coincide at $t=t'=0$. The purpose of the solutions is to determine the relations governing the transformation of the x and t coordinates according to the Lorentz transform equations.

We determine the simultaneous solution of the four equations when we specify the condition that t=0. The results are as follows:

(11) $x'=\beta(x-vt)=\beta x$
(12) $t'=\beta(t-vx/c^2)= -\beta vx/c^2$
(13) $x=\beta(x'+vt')=\beta x'(1-v^{2/}c^2)=\beta^{-1}x'$
(14) $t=\beta(t'+vx'/c^2)=0$, therefore $t'=-vx'/c^{2.}$

Notice that (11) is the same as equation (3) and that (13) is the same as equations (4) and (5). But to obtain equations (4) and (5) we did not invert equation (3). Instead a different procedure was used. With t=0, equation (9) was solved to obtain equation (14). This was substituted into equation (13) to obtain the result. But now we have a problem. We have two possible solutions. Equation (11) and equation (13). If we make the substitution which Rindler uses, i.e., setting $x'=L_0$ and $x=L$, the result is a contradiction, because we obtain $L_0>L$ from (11) which contradicts the traditional result $L<L_0$ given by (13). Hence, we are unable to decide if the correct result is the dilation (11) or the contraction (13). This problem will be resolved in the following sections.

5.0 Proof That Einstein's Method Is Incorrect

It is well known that Einstein's method produces a contradiction. The contradiction is one of the most celebrated results of the special theory of relativity. The contradiction arises as follows. A rod whose length measure is defined as L_0 in the stationary frame is placed in motion so that is has velocity v relative to that frame. We call the stationary frame S and the moving frame S' as above. (In the 1905 paper these frames are designated K and k respectively. In later papers the notation is the same as used here.) Einstein claims, as discussed above, that the rest length in frame S' is the same as in S. So using primes to denote the different rest lengths, we have that L_0'=L_0. This implies the transformation law is x'=x. This law is contradicted by the result which is deduced in equations (4) and (5), i.e., $L_0=\beta^{-1}L_0$'. Therefore, there is a contradiction. This is clearly shown by the fact that Einstein's assumption of step 3 is contradicted by equation (11). Furthermore, it is clear that the famous result of Lorentz contraction is contradicted by the use of the correct method of section 4.0.

The reason that Einstein's method is false is that he encounters a major mathematical difficulty which he avoids by a clever manipulation, which however is an illegitimate mathematical procedure. The root problem is the fundamental assumption that length is defined by simultaneous measure of both ends of the moving rod. To see the problem consider the Lorentz transform equations (6) and (7). These equations give the codomain coordinates as functions of the domain coordinates. The codomain coordinates are written of the left side and the domain coordinates are on the right. Looking at these we see that if we are to perform a simultaneous measure of the transformed space coordinates, i.e., the x coordinates at times defined by codomain coordinates, i.e., the t' coordinates, there is a problem because the transformation equation is not a function of the t' coordinates that we can set to be equal or simultaneous.

However, we can measure the codomain length in terms of the simultaneous measure of the ends of the rod in S by setting t=0. Hence we have that transformation of length is defined in the domain while measure of length of the codomain can not be defined in terms of the Lorentz transform equations. The Einstein method attempts to circumvent this difficulty by exchanging the domain and codomain by transferring from the S to the S' frame making the S' frame a rest frame. The transformation is solved going backwards and then the domain and codomain are exchanged or reversed again to obtain the desired result.

The proof that this procedure is false is the correct proof given above. In the correct method the problem is resolved differently by solving for the simultaneous t coordinates as functions of the t' coordinates. The solution is equation 14, which is substituted into 13 to obtain the transformation of length from S' into S.

6.0 New Interpretation Of Transformation Equations

Rindler gives the traditional interpretation of the Lorentz-FitzGerald contraction. He says that the contraction is real. On the other hand in 1905, Einstein seemed to say that the effect was merely apparent, a result of perspective caused by viewing a moving rod from a relatively moving frame. In later papers he was sufficiently vague about the interpretation. Then in 1911, there was a famous exchange in which Einstein responded to a publication by V. Varicak. Varicak published a short article in which he asserted that, "In the present case the center of the controversy was the question: Is the Lorentz contraction a genuine or only an apparent phenomenon? In relativity, the Lorentz contraction comes about purely as a perspective effect, because for an observer at rest the points of a moving body appear as simultaneous when in fact they are not simultaneous in the system in which the body is at rest. Einstein objects to the statement that ‘therefore the Lorentz contraction is not a physically objective phenomenon’.” Historians agree that Einstein’s intervention in the controversy settled it in his favor when he responded to Varicak with a short note in which he stated that ”The question of whether the Lorentz contraction does or does not exist in reality is misleading. It does not exist ‘in reality’, i.e., in such a way that, in principle, it could be detected by physical means, for a co-moving observer. This is just what Ehrenfest made clear in such an elegant way.” Thus he avoided a clear pronouncement which failed to clear up the problem and generated much debate and confusion which has persisted until today.

Examination of the equations of section 4.0 shows that there are two equations (11) and (13) which govern transformation of length measure between frames S and S’. The law given in (11) transforms length from S into S’, and the law (13) transforms from S’ into S. These transformations are bijective since taking (13) we substitute (11) and obtain

(15) $x=\beta^{-1}x'=\beta^{-1}\beta x=x$

A similar result is obtained by substituting (13) into (11).

(16) $x'=\beta x=x'=\beta\beta^{-1}x'$

Thus it is proved that the transformations are bijective.

The traditional relativist is sure to object to these conclusions by using the argument that they were obtained with the condition that t=0, or that simultaneity is defined in frame S, only and that this analysis has failed to take into account that measurements must be simultaneous in S’, or that we have the condition t’=0. Hence, lets consider the solution for the Lorentz equations for the condition t’=0 using the above method. The results are as follows:

(17) $x'=\beta(x-vt)=\beta x(1-v^{2/}c^2)=\beta^{-1}x$
(18) $t'=\beta(t-vx/c^2)=0$, therefore $t= vx/c^2$.
(19) $x=\beta(x'+vt')=\beta x'$
(20) $t=\beta(t'+vx'/c^2)= \beta vx'/c^2$.

In this case the solution of equation (18) is substituted into (17). Equations (17) and (19) form a bijective transformation pair as was demonstrated for (11) and (13) above. Notice that in the traditional interpretation equations (13) and (17) are taken as the reciprocal solution set. However, these solutions are contradictory and result famous in paradoxes of relativity. The solution to the paradox problem is to apply different physical interpretations to the equation pairs (13) and (17) and (11) and (19). We interpret (11) and (13) as the basis transformation laws and the pair (19) and (17) as the coordinate transformation laws. The two pairs of laws are defined under different conditions of simultaneity; t=0 for the first pair and t’=0 for the second. This removes the contradictions which result in traditional relativity. Furthermore, since the basis or standards of length change inversely to the coordinates of measure the actual physical length of rigid rods does not change. What changes are the basis of measure or measure units and the number of measure units (coordinates) in which the measurement results are reported. This is discussed in more detail in my paper “Light Velocity is a Ratio, It Can Not Be Absolute.”

7.0 Review Summary and Discussion

This paper has shown that the traditional derivation of the Lorentz-FitzGerald contraction is false. The error arises from Einstein’s requirement that the length of a relatively moving rigid rod is to be measured simultaneously in a rest frame which has synchronized clocks. This requires that the transformation equations be functions of codomain time rather than domain time. This difficulty was circumvented by a procedure that used the questionable assumption that a rest frame length could be defined in the moving frame the same as in a rest frame and using the Lorentz transformation equations going backwards. This led to the conclusion that the moving length was different from rest length. This conclusion was one of the controversial results of relativity.

In this paper a new method was used that removes the difficulty by formulating the codomain length as a function of domain length and codomain time coordinates. However the solutions derived from the new method do not support the traditional interpretation of the theory of relativity that there is a physical contraction of space.