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# Herbert Dingle Was Correct!

## An Investigation of the First Refutation of Relativity

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## 1.0 Introduction

Herbert Dingle is well known for his claims of inconsistency in Einstein's Special Theory of Relativity. One of the most interesting of Dingle's arguments appeared in the September 8, 1962 issue of Nature under the title "Special Theory of Relativity"[1]. This short note by Herbert Dingle points out "what appears to be an inconsistency in the kinematical part of Einstein's special theory of relativity." Here the thesis is presented that Dingle's modest claim is based upon a mathematically correct derivation of the transformation of time from a moving frame into a rest frame following Einstein's methods. It is concluded that Dingle's assertion of an inconsistency in Einstein's 1905 paper on relativity is correct. Furthermore, in addition to the inconsistency, there are errors in Einstein's 1905 proof that he attempted to rectify in his 1907 and 1910 papers on relativity.

This claim may seem outrageously absurd to a physics community which is convinced of the absolute correctness of Einstein's special theory. The purpose here is to take up Dingle's challenge, and to examine with mathematical rigor the mathematical derivations which lead to the famous conclusion in the special theory that "moving clocks run slow". In taking up Dingle's challenge, the focus will be on Einstein's three fundamental papers which establish the theory[2,3,4]. An examination of the proofs of time dilation which appear in these papers indicates that three different methods are presented. Einstein presented a different proof in every one of the three papers. The final method of proof given in 1910 is completely different from the proof of 1905. The reason for these different proofs is interpreted here as an attempt to correct deficiencies in the 1905 paper in light of Stark's discovery of spectral line shifts of canal rays. The 1907 method was further revised in 1910 which is taken as Einstein's final version of the proof. The argument will be advanced that mistakes and inconsistencies in the 1905 paper contributed to the inconsistency arguments and confusion which has accompanied this subject for nearly 100 years.

In the final analysis, it is determined that Dingle's argument fails to prove that moving clocks run fast. Instead, his proof correctly demonstrates that clocks at rest in the moving frame run slow compared to clocks at rest in the rest frame. But, Dingle's assertion that moving clocks run fast according to Einstein's proofs is correct. Because of mathematical errors, Einstein incorrectly derives the transformation equations from a moving frame into a rest frame. He misinterprets them as indicating that moving clocks run slow, instead of the correct result, which is that they run fast. The correct conclusion which is given here is that moving clocks run slow, however, proofs based upon Einstein's 1905 method are incorrect because the equations used actually indicate that moving clocks run fast.

## 2.0 Objective

The main objective of this paper is to take up Dingle's challenge to examine Einstein's proofs of time dilation and resolve the inconsistency that Dingle has discovered. This analysis has revealed mistakes in Einstein's method of analysis that will be discussed and explained. This paper will demonstrate that Dingle's criticism of Einstein's 1905 derivation of time dilation is valid and that his 1905 paper contains mathematical errors. In the 1907 and 1910 papers, Einstein attempted to rectify the errors of the 1905 paper by presenting revised proofs. Einstein was not satisfied with the 1907 result and presented a further revision in 1910. The main purpose of this paper is to demonstrate that Dingle's 1962 demonstration of an inconsistency in special relativity is an attempt to point out the mistakes in Einstein's 1905 paper. While Dingle does not assert this, it is shown that his solution is the mathematically correct derivation which Einstein should have obtained.

## 3.0 Approach

The thesis advanced in this paper is that Dingle's demonstration of an inconsistency in special relativity was a low key, or quiet urging to the physics community, to examine Einstein's 1905 derivation of time dilation. Forewarned of the hostility to his views, Dingle did not assert that Einstein's 1905 presentation was wrong, he merely pointed to an inconsistently, which he hoped would prod an examination of Einstein's method and conclusions. The means by which this was to be achieved was to compare Einstein's incorrect statements with his own correct statements.

The essence of Dingle's argument is that based upon the mathematical method of Einstein's 1905 paper, the correct conclusion is that moving clocks run fast. However, Einstein reached the opposite conclusion that moving clocks run slow. Dingle shows that if we follow Einstein's 1905 method using the inverse Lorentz transform instead of the Lorentz transform which Einstein uses, then we must conclude that moving clocks run fast. This argument will be analyzed further here.

One result of the analysis is the discovery that Einstein's 1905 paper contains a major blunder. He incorrectly concludes by his method that moving clocks run slow. This error is corrected in the 1907 and 1910 papers by a revision of the method of proof and by deleting all references to the conclusion that moving clocks run slow as a result of their motion. It is argued that Einstein recognized his errors and attempted to rectify them in his 1907 and 1910 papers. A further revision in the 1910 paper attempted to correct the deficiencies of the backward method used previously by substituting the inverse Lorentz transform in place of the Lorentz transform used in 1905 and 1907. However, he continues to conclude that moving clocks run slow. The reason that Einstein continues to insist that moving clocks run slow is a mathematical error that will be discussed.

The analysis method used here presents a comparative analysis of the two derivations and identifies two main mistakes in Einstein's 1905 paper; the incorrect use of the Lorentz transform and the incorrect interpretation of the result. Dingle's presentation corrects these errors while maintaining the other aspects of Einstein's method. His result is therefore interpreted as the correct result that Einstein was unable to obtain. Dingle correctly uses the inverse Lorentz transform and correctly recognizes that the resulting contraction of time interval indicates that moving clocks run fast. The conclusion is that, based on Einstein's principles enunciated in 1905, the correct conclusion should have been that moving clocks run fast, and not slow as Einstein mistakenly concluded.

The evidence to support this argument is divided into two main lines of approach. One is historical. It proceeds by an analysis of Einstein's three papers on special relativity and demonstrates that Einstein recognized that the 1905 derivation was incorrect. But he never recognized that his 1905 conclusion regarding moving clocks running slow might be in error. The second approach investigates the rigor of the mathematical method and points out mathematical errors in the 1905 paper.

**4.0 Background**

Here the brief note presented by Dingle is described[1]. Dingle states that "The alleged inconsistency lies in the fact that the argument used to prove that 'moving clocks run slow' (with which all the kinematical implications of the theory are bound up) proves with exactly the same validity, that moving clocks run fast. Both cannot be right, so the basis of the theory must be faulty." Dingle uses Einstein's notation and copies his exact words in a comparison of Einstein's solution for the transformation of time with Dingle's alternative solution based on the inverse Lorentz transformation.

Dingle's demonstration consists of four parts. The first part defines the reference frames and the coordinates in accordance with Einstein's original terminology. Dingle makes an addition by defining the inverse Lorentz transform which Einstein does not use. The second part presents a statement of the desired solution in Einstein's exact words by quotation. The third part states Einstein's solution as a direct quotation. In the fourth part, Dingle presents his alternate solution, exactly as Einstein presents it with the appropriate changes in mathematical symbols and words to make the necessary corrections.

After demonstrating that the two solutions are inconsistent, Dingle poses the following problem: "...it must be explained not why the two cases are different-that is obvious-but why, consistently with the theory, the former result must be accepted as true while the latter must be rejected as false." He then sent copies of the reprints to relativity experts with the enclosed handwritten remark: "With kindest regards. Test case for the integrity of scientists." This goaded Max Born to write a response which dismissed Dingle's argument with a snide and curt refutation. While Born's argument has been accepted as a correct refutation of Dingle, it has not been generally regarded as completely satisfactory.

Dingle's method and his approach is curious. At first glance, the problem he poses appears to be merely a semantic gambit; not worthy of notice. This is apparently why Born responded by saying: "Dingles objections are just a matter of superficial formulation and confusion." Born attributes the confusion to Dingle and not to the theory which he is being urged to critically examine. Dingle does not assert that Einstein's statements are incorrect. Instead, Dingle merely asserts an inconsistency. But the inconsistency can only be resolved in one of two ways. Admit that Einstein's derivation is wrong, or assert that Dingle's derivation is wrong. Born, and all other commentators, choose the latter course of action.

Dingle's approach was to goad Born and others to critically examine Einstein's solution. He expected them to discover that Einstein's answer was wrong. This is clear from his admonition to explain, not why the answers were different, but to justify why Einstein's answer is considered the correct one. The response was not as required. Born evaded the injunction to explain, and others have asserted simply that Dingle's paradox is a fallacy.

**5.0 Comparison of Dingle And Einstein**

The method used by Dingle, which permits a word-for-word step-by-step comparison of Einstein's solution with the solution by Dingle, is analyzed here. This comparison reveals the following significant differences. First, Dingle uses the inverse Lorentz transformation equations of coordinates from the moving system into the stationary system, while Einstein uses the Lorentz transformation equations. Dingle exchanges the moving and stationary system coordinates where necessary in the solution. Dingle follows every step of the solution exactly as Einstein, so that the principles used in the solution are the same. Dingle reduces the solution in the same manner as Einstein. At the conclusion, Dingle states that the solution indicates that time marked by the clock in the moving system runs fast when viewed from the stationary system, while Einstein says that it runs slow. This summarizes the only differences in the two proofs. So we see that the main differences are the use of the inverse Lorentz transform and the conclusion that the result indicates that moving clocks run fast.

Dingle defines the problem exactly as Einstein; a transformation from the moving frame into the rest frame. Dingle uses the inverse Lorentz transform; Einstein uses the Lorentz transform. Both Dingle and Einstein use the same method of evaluation, but-this is significant-with different conditions. Einstein's condition specifies the coordinate of the clock at rest in the moving frame; $\xi=0$ hence $x=vt$, and Dingle uses the condition that measurement is in the rest frame at $x=0$, hence $\xi=-vt$. Dingle uses the condition appropriate to the inverse Lorentz transformation. Dingle and Einstein obtain the same basic equation with the variables t and $\tau$ exactly reversed as required. Dingle interprets the result as showing that the moving clock runs fast, while Einstein says that it runs slow. We will see that Dingle's solution can be viewed as the correct one. He has exactly corrected all of the mistakes in Einstein 1905 paper, but as noted before, he does not make this claim.

Parsing Dingle's method, we see why his method of solution is correct and Einstein's is in error. The problem is defined as a transformation from the moving frame as domain into the stationary frame as codomain. The first step defines the domain as the moving frame with time coordinate $\tau$ and codomain as coordinate t. Dingle writes the inverse Lorentz transform with the domain coordinates on the right and codomain on the left. Einstein writes a Lorentz transform with domain coordinates on the left and codomain on the right. Dingle performs evaluation in the codomain using a substitution of codomain coordinates on the right. Einstein performs evaluation in the domain using a substitution of domain coordinates. Dingle's result is a transformation with the codomain on the left side expressed as a function of the domain coordinates on the right: $t=\tau\sqrt{(1-v^2/c^2)}$. Einstein's result expresses the domain coordinates as a function of the codomain coordinates: $\tau=t\sqrt{(1-v^2/c^2)}$ . Clearly Einstein's result is in the improper format, because the domain and codomain are reversed.

Dingle's equation expresses time-as indicated by seconds "marked" on the clock in the moving system- as it is measured in the stationary frame in terms of a function relating time in the moving frame to measured time in the stationary frame. Einstein's equation expresses time in the moving frame in terms of time in the rest frame. This is the mistake that has caused nearly 100 years of confusion and controversy. Einstein concludes that the moving clock runs slow. But this seems wrong because the equation which Einstein presents does not support this conclusion. Correctly understood, it states that the rest frame clock runs "slow" when viewed from the moving frame. A result which partly explains the confusion.

Einstein's method of analysis is the following. It is characterized by the peculiar method in which he performs the analysis backwards, going from the moving system into the stationary system. Time is defined in the moving system as being the same as marked by a clock in the stationary system. This is a strange approach, and it is the source of considerable confusion and argument.

The definition appears to establish that the clocks at rest in both frames run at the same rate. But there is sufficient ambiguity to reach the opposite conclusion. Another difficulty is that the analysis proceeds backwards, there does not appear to be any reason why this is required, it seems to only deepen the confusion.

Einstein solves the Lorentz transform -which goes backwards relative to the desired transformation- and obtains an equation for the transformation of time going from the rest frame into the moving frame. He then solves for time in the moving frame when the rest frame clock dial indicates one unit of time. The result is that when the rest clock reads one time unit, the moving clock reads $p(1-v^2/c^2)$ units of time. Since this has a value less than one for any velocity v greater than zero, Einstein concludes that the moving frame clock is slow. Dingle uses the same procedure on his resulting equation. Setting the rest frame time equal to one unit of time, he solves for the time on the moving frame clock as $1/\ p(1-v^2/c^2)$; the inverse of Einstein's result. Hence, the moving clock runs fast in Dingle's solution.

Dingle's challenge is to explain why the solution obtained by Einstein should be considered the correct one. When we consider the strange method of solution-going backwards-, the ambiguous definition of time marked by the moving frame clock, and the awkward method of clock comparison, Dingle's demand that the validity of Einstein's solution be explained is amply justified.

## 6.0 Analysis of Mathematical Method Of Solution

Before we can determine the mistakes or errors in either Einstein's or Dingle's methods of solution, we must have some method or approach to determine what constitutes a correct solution. This seems to consist of at least two basic considerations. A method of clock comparison and a method to implement this comparison of clocks mathematically. The first involves a theory of how we compare the clocks, and the second addresses the mathematical procedure to accomplish the comparison. The theory of clock comparison is addressed first.

## 6.1 Theory Of Clock Comparison In Special Relativity

A clock is generally defined today as a system which produces an output without an input. What this means is that a clock measures time by counting the vibrations or beats of a self sustaining oscillatory system. The type of self sustained oscillation used in 1900 was a mechanically vibrating system which performed repetitive harmonic motions. The specific requirement of the vibrations, which was required to make a good clock, was the exact repetitive nature of the harmonic motion so that the cyclic motion always repeated itself within the same interval of time. This time interval is called the period of oscillation, and the reciprocal of the period of oscillation is called the frequency of oscillation.

Time measurement circa 1900 was performed by counting the oscillations or beats of the clock using a dial mechanism which converts the vibration counts into dial readings, calibrated in terms of measures of time intervals; usually days, hours, minutes and seconds. The beats measure tiny intervals of time, these are counted and related to the dial movement or ticks, so that a certain number of beats corresponds to a standard interval of time. Since the counting of beats is fixed in relation to the dial reading, the main error that effects the time measure is the change in period of the harmonic vibrations.

The method used to display time related the clock beats to time intervals inversely via the dial readings. Clocks are calibrated in terms of time by comparison to a standard of time. This standard is a clock maintained by the government at an astronomical observatory. A clock is said to run slow when it reads time behind the reference standard of time. It runs fast when it reads time ahead of the reference. This process requires that the readings of two clocks be obtained and compared when the standard clock has run for a defined interval of time by its own measure. The terms fast and slow therefore refer to the rate of beats of the clock compared with the reference. Rate is a measure of the frequency of vibration. When the clock runs fast, its vibration frequency is higher than the reference clock frequency, and when it runs slow its frequency is lower than the reference clock frequency. The reverse is true for the time interval or period of the vibrations. The clock runs fast when the time period is smaller or less than the reference time period, and it runs slow when the time period is longer or greater than the reference time period.

The reader should notice the following important result. With the definitions used here, the dial readings of clocks are inversely related to the changes in time interval. (The dial of a slow clock reads less than the reference because its period of oscillation is longer than the reference.) A slow clock reads behind the reference because its period is longer. The clock is slow because there are fewer beats or ticks of the clock during the reference time period. The frequency is lower, and the time required to complete one period of oscillation is longer than the reference. A clock reads fast because more ticks are recorded during the reference time period. The frequency is higher, and the time to complete one period of oscillation is less than the reference period. Hence while the dial readings of clocks are directly related to the clock rate, they are inversely related to the transformation of the time intervals.

When discussing the transformation of clock rate between reference frames, it is not necessary to consider dial readings. We can compare the clocks based upon the transformation of time intervals directly. Because the period is inversely related to clock rate, the direct transformation of time intervals gives the necessary result. A clock runs fast when the transformation contracts the time interval; numerically a smaller measure, and it runs slow when the time interval is dilated; a numerically larger measure. In the following sections, we will see that Einstein makes a serious blunder when he confuses the time as defined by clock dial readings with the transformation of real time intervals.

The procedure for clock comparison is as follows. A reference clock is defined, and it runs for a unit of time by its own measure. The comparison clock is defined and it is started or adjusted so that its start time coincides with the start of the reference clock. The dial readings which represent time intervals are compared at the end of a unit of time run on the reference. But, we can not use this procedure unless we assume that the time that is measured is the same for both clocks. When we are trying to determine if there is an actual change in clock rate due to possible differences in time scale, the comparison of time intervals is preferred. Since the supposed relativistic transformations change the time scale, the preferred method is to directly compare the frequencies of oscillation as the measure of clock rate.

## 6.2 The Theory of Mathematical Transformation

The deceptively simple procedure for clock comparison is complicated by the problem of transformation of time in the theory of relativity. The question arises: What is to be compared when we do the measurement? Mathematically we need to define the symbols that represent the intervals used in the comparison of clock rate measurements. There are two symbols used in the transformations; one for the domain of the transformation, and one for the codomain. But they don't always represent the same concept in terms of clock time. In some cases, the symbols represent the time determined by the reference clock in the rest frame, in others the time generated by the moving clock and measured in the rest frame, and in others the time obtained from the clock situated at rest in the moving frame. The different cases depend on how the Lorentz transform is interpreted. Three specific cases can be identified: projection as a contraction, projection as a dilation, and a change of basis or time scale change. The interpretation of the symbols used is different in each of these cases.

The solution to the problem of transformation of time intervals is based on the use of the Lorentz transform. This is a relation that expresses coordinates defined in what Einstein calls the stationary frame into coordinates of the moving frame. Mathematically it is a relation that transforms the rest frame space-time events taken as the domain into the moving frame space-time events taken as codomain. In order to express the transformation in terms of time coordinates only, a procedure termed "evaluation" is used to eliminate the spatial coordinates from the solution. Einstein does this by substitution of the equation $x=vt$, into the Lorentz transform. Einstein's evaluation is mathematically equivalent to evaluation by solving the Lorentz transform for space-time events located at the coordinate $x'=0$ in the codomain or the moving frame. The resulting equation is then reduced to its simplest form. Einstein obtained the result: $\tau=t\sqrt{(1-v^2/c^2)}$ . We see that this equation expresses the moving frame coordinates in terms of the rest frame coordinates. But, the solution requested the reverse of this; rest frame time in terms of moving frame time.

An unrecognized complication is that there is an alternative method of evaluation of the Lorentz transform. This procedure is to evaluate the Lorentz transform for the space-time events located at x=0 in the domain. The first procedure results in a contraction of time in the codomain and the second in a dilation in the codomain. Unfortunately these solutions are not what is required if we are asked to determine what is the rate of the clock in the moving frame when it is viewed from the rest frame. This problem is solved by inverting the results of the previous evaluations. Then the contraction becomes a dilation and the dilation a contraction. The third alternative is that the Lorentz transform is a scale change or basis transformation between reference frames. The result is either a contraction or dilation as determined by the evaluation, but the procedure for comparison of the time intervals is different in this case. The result of the first two cases is a comparison of the apparent transformation of time, while in the last case, the results represent a real change in the rate of the moving frame clock. With this brief introduction to the issues that must be confronted, we proceed to the examination of Einstein's method of solution.

### 6.3 Einstein' s Method Of 1905

This section presents an analysis of Einstein's derivation of time dilation in his first paper on relativity published in 1905. Einstein's 1905 derivation of the transformation of time is analyzed and the errors in his derivation are discussed.

Einstein's derivation of time dilation is presented in section 4 of the 1905 paper. After discussing the transformation of space which leads to a contraction, Einstein takes up the transformation of time. The first step is to define the domain and codomain coordinates for the transformation. Einstein does this as follows. A clock which marks the time $\tau$ at rest in the moving frame is located at the origin of this system. Einstein asks for the transformed rate of this moving clock when viewed in the stationary or rest frame. The problem is defined as a transformation from the moving frame as domain into the rest frame as codomain. Therefore, we expect the result to be in the form of a functional relation which expresses the measured or observed rest frame time as a function of the moving frame time.

The second step is to select a Lorentz transformation which expresses the equivalence of moving frame coordinates as domain with rest frame coordinates as codomain. Einstein uses the Lorentz transformation going from the rest frame as domain into the moving frame as codomain. This is an error that leads him astray. It is ironic that the solution is in the correct form for transforming time from the rest frame into the moving frame, but that is not the solution which was specified. So it must be judged as incorrect. The proof of this error is that the result which Einstein gives, is an equation which expresses the moving frame time as a function of the rest frame time. Hence it is expressed incorrectly, because the required solution is an equation expressing the rest frame time as a function of moving frame time.

Einstein proceeds to use this incorrect result to support the conclusion that the moving clock runs slow. This approach is wrong because it uses an incorrect derivation of the Lorentz transform. A second problem is the question of the correct method of evaluation. There are two possibilities; evaluation for x=0 and evaluation for x'=0. The answer requires a knowledge of which evaluation condition is correct. The correct evaluation occurs when we transform from the moving frame into the rest frame, with rest frame measurement performed at the same place -- this is where the reference clock is located in the rest frame-- but at different times. This condition is only achieved by evaluation at x=0 as correctly specified in Dingle's method of solution. Einstein uses an incorrect method which performs evaluation in the moving frame of reference, and not the rest frame as required.

Einstein's use of an incorrect evaluation method is apparently motivated by his desire to show that the rate of a moving clock runs slow. This being the reason for transforming from the moving frame into the rest frame and his use of the evaluation method which employs substitution of the condition x=vt into the Lorentz transformation formula. Unfortunately, this method is not properly matched to the use of the Lorentz transformation. But, at this stage, Einstein has not developed the inverse transformation formula that would have permitted the use of the evaluation procedure which he uses. The correct procedure would have been to use the Lorentz transform evaluated at x=0. Instead he uses a confusing and incorrect procedure that leads him into error.

The first mistake is now clear, for the mathematical solution that is presented, the problem is defined incorrectly. The correct definition would have been to ask, what is the rate of the rest frame clock when viewed from the moving frame. Presented this way, the mathematical solution given is correct. The second mistake cited notes that the Lorentz transform is used incorrectly relative to the problem definition. The inverse Lorentz transform should have been used instead of the Lorentz transform. With this change, and the appropriate change in the evaluation condition, a correct derivation results as demonstrated by Dingle.

An analysis of the method used by Einstein reveals the following points:

- The problem is defined by asking for the rate of the moving system clock when viewed from the stationary system
- Einstein uses the Lorentz transform formulation of the problem which provides the transformation of time defined in the stationary frame into the moving frame
- The solution is written with the moving frame time written as a function of the stationary frame time

- Einstein interprets the solution, which shows a time interval in the moving frame is contracted relative to the stationary frame, as demonstrating that the moving frame clock runs slow
- Einstein uses this result to assert that when a clock at rest in the stationary system is moved with a velocity v from A to B in the stationary system, it will lag behind the clock at B when it arrives because of this motion
- Einstein then asserts that a clock at the equator "must go more slowly ...than a similar clock situated at one of the poles"

The first two points address the problem mentioned before that the resulting equation used to compare the clock rates is backwards. Hence, it can not be used to represent a projection of the time marked on the moving frame clock into a measurement of time in the stationary frame. So it must represent a direct comparison of the rate of a moving clock with a clock at rest. Points four, five and six can now be seen as following directly from the demonstration that a moving clock runs slow. The comparison assumes a time scale or basis change from the moving frame into the rest frame with time in the rest frame defined as reference time. The result that the moving frame clock runs slow is then used to support the conclusions given in the last two points.

### 6.4 Einstein's Big Blunder

Einstein's blunder is as follows. He says that when we have a Lorentz transformation between two time intervals, representing time marked by clocks, when we set the time indicated on the reference clock to 1 unit and calculate time on the opposite clock, if the result is less than 1 unit, the subject clock is slow. This is incorrect.

The equation for the transformation of time intervals is in the form of a function as follows: $t_s=L_t(t_r)$. Here $L_t$ is the transform function, $t_s$ is the subject time interval, and $t_r$ is the reference time interval. The subject time interval is a representation of the clock time subject to the transformation. We calculate the comparison of the clock rates as follows. When $f_s>f_r$, the subject clock is fast. Solving for the times we obtain: $f_s>f_r$, hence, $1/t_s>1/t_r$, so $t_r > t_s$, and $t_s<t_r$. Using the functional notation, the result can be expressed so that when the function $L_t$, usually written as $\beta$ or $\beta^{-1}$, results in a value less than unity, the subject clock is fast, in other words when $t_s= \beta^{-1}t_r$, then $f_s>f_r$. The subject clock is slow when we have the result $t_s= \beta t_{r.}$ Because in this case, $\beta$ is greater than unity , and we have the result that $f_s>f_r$, because $t_r < t_{s.}$ These results are exactly opposed, i.e. inversely related, to the method Einstein uses.

Einstein's blunder is confusing dial readings with time intervals. He assumes a direct 1 to 1 relation such that clock dial or display readings represent time. This is not true in the theory of Lorentz transforms, which act directly upon the time itself, and not dial readings as assumed in the operational method adopted by Einstein. Dial readings are counts of clock beats. Beats are oscillations with a fundamental cyclic frequency of occurrence. They are usually assumed uniform of fixed period. Clocks keep time by counting these periods of oscillation. Oscillations with shorter periods, i.e., contracted time intervals, mean the clock runs fast because more beats are counted during a given reference period as measured by the reference clock.

Einstein's mistake, interpreting a time interval contraction as indicating that a clock runs slow, must be recognized as a blunder of major significance for the following reason. It led to an incorrect interpretation of the theory and the wrong equations. To explain this lets take up an analysis of the 1905 paper from a different viewpoint. In 1905, Einstein performed an analysis of the Fitzgerald-Lorentz contraction going backwards. The method can be summarized in modernized form as follows. Starting with the Lorentz transform for distance as $x'=\beta(x-vt)$, where x' is the distance defined in the moving frame and x the distance in the rest frame, and $\beta=(1-v^2/c^2)^{-1/2}$ .We have for t=0 the resulting solution, $x'=\beta x$, which is solved for $x=\beta^{-1}x'$. The result being the required Lorentz contraction going from the moving into the rest frame.

Suppose we utilize an analogous method for the transformation of time. The Lorentz transform is $t'=\beta(t-vx/c^2)$. Now solving with the condition that x=0, the solution $t'=\beta t$ is obtained. Apparently Einstein rejected this solution because it leads to the conclusion that the clock in the moving frame runs fast -remember he thinks that a dilation of the time means that the clock runs fast because he interprets a contraction of the interval as a slow clock. But, this is the correct solution for the dilation of time experienced by a moving clock. It is the solution that he was searching for. However, because he misunderstood the correct interpretation of time intervals, he obtained the incorrect result of 1905. Hence Einstein missed the correct solution and the consequence was that he introduced a different interpretation which explained the slowing of moving clocks in his 1907 paper. The failure to clearly retract his errors in the 1905 paper has resulted in confusion, misinterpretation, bitter debate, argument, and dissent for nearly 100 years.

### 6.5 Einstein's Blunder Vindicated - The 1907 Method Of Solution

In 1907 the blunder of the 1905 paper is erased with the presentation of a new interpretation and method of proof. The 1907 method of proof vindicates the blunder of 1905 by showing that moving clocks run slow just as concluded in 1905. This revised proof was motivated by new experimental results obtained by J. Stark. This suggested to Einstein that moving atomic clocks run slow because " ...the oscillation process that corresponds to a spectral line is to be considered an intra-atomic process whose frequency is determined by the ion alone, we may consider such an ion as a clock...the effect of motion on the light frequency...reduces the (apparent) proper frequency of the emitting ions..." The revised theory presented in 1907 demonstrated the mathematical proof of this supposition by showing that the Lorentz transformation of the frequency of a moving clock observed from the stationary frame was less than the observed frequency of the same ions at rest. Hence the clocks of the moving ions appeared to run slow.

Another revision was that the assertions given in the last two points of the summary of section 6.3 - the conclusions that moving clocks actually run slow- are dropped. We do not see any reference to them after this. This suggests a change in viewpoint is undertaken in the 1907 revision. This new viewpoint stresses the interpretation that the rate of a moving clock only appears slow when viewed from the rest frame. Hence in the 1907 revision, the interpretation that the slowing of moving clocks is a real change in clock rate gives way to the interpretation that the change is in the apparent rate of the clock observed from the moving frame. In the following, this altogether different interpretation will become apparent.

It is clear that by 1907 Einstein believed his derivation of time dilation in the 1905 paper was unsound. The new interpretation required a different approach. For the new interpretation, it was necessary to show that the time marked by the clock in the moving frame was shifted in frequency. This led to a more elegant and convincing method of proof than that used in 1905. Hence, in 1907, the derivation of the transformation of time intervals was dramatically revised and appeared in a completely different form. The new form posed a completely different problem. It addressed the transformation of clock rate, as opposed to the previous approach based only on the transformation of time interval.

Einstein's method for demonstrating the transformation of clock rate begins by defining a clock in the moving frame which runs or beats $\nu_0$ times faster than the other reference clocks used to measure time in the moving and rest frames. Hence this clock beats $\nu_0$ times while every other clock beats only once. The method is to calculate the transformation of this clock rate from the moving frame to the rest frame. Here we see that the problem is defined backwards just as in 1905. Hence he again defines the transformation going from the moving frame to a rest frame, but the 1907 method proceeds differently after obtaining the same result as in 1905; $t'=\beta^{-1}t$, where new notation is used (primed variables refer to the moving frame and unprimed to the rest frame).

Although Einstein demonstrates that the clock rate measured in the rest frame appears slow compared with the rate defined in the moving frame, he does not present a complete mathematical derivation. He skips the steps which calculate the transformation of the time interval. He states this procedure as follows: "Using the first two transformation equations, one obtains". He then proceeds to write the results: $t=\beta t'$. The reader is left with the task of filling in the intermediate steps. The implication here is that the same method that is used in the 1905 paper is to be applied, because he refers to the two transformation formulas, instead of only one which is needed for the transformation. This involves solution of the spatial equation with the condition $x'=0$, and following that with the substitution of $x=vt$ into the Lorentz transformation equation for time.

The proof now proceeds by inverting the result of the 1905 solution, and the solution becomes a dilation of clock rate transformed from the moving frame into the rest frame where it is measured by comparison with a rest frame clock. Hence he obtains: $t=\beta t'$. But t now represents the observed time interval, as viewed from the rest frame, marked by the clock in the moving frame, and not the time marked by the rest frame clock. The symbol t' now represents the reference time which is assumed the same in both the moving frame and the rest frame. This change in the meaning of the symbols is crucial in understanding the difference between the transformation of reference clock rates -used to interpret a real change in clock rates- as opposed to the transformation of apparent clock rates.

The proof follows immediately from the transformation formula obtained for the transformation of time by converting the time intervals into frequency. The method used however is awkward and difficult to follow. Defining frequency observed in the rest frame as $\nu$ and the moving frame frequency as $\nu_0$ the transformation of time intervals can be converted into a transformation of frequency as follows: $t=\beta t'$, hence $1/\nu = \beta\ 1/\nu_0$, and the resulting solution for frequency observed in the rest frame is $\nu=\beta^{-1}\nu_0$. Hence, $\nu_0=\beta\nu$. This result shows that the frequency generated in the moving frame is greater than the observed frequency in the rest frame. Therefore, the apparent clock rate is slow as viewed from the rest frame. As Einstein puts it " The motion... reduces the (apparent) proper frequency of the emitting ions..."

## 6.6 Einstein's 1910 Relativity Paper

Einstein was not satisfied with his solution of 1907, because a new but not significantly different solution is presented in his 1910 paper. The main change provided a justification for the equation, $t=\beta t'$, which was not presented in the 1907 paper. The method used reveals a completely different approach to the problem.

The new feature of the 1910 derivation is the use of the inverse Lorentz transform equation for time. This, however, is written incorrectly as: $t=\beta(t'-vx'/c^2)$. (The error is using a minus instead of a plus sign.) However, this is clearly intended to be the inverse Lorentz transform for time, despite the mistake. Einstein proceeds by saying: "and since clock H' is at rest at the origin of S', we must always have $x'=0$, which yields $t=\beta t'$." Here evaluation is performed by substitution of $x'=0$ into the inverse Lorentz transform equation. This procedure is equivalent to the evaluation method used in 1905, so the same result is obtained. Using this evaluation method gives the same results as in 1905 and 1907, but it is another blunder. He misses another opportunity to discover the correct equation that proves moving clocks run slow.

To see why, we return to the 1905 derivation. There the substitution, $x=vt$ was used. This equation results when we solve the spatial Lorentz transform for the condition $x'=0$ ($\xi=0$ in the 1905 notation). But the inverse Lorentz transform is the correct equation, so the correct condition is actually $x=0$. Analysis of the 1910 derivation of time dilation brings Einstein's erroneous misconception regarding evaluation into focus. He uses the specification $x=vt$ in both the 1905 and 1907 papers. This results from the solution of the Lorentz transform for space when the condition $x'=0$ is imposed in the domain instead the codomain. This same incorrect specification is also used in 1910, but in a different way. This requirement is wrong, because the solution asks for the transformation of time going from the moving frame into the rest frame. Einstein's evaluation condition specifies the location of the clock in the moving frame but does not specify where the observer is to be located in the rest frame. This specification is necessary to correctly transform the clock rates. Einstein does this incorrectly. His specification that $x'=0$ is only valid for the transformation from the rest frame into the moving one.

### 6.7 All Of Einstein's Derivations Of Time Dilation Are Incorrect

The purpose of the section is to show that Einstein's derivations of time dilation in all three of the fundamental papers on relativity are incorrect. The proof follows when the equations used for transformation of time intervals are converted to frequency. To do this we define the clock rate in terms of frequency as follows: the clock at rest in S has the rate $f=1/t$, and the clock at rest in S' has the rate $f'=1/t'$. These equations relate unit time intervals defined in S and S' into clock rate or frequency to permit comparison of clock rates.

To calculate the rates of the clocks we use equations $t'=\beta^{-1}t$ and $t=\beta^{-1}t'$and convert time to frequency using $f=1/t$ and $f'=1/t'$. The results for the 1905 result given in the first equation are as follows: $t'=\beta^{-1}t$ or $1/f'=\beta^{-1}1/f$. Therefore $f=\beta^{-1}f'$ or $f<f'$. The result shows that the clock at rest in S, the stationary system, is slow relative to the clock at rest in S', the moving system. The calculation for the 1907 result is: $t=\beta t'$ or $1/f=\beta 1/f'$. Therefore, $f'=\beta f$, or $f'>f$. This result shows that the clock at rest in S', the moving system, is fast relative to the clock at rest in S, the stationary system.

These results prove that the conclusions given in the fundamental papers on relativity are incorrect. The contradiction of the 1905 result is obvious, but that the 1907 result is also wrong is not as clear and requires an explanation.

In the 1907 and 1910 papers the equation $t=\beta t'$ was used. Einstein obtained the following result, which was calculated above based upon the 1905 equation: $t'=\beta^{-1}t$ or $1/f'=\beta^{-1}1/f$. Therefore $f=\beta^{-1}f'$ or $f<f'$. He incorrectly interpreted this to mean that the frequency of a moving clock, represented by the symbol f was slow relative to the clock in S which was now represented by the symbol f'. So the proof was accomplished by a definition of the meaning of the symbols form the

correct meaning and interpretation. Hence the proof is incorrect. But the later textbook writers corrected the "mistake" and promulgated the now accepted interpretation that was given in sections 6.5 and 6.6, even though the derivation given in the 1907 and 1910 papers is clearly incorrect.

The result has been a major problem for the theory, its interpretation, and understanding. The correct equations for time dilation using the notation and definitions given by Einstein are the equations $t' = \beta t$ and $t = \beta^{-1} t'$. The first gives the result: $t' = \beta t$ or $1/f' = \beta 1/f$. Therefore, $f = \beta f'$, or $f > f'$. Which says that the clock at rest in S, the stationary system, is fast relative to the clock at rest in S', the moving system. The second equation, which is Dingle's result, gives: $t = \beta^{-1} t'$ or $1/f = \beta^{-1} 1/f'$. Therefore, $f' = \beta^{-1} f$, or $f' < f$. The clock at rest in S', the moving system, is slow relative to the clock at rest in S, the stationary system. Therefore, we conclude that it is Dingle's solution that gives the well known result, which is experimentally verified, that the moving clock runs slow relative to the rest frame clock. We must conclude from this that Dingle's result, given in his 1962, paper is correct.

**7.0 Special Relativity After Einstein**

Einstein had begun to look beyond the special theory of relativity during preparation of the 1907 paper. It contained not only the completion of special relativity, but the seeds of general relativity as well. Following its publication, Einstein devoted his efforts to general relativity and the advancement and perfection of special relativity was left to others. Unfortunately, the damage of the 1905 blunder was not erased, and in the following years the two different interpretations of special relativity based on the 1905 and 1907 theories continued to flourish. Here we will briefly review these developments which are closely tied to the clock paradox.

To clearly understand the two different versions, it is important to understand that the clock paradox arises from the 1905 version and not the 1907. The 1905 version clearly states that a clock at rest set in motion then brought back to rest runs slow. This is the basis of the clock paradox. The 1907 version is incompatible with this interpretation because it is clearly assumed that the clocks in the moving frame always run at the same rate as clocks in the rest frame. Consequently Einstein dropped this conclusion from the 1907 and 1910 papers on relativity. The failure to clearly distinguish between the different assumptions and derivations of the two different versions of relativity has resulted in endless controversy and confusion.

We can see this when we compare W. Pauli's Theory of Relativity[5] with Max Born's interpretation in Einstein's Theory of Relativity[6]. In his book first published in 1921, Pauli gives the 1905 version which is based on the idea that "the time scale is changed by the motion". The equation for time dilation is derived by the 1910 method using the inverse Lorentz transform, but he then reverts to the 1905 interpretation when he says "...therefore, a clock moving with velocity v will lag behind one at rest..." He specifically discusses the clock paradox and asserts that the retardation of the moving clock is a real effect, despite the paradox that arises from the reversal of reference frames. He tells us that the paradox is merely apparent.

Born's book first published at about the same time in 1924, gives the 1907 version of the theory in terms of the Minkowski space-time geometry. He states the conclusion as "...viewed from any one system, the clocks of every other system moving with respect to it appear to be losing time." Unfortunately Born then reverts to the 1905 version, when he says "The course of events in time in the systems in relative motion are slower, so that all events in a moving system lag behind the corresponding events in the system regarded at rest." This bewildering contradiction is followed by: "We shall return later to the consequences which arise from this fact and which are often regarded as being paradoxical." Born's solace for the reader is a discussion which follows in a later section which describes the clock paradox and explains "that absolute time has no physical reality".

Both the Pauli and Born books attempt to accommodate the different versions without clearly distinguishing them. Pauli, uses the mathematical method of 1907 but adheres to the interpretation of 1905. Born on the other hand, stresses the method and approach of the 1907 version, and does not really stress the 1905 viewpoint but accommodates it in his final remarks. Neither is clear upon which interpretation is correct. Ultimately, the contradictions appear in the clock paradox which both versions are apparently able to explain, although the methods are not always the same or equally satisfactory.

The history of relativity following Einstein's withdrawal from the discussion is characterized by an attempt to reconcile the clock paradox, which represents the contradictions between the two different versions. The usual procedure involved the presentation of the formal theory in terms of the 1907/1910 version while maintaining the basic idea that time in the moving system is retarded or dilated by motion. The effect is regarded as reciprocal since being relative, a reversal of reference frames reverses the conclusions of the observer. The main problem of the textbook writers was to express both of the viewpoints in a consistent way. The attempts to do this were not always successful. The resulting explanations were often confusing and contradictory, leaving the reader bewildered and confused. But, as time progressed the dominant interpretation became that of the formal system of the Minkowski space-time. No one noticed that the contradictions were not fully suppressed.

The problem for textbook writers and interpreters of relativity was to preserve the integrity of both versions of relativity. In his 1923 book Four Lectures on Relativity and Space, Charles Proteus Steinmetz[7] solved the problem by giving a through presentation of the 1907/1910 theory, complete with a discussion of the Minkowski geometry and Lorentz group rotation. The contradiction with the 1905 version was resolved by simply avoiding any mention or reference to it. There was no reference to the twins paradox despite the recent controversy between Einstein and Henry Bergson following Bergson's critique of the relativistic time concept in his book Duration and Simultaneity.

In the book Introduction To Modern Physics[8] first published in 1928, Richtmyer, Kennard, and Lauritsen give a completely different solution to the problem of reconciling the results of the different versions of relativity by rejecting the 1907/1910 version and reinterpreting its results in terms of the 1905 version. They begin by assuming a vibratory system such "as a good crystal clock or a radiating atom". By interpreting the time intervals in the Lorentz transforms as the periods of an oscillator in different reference frames they obtain the conclusion: "Thus, relatively to S, setting it in motion has lengthened its period of vibration in the ratio $\gamma$. More generally, when any two physical systems are in uniform relative translation at speed u, the effects produced by system A on system B are modified just as if all natural processes on A were slowed down in the ratio $(1-u^2/c^2)^{1/2}$, and similarly for the effects of B on A... Spectral lines from moving atoms, for example, should show a slight displacement toward the red...Such an effect appears to have been observed by Ives, working with canal rays in hydrogen." This interpretation is based on the idea that motion causes a time scale change for the laws of physics in the moving frame. This is the hallmark of the 1905 interpretation. Notice however that it is not the result obtained by Einstein in 1907/1910.

In his 1950 book titled Classical Mechanics[9], Herbert Goldstein came up with a totally different interpretation. By this time a number of textbooks were giving the time dilation formula in a manner different from the original version of 1907/1910; i.e. $t=\beta t'$. This involved reversing the reference frames so that the formula appeared in the form $t'=\beta t$. Goldstein gives the dilation equation in this form and then says: "When the stationary clock has turned one hour the moving observer will find his clock has gone $\beta$ hours. He will say the stationary clock is slow, that it is losing time; hence the name 'dilation of time' given to this phenomenon. But it should be emphasized that observers in the unprimed system examining the rate of a clock fixed in the primed system likewise come to the conclusion that it is running slow compared to theirs."

At about the same time, 1952, Leigh Page published his Introduction To Theoretical Physics[10]. This publication gives the now "classical" version of the time dilation formula from 1907/1910 as $t=\beta t'$. Page interprets this as follows: "Therefore the interval of time which has elapsed in S is greater than that which has elapsed in S'. Observers in S conclude, then, that the moving clock is running slow. Again the effect is reciprocal. If a clock is at rest in S observers in S' find that it runs slow as compared with clocks at rest in their system."

Both statements by Goldstein and Page seem fine when interpreted separately, but when placed side by side it becomes difficult to make sense of either of them. Which one is correct. They seem to contradict each other. Goldstein says that it is the stationary clock that runs slow, while

Page says that the moving clock runs slow. The answer of the relativist to this "apparent contradiction" is that the effect being reciprocal does not involve a contradiction, since it merely involves a reversal of the observers reference frame. But the truth of this statement also depends upon which version of relativity is being used in the interpretation.

Technically both statements are wrong, because they have allowed the 1905 version of relativity to slip into the interpretation. We can see this by comparison with the correct statement by Donald Menzel, from his book Mathematical Physics[11] first published in 1947. Menzel gives the time dilation formula in the classical form, just as Page, but his explanation is more precise than Page. He says: "Thus the interval ($t'_2$-$t'_1$) as perceived in the unaccented system will appear longer than the interval $t_2$-$t_1$. The clock in S' will appear to run slow, as checked by an observer on S, and vice versa." This is an exactly correct interpretation based on the formal interpretation of the 1907/1910 version relativity.

The usual relativistic answer, that reference frames are reciprocal and therefore equivalent doesn't solve the discrepancy between the statements of Goldstein, and Page. The statements are ambiguous and confusing. The usual interpretation (an example being the statement by Menzel) is that the clock in the moving system appears to be slow when viewed from the stationary system. But Goldstein says that the stationary clock is slow. It is a bewildering explanation. Primarily because we lose track of which reference frame belongs to the observer and which clock is running slow. But, the main points are that Goldstein says that the stationary clock runs slow according to the equation $t'=\beta t$ and also that he is using the 1905 version of relativity in his interpretation.

Correctly interpreted, the two statements by Goldstein and Page are inconsistent. Goldstein claims that the stationary clock runs slow as viewed from the moving frame, and reversing viewpoints, that the moving clock runs slow as viewed from the stationary frame. This is not the same as the statements by Page which assert that the moving clock runs slow as viewed from the stationary frame, and reversing viewpoints, that the stationary clock runs slow as viewed from the moving frame. Mathematically these are not equivalent statements, but the recognition of this fact is difficult given the lack of rigor used in the discussion of the problem. The statements by Goldstein arise from the equation $t'=\beta t$ and the statements by Page arise from the equation $t=\beta t'$. The assumption is that these equations are reciprocal such that when reference frames are reversed, the equation $t=\beta t'$ becomes $t'=\beta t$. This is only a mathematically true statement within the 1907/1910 version of relativity and not the 1905 version. At this point we turn to Dingle and his discovery of an inconsistency directed at Einstein's 1905 version of special relativity.

## 8.0 Dingle's Inconsistency

The reader who has carefully read and compared the different interpretations given in the previous section can see that the various interpretations are inconsistent and confusing. The objective of this section will be to explain how these confusing interpretations arose and how Dingle was able to prove an inconsistency in the special theory of relativity. We begin by returning to Einstein's 1905 paper.

There are two main errors in the 1905 paper that are the source of the confusion which the textbook writers vainly endeavored to clear up. The first is that the method works backwards with domain and codomain reversed. This causes a logical confusion in the wording and explanations. Combined with the clock dial blunder, the result was a mountain of confusion.

Einstein tries to erase the blunder contained in the 1905 version of relativity by making the theory consistent with Stark's canal ray measurements. He tries to do this without radical changes to the structure of the theory. He has two equations to work with; the first one derived in 1905 $t'=\beta^{-1}t$, and its mathematical inverse $t=\beta t'$. The clock dial method doesn't work because of the clock dial blunder. As we see in section 6.8, calculation of the clock rate in the moving system shows that the moving clock is fast.

When we set $t'=1$ in the 1905 equation, $t'=\beta^{-1}t$, the result is $t=\beta$. A result that seems to show that when the clock in S' is viewed from S, the apparent period in frame S is greater or longer than the period in S'. This result suggests how Einstein may have gotten the idea used in the 1907 paper on relativity. The correct conclusion however, is that the clock in S' appears fast when viewed from S, because it is the reference clock in S that is slow, and not because the period of the clock in S' appears longer when viewed from S.

According to Einstein's interpretation of clock rate, the result $t=\beta$ contradicts the assertion of the 1905 paper that the clock in S' runs slow, because when the clock in S reads $t=1$, the S' clock reads $\beta^{-1}$. Hence the conclusion that the clock in S' is slow depends on the interpretation that its dial reading is behind the dial reading of the clock in S. But this method is contradictory to the method which compares clock oscillation periods, since a longer time interval indicates a greater period and hence a lower oscillation frequency and a slow clock. The two methods are incompatible.

Einstein discovered that the contradiction can be avoided by using a different method in which the frequency of the clock is compared instead of clock times. This appeared to give the desired result that the frequency of a clock at rest in the S' reference frame appeared to be reduced when viewed by an observer at rest in the S reference frame. Hence the desired result that moving clocks appear to run slow. This resolved the problem as long as the subtle change in the meaning of the symbols used was not contested. The mistake and the change in the meaning of the symbols was not noticed and the theory with all its flaws became accepted physics.

The confusion results from the fact that the textbook writers failed to follow Einstein's lead by shifting to a frequency method of clock rate comparison. They continued to use the 1905 method of clock dial comparison. To see how this created a major problem, return to the solution of the problem of comparison of the time intervals. Using the equation $t=\beta t'$, when we set time in the domain to one second, the time in the codomain is $\beta$ seconds. Hence, the time interval transformed from frame S' into S in longer in S, compared with S' and the clock appears slow in S. But consider the statement given in the previous section. There we are to determine the rate of the clock in S relative to S' by setting the time in the codomain equal to one second and we conclude that the time in the domain is $\beta^{-1}$ seconds. This procedure is backwards and incorrect, but it leads to the supposedly correct conclusion that the clock in S' appears to be slow. This error leads to tremendous confusion and to the clock paradox.

Dingle exploits the method described above in the following way to demonstrate an inconsistency. He derives the equation $t=\beta^{-1}t'$. This is done exactly correct. But the next step is the clever move. He follows the procedure used above, sets the codomain time t=1, and solves for the domain time as $t'=\beta$. Hence the clock in S' is fast according to the Einstein interpretation of clock rate. This is clearly a contradiction to the Einstein result of 1905 which concludes that the clock in S' is slow. What is really interesting is that by this procedure, Dingle points out that the conclusion in Einstein's 1905 paper is wrong. The clock really runs fast instead of slow. A mistaken result that arises from the dial reading blunder. There really is an inconsistency, but more than this, the conclusions of the 1905 paper are incorrect. Because of the clock dial blunder, Einstein says that the clock in S' runs slow when the proof really shows that it runs fast. Since experiments show that moving clocks run slow, this is clearly an embarrassing mistake.

Dingle did not present a proof of contradiction. His argument was in terms of an inconsistency. This inconsistency can be resolved by either the recognition that Einstein's proof is in error, as shown above, or by proof that Dingle's solution is in error. Otherwise the following proof shows the theory contains a fatal self contradiction. To prove this we take the two solutions. Einstein's $t'=\beta^{-1}t$ and Dingle's solution $t=\beta^{-1}t'$. We show that the only solution is t=t'. Since $t'/t=\beta^{-1}$ and $t/t'=\beta^{-1}$, we have that $t'/t=t/t'$, and $t'^2=t^2$ or that t'=t. This solution clearly is contradicted by either of the equations obtained by Einstein or Dingle unless v=0 and $\beta=1$. Hence the solutions have no common solution consistent with each other except when v=0 which means there is no relative motion. A solution which has no usefulness. Hence unless it can be shown that Dingle's solution is demonstrably wrong, the theory is refuted.

As noted above, the conclusion which Dingle uses, that the clock in S' is fast, is incorrect. But in obtaining his results he used the accepted method for comparison of clock time in special relativity. It showed that one of the following possibilities existed:

- That Einstein's 1905 calculation for the clock rate in the moving frame was incorrect; it runs fast instead of slow.

- That the two different calculations by Einstein and Dingle are both correct but give contradictory and inconsistent results; one calculation shows the clock runs fast while the other shows it runs slow.

Dingle's initial purpose was to demonstrate the second possibility, but he hints that the first should be examined as well. (See the background section above.) That both possibilities have proven correct is a bonus for relativity critics. The correct result is the following. Einstein's 1905 calculation is incorrect, but by using his method the correct result is that the clock in the moving system runs fast when viewed from the stationary system. Dingle's result is also incorrect. However, his method demonstrates that the clock in the moving system runs slow when viewed from the stationary system. Hence, his corrected results demonstrate a contradiction with the result obtained by Einstein, so the inconsistency or contradiction is proved to be correct. This result is easily obtained by simply reversing the conclusions based on the incorrect interpretation obtained using the dial reading method.

The conclusion of this section is that the two different methods give contradictory conclusions that are diametrically opposed physically. The calculations show that for the observer in S, the clock in S' runs either slow or fast depending upon which solution you believe is correct. Both can not be true so either Einstein is wrong, and the theory fails on that account. But, if the reader refuses that result, and insists that Einstein is correct, then he must also admit that the theory is logically flawed and the theory is refuted on that basis.

**8.1 Extending Dingle's Argument- The Flaw In The 1905 Version Of Relativity**

Clearly in order for an inconsistency or logical contradiction to exist, it is necessary to show that when two different calculations for the same physical situation reach different conclusions, then either one of the calculations is wrong or the theory used in the calculations is inconsistent. This is exactly what Dingle set out to prove. Using Einstein's method, he shows that viewed from the moving frame S' the clock in S appears fast, while Einstein concludes it is slow, hence the inconsistency is proved.

In this section Dingle's argument is modified and extended in order to show exactly the problem which he has discovered. Refutations of Dingle's argument have not correctly addressed the main issue which he raises. This is an inconsistency in the conclusions obtained from two different "proofs".

The mathematical method used here is to solve the system of Lorentz and inverse Lorentz equations for time simultaneously using a specified condition of evaluation. Here the term evaluation is used in the same sense as it is used when a polynomial equation is solved for its roots by setting the equation to equal zero and solving for the indeterminate. The procedure used here is similar. A selected variable is set to zero, and the resulting solutions are obtained.

The Lorentz transformation equations in a simplified form are assumed as follows:

$x'=\beta(x-vt)$ $t'=\beta(t-vx/c^2)$ $x=\beta(x'+vt')$ $t=\beta(t'+vx/c^2)$ $\beta=(1-v^2/c^2)^{-1}$

Here there are four equations which express the simultaneous solutions for the transformation of coordinates. These equations are defined in the usual way in terms of two relatively moving reference frames S and S'. Where the origin of frame S' is in motion with velocity v in the positive x direction of S.

To consider the role of evaluation in space, we determine the simultaneous solution of the four equations when we specify the condition that x=0. The results are as follows:

Equation 1: $x'=\beta(x-vt)=-\beta vt$

Equation 2: $t'=\beta(t-vx/c^2)=\beta t$

Equation 3: $x=\beta(x'+vt')=0$ , Therefore $x'=-vt'$

Equation 4: $t=\beta(t'+vx'/c^2)=\beta t'(1-v^2/c^2)=\beta^{-1}t'$

Notice that equation 4 is solved by substitution with the result from equation 3. Therefore, from equations 2 and 4 we have the following two solutions for the condition x=0: $t'=\beta t$ and $t=\beta^{-1}t'$. The solutions for equations 1 and 3 gives the results $x'=-\beta vt =-vt'$, from which we conclude that $t'=\beta t$. A result which is the same as obtained from equation 2 which will be called the primary result for the condition x=0. The primary result has an inverse, called the reciprocal solution, given by equation 4.

To further consider the role of evaluation in space, we determine the simultaneous solution of the four equations when we specify the condition that x'=0. The results are obtained as follows:

Equation 5: $x'=\beta(x-vt)=0$, Hence $x=vt$

Equation 6: $t'=\beta(t-vx/c^2)=\beta t(1-v^2/c^2)=\beta^{-1}t$

Equation 7: $x=\beta(x'+vt')=\beta vt'$

Equation 8: $t=\beta(t'+vx'/c^2)=\beta t'$.

Notice that equation 6 is solved by substitution with the result from equation 5. Therefore, from equations 6 and 8 we have the following two solutions for the condition x'=0: $t'=\beta^{-1}t$ and $t=\beta t'$. The solutions for equations 5 and 7 give the results $x=vt =\beta vt'$, from which we conclude that $t=\beta t'$. A result which is the same as obtained from equation 8 which will be called the primary result for the condition x'=0. This solution has an inverse, called the reciprocal solution, given by equation 6.

When an evaluation condition is imposed upon the system of Lorentz transformation equations, two simultaneous solutions result. One solution, which we call the primary result, and its inverse, which we call the secondary result. The primary results appear in equations 2 and 8, with the secondary results given by equations 4 and 6. The primary results always give a dilation of the transformed variable while the secondary result is always a contraction. The primary result is distinguished from the secondary because it is also obtained as a solution of the remaining two equations which are redundant with the result of the primary solution.

Primary solutions arise when evaluation is specified in the domain and they always transform time as a dilation from the domain into the codomain. The primary solutions are the same as obtained in the 1907/1910 version of relativity, while the secondary solutions arise from the 1905 version and are the same as the results discussed by Dingle in the 1962 note in Nature. The primary solutions always result in a dilation of time, because the domain time is considered the reference or proper time and the codomain the apparent or observed time.

However the situation for the case of the 1905 version of relativity is not as clear. There we have the situation where it is implicitly assumed or implied that the stationary frame S is the reference frame for calculation of the time scale change in the moving frame S'. So using equations 4 and 6 we set t=1 and solve for t'. The results as we already know are $t'= \beta$ for equation 4, Dingel's equation, and $t'=\beta^{-1}$ for equation 6, Einstein's equation. Hence the conclusion that the moving clock is both slow and fast depending upon the evaluation condition.

But which evaluation condition is the correct one? This depends upon what the process of evaluation means physically. The answer implied by the results for the 1907/1910 version of relativity is that evaluation defines the frame which contains the reference clock. In other words, it defines the rest frame. But when we do this, evaluation with x'=0 makes the S' frame the rest frame in place of the S frame. This leads to the revealing result that the two inconsistent equations are really the same equation with the symbols representing the rest and moving frames reversed. This may seem to imply that this result means that the frames are reciprocal in a relativistic sense, but this is not the case. When we actually measure the rate of a moving clock, if it is slow, then reversing the reference frame does not mean that the opposite clock runs slow as well. Just the opposite, it runs fast just as Dingle showed. It appears then, that the 1905 version of relativity isn't relative at all, it implies the existence of an absolute rest frame.

**8.2 The Reason Behind Dingle's Inconsistency**

The main flaw in the 1905 version of relativity theory is the inability to define an adequate clock reference. What this means is that despite the seemingly consistent nature of the mathematical formalism, we are never able to clearly identify which clock is the reference and which is the clock subject to measurement. We always can assume a reference clock which gives correct time, the problem is to make valid conclusions regarding the result of a measurement in the opposite reference frame. But we are unable to do this in practice because in principle there is no way to distinguish whether the clock we designate as the reference, is actually slow or fast. Since we don't know whether the reference clock is fast or slow, then we can't say that the observed or measured clock rate is fast or slow. It is a logical contradiction. But we can escape it by defining a reference frame. But once we do so, this becomes an absolute reference frame, and the 1905 version becomes the theory of Larmor and Lorentz. Dingle exploits the logical inconsistency of the 1905 version of relativity. It is not difficult to prove that Einstein is wrong, the moving clock runs fast and not slow as Einstein claims.

We must conclude that Dingle's argument is an effective refutation of Einstein's 1905 version of relativity which relies upon the idea that clocks at rest in frames S and S' run a different rates, such that the clock in the opposite frame is always slow. This last requirement is the a major source of difficulty. In this section we will see how textbooks writers use it to apparently escape the clock rate contradiction.

The conflicting clock rate problem is neatly avoided by the 1907/1910 version of relativity. This version assumes that all clocks at rest run at the same rate. Put differently, all proper times are equal in all reference frames. The logical contradiction encountered in the 1905 version of relativity is avoided because a rest clock always runs at the proper time rate and this is a universal rate valid in all reference frames. Hence, when we choose the proper time as reference time we are always able to say the clock in the opposite frame appears to run slow.

Now by examining the inconsistency in the results obtained using the 1905 versus the 1907/1910 methods we can see the problem. The methods are different in the following way. When correctly applied, the 1905 method gives the result that moving clocks run fast. This is the result that is demonstrated by Dingle. It is clearly inconsistent with the result of the 1907/1910 method which seemingly predicts that moving clocks run slow. This brings into focus the reasons why Dingles 1962 publication has been incorrectly dismissed as fallacious. Born says regarding the symbols for the times t and t' ($\tau$) that they "refer to different physical situations; t and $\tau$ have not the same meaning ..." But as we have seen in the discussion regarding Einstein's 1907 paper on relativity that the meaning of these symbols is deliberately obscured. Born uses the same device to avoid admitting that there is an error in the theory.

The problem with taking refuge in the 1907/1910 version of relativity, in order to avoid Dingle's proof of inconsistency, is that the theory is built upon inconsistency in the meanings of the symbols to begin with. These symbols really mean the rates of clocks at rest in S and S' and not the rates as they appear to an observer in the opposite reference frame. There is nothing within the theory that supports this interpretation of the meaning of the symbols given in the Lorentz transforms. The 1907/1910 interpretation is therefore based upon an incorrect assignment of meaning to the symbols t and t' which is contrary to their correct meaning.

This is another one of the causes of confusion in relativity. When experiments show that clock rates actually are different in frames S and S' it is claimed to prove relativity. But the experiments actually refute relativity because in the 1907/1910 version of relativity the clocks rates are required to be identical. The problem here is basically a question regarding satisfaction of the requirements for a physical theory. The theory must have clear meanings for the symbols it uses in terms of their representations of physical reality. The theory of relativity fails this requirement.

To make this assertion rigorous consider the results of evaluation given in section 8.1. The symbols t and t' in the Lorentz transformation equations refer to the "universal" time scales in reference frames S and S'. They are the times which are associated with events which occur in frames S and S'. According to Einstein's operational method, t and t' are the times indicated upon any clock in frames S and S'. The symbol t refers to time in S and the symbol t' refers to time in S' in the following way. At time t in S equation 1 shows that the origin of S is located at $x'=-\beta vt$, while equation 7 indicates that at time t' the origin of S' is located at $x=\beta vt'$. So the symbols represent real parameters of the motion of the reference frames relative to each other. Thus we see that they represent the times scales in S and S', not the apparent times observed in the other reference frame.

A concise summary of the results. Examination shows that Einstein's and Dingle's proofs are not identical. They differ with respect to a crucial assumption. In the 1907/1910 proof, Einstein asserts that the time equation is solved or evaluated using the condition $x'=0$, while the alternate proof by Dingle asserts that the time equation is evaluated under the condition that $x=0$. The question now can be stated in the following way: Which of these evaluation conditions gives the correct answer consistent with the principles of the theory? The answer depends upon which version of relativity is being used. The 1907/1910 version gives the consistent result that both evaluation conditions indicate that a clock appears slow when viewed from the opposite frame. But, the 1905 theory gives the result that the clock runs either slow or fast depending upon which evaluation condition is used. This is a fault in the 1905 version that Dingle sought to point out.

There is also a second problem to be addressed. The physics community must admit that Einstein's 1905 derivation of time dilation is faulty. As shown by Dingle, the argument is incorrect. The correct argument which justified the conclusion that moving clocks run slow is only given in the 1907 and 1910 papers on relativity. The previous proof given in 1905 should be recognized as defective.

### 8.3 Is Dingle's Argument A Fallacy?

Before ending this paper, it is necessary to consider the arguments used to prove that Dingle's paradoxes and inconsistencies are fallacious. Clearly if these arguments are correct, then the proof of inconsistency given here fails. The essence of the claimed refutations of Dingle's inconsistency argument are based upon the syllogism: The theory of relativity is correct, therefore, Dingle's proof is invalid. This had become a standard tactic of the defenders of relativity since the general acceptance of the theory. No proof of its correctness need be given, it is correct therefore, any criticism of it is wrong. This is not how we expect scientific questions to be answered in the course of scientific enquiry.

The curious result is that an examination of the arguments used to refute Dingle actually prove useful towards advancing the argument given here. McCrae and I.J. Good assert that the solutions given by Dingle are contradictory. They turn the argument around. Since there can only be one valid solution, and Einstein's solution is obviously correct, therefore Dingle's solution and his assertion of an inconsistency is false. McCrae says "Dingle's assertion is obviously and demonstrably wrong. Using no more than the Lorentz transformation in his algebra, he claims to derive two different values for the same quantity. But the transformation is linear and any result that it gives can only be unique. It is trivially impossible for it to give two different answers to the same question. If Dingle obtains two different answers it must be because a) he has made a slip in the algebra, or b) his quantities are not well defined, or c) what he treats as the same quantity are two different quantities." Since it was shown here that Dingle's solution is actually the correct one, then this argument proves that Einstein's solution is wrong and the theory fails. But McCrae also claims a different type of mistake and this becomes the main line of attack since as McCrae says "Dingle has not made any mistake in his algebra".

Born, McCrae, and Good were unable to demonstrate a mathematical error in Dingle's solution. McCrae ruefully admits that. But, they open a new line of attack. Born asserts that the symbols t and t' "have not the same meaning" and that they refer to two different physical situations. But he doesn't explain what this means. McCrae says that Dingle "deals with objects to which the theory especially denies a meaning". This statement can not be a creditable remark. The objects with which Dingle deals are the physical measures of time in the frames S and S'. Surely McCrae doesn't assert that the theory denies a meaning for time in two different reference frames.

Since time in frames S and S' does have a meaningful interpretation, the only solution that avoids the inconsistency is the "alias solution". This can mean either, the two solutions are the same solution with the names changed to appear to be different solutions, or that the symbol names have been changed and apply to two physically different situations that are meaningless. Dingle's critics have used the second method, and tried to prove that Dingle's inconsistencies were fallacious because they apply to different physical situations. McCrae does this. He says that Dingle's equations "are meaningless as they stand". This approach seems peculiar because he does not make it clear what exactly the other different physical situations are and why the equations are meaningless. McCrae's attempt to do this shows that it is relativity and the meaning of its symbols that is meaningless. Referring to the rates of clocks McCrae tells us that "They become simply two different ways of putting the readings of A,B into 1-1 correspondence with each other. There are infinitely many different ways of doing this! Being no more than ways of attaching labels, there can be no question of any two of these ways being “contradictory”.

Born tries to make a similar argument by saying that the symbols t and t’ “have not the same meaning” in equations 2 and 6 of section 8.1. Born goes on to say that, the symbols t and t’ refer to different physical situations, and “these are inverse and must of course correspond to an exchange of the symbols”. But these solutions are not inverses. Hence they must either represent an inconsistency or the same physical situation as in the alias solution. In either case Dingle’s claim of an inconsistency remains valid.

At this point, the reader can see that the proof of inconsistency does not rely as much on the contradictory nature of the solutions as it does upon the failure of the solutions to be bijective inverses. If the solutions are bijective, then we can be sure that the same physical situation is being represented mathematically. If they are not bijective inverses, we can not be sure what the equations actually represent physically. Therefore, the arguments against Dingle’s inconsistencies leave the problem as it stood. The solutions have no physically consistent meaning and the contradiction stands unresolved.

We end up with another ironic situation. In their attempts to refute Dingle, the offered proofs reinforce the conclusion that there is an inconsistency. The two equations $t'=\beta^{-1}t$ and $t=\beta^{-1}t'$ are shown to have no common solution. Hence they are mathematically inconsistent. But by taking their inverses, we obtain $t=\beta t'$ and $t'=\beta t$
and these equations must also not have a common solution. Hence the assertion that in the theory of relativity these equations represent reciprocal solutions must be false.

**9.0 Conclusion- Dingle’s Challenge**

The problem of Dingle’s challenge is a difficult one for the defenders of special relativity as currently formulated. If they admit that Einstein’s 1905 derivation of time dilation is incorrect, they open the door to admitting other errors. The approach has been to deny that Einstein ever made a mistake. But this leads to criticisms of inconsistencies and paradoxes in special relativity based on the mistakes in the 1905 paper. This has been exactly what has happened. Critics have correctly attacked the errors in 1905 but have always been rebuffed by the physics community. The result has been bitter controversy and a suspicion that something is seriously wrong.

A further problem is posed by the situation. How can we be certain that Einstein’s conclusions really are correct? The problem is this: Which Einstein method is correct? Is the 1905 method correct or the 1907/1910 method? This is a serious dilemma. But relativity has an answer. Pick the method that is consistent with the experimental results. Unfortunately, this appealing option doesn’t resolve the problem. As shown here the 1905 version is supported by experiments while the 1907 version is not consistent with the experiments. But, as shown by Dingle in 1962 and confirmed here, the 1905 solution is mathematically contradictory. What is badly needed is a complete understanding of the correct mathematical derivations so that the mathematics is absolutely correct.

One of the most interesting aspects of Dingle's approach is that he does not insist that Einstein's 1905 paper contains an error. His approach is more fundamental than this. He is seeking an examination of the fundamental basis of the theory. But relativists have not even admitted that Einstein's 1905 paper contains an error. The failure to do this continues to fuel the criticism and debate regarding the correctness of the theory. This effort clearly favors the critics, because traditionally Einstein's 1905 paper has been held up as absolutely correct. But, this is a false conclusion as shown here. That the mathematics is incorrect is a bonus for the critics. Clearly this needs to be corrected. But there is a bigger problem. As shown here, it appears that Einstein's conclusion, moving clocks run slow, based on his 1905 paper is definitely incorrect. But in order to preserve the theory, with all its faults, Einstein's work and his reputation must be vigorously defended, even though his conclusions are based upon incorrect mathematical proofs.

When a person dies at a young age we say that he "lived life fast". This illustrates the error that has plagued Einstein's theory of relativity. Einstein would change this statement to be that a person who dies young "lives life slow". It illustrates the danger in using the operational method. We need to be sure that mathematical representations of physical concepts really do describe them correctly, and behave the way they are supposed to. When we don't, paradoxes and other inconsistencies are sure to arise along with controversy and acrimonious debate. The proper method is not to ignore the problem but try to solve it.

**Appendix A: Refutations of Dingle's "Paradox"**

The purpose of this appendix is to examine attempts to refute "Dingle's Paradox". The refutations[12,13,14] have considered Dingle's problem as a "contradictory paradox" and have treated it from that viewpoint. The approach has followed Born's initial response with slight improvements and alterations.

Born's attempt at refutation[12] is a typical example of the relativity establishment's response to Dingle's thoughtful criticisms. Born deliberately misinterprets and misrepresents the problem as posed by Dingle. Born presents Dingle's claim of an inconsistency as a claim of contradiction, as in a paradox. This allows Born to refute Dingle's claim by misrepresentation of Dingle's argument. Born asserts that the problem lies with Dingle's definition or statement of the problem, while ignoring that it is essentially Einstein's formulation of the problem that is wrong. Born redefines the problem so that Einstein's solution now appears to be correct, and Dingle's argument incorrect. But of course that is not the issue. Redefining the problem so that Dingle's solution is now incorrect is not the way we expect it to be resolved. Born caps this with the statement that the formula “refer to different physical situations; t and $\tau$ have not the same meaning ...” This is a curious statement. Dingle never made this claim. It is Born’s attempt to confuse and misdirect the readers understanding of the problem.

Born’s solution is as follows. It contains a serious mathematical error which will be considered later. He redefines the problem addressed by Einstein in 1905 as follows: “What is the rate of the clock in k, when viewed from the ‘stationary system’ K?” This redefinition makes Einstein’s equation (1), $\tau$=tp(1-$v^2/c^2$) to be correct. We can now interpreted it as a time dilation which has been written backwards. This is what Born intends to accomplish by his redefinition. He neglects to point out, however, that Einstein’s equation is written backwards - Einstein’s mistakes are never acknowledged or pointed out. Born redefines Dingle’s result as follows: “What is the rate of the clock in K, when viewed from the ‘moving’ system k?” As presented, Dingle’s equation is also now backwards relative to the correct solution just as Einstein’s. Why, because Born redefined the problem so that now both Einstein and Dingle are correct so there is no inconsistency or contradiction.

In order to understand this misdirection by Born, consider the original problem specification as given by Einstein “What is the rate of this clock (referring to the clock in k) when viewed from the stationary system.” Dingle’s solution address this question not the second question as Born asserts above. Clearly the solution must express the time t as a function of the time $\tau$. This is what Dingle does and Einstein does not do.

The essence of Born’s method is to obfusticate Dingle’s critique of Einstein. Dingle’s approach was not to present a contradiction, but to show that Einstein’s solution was mathematically incorrect with respect to the definition of the problem to be solved. Dingle provides the correct solution and then asks why Einstein’s incorrect solution is considered to be correct. Born responds by redefining the problem so that Dingle’s solution is wrong there is no contradiction or inconsistency. He accomplishes this by confusing the problem and misrepresenting Dingle’s criticism. He concludes by stating: “ The two cases are therefore different, the symbols t and $\tau$ in expressions (1) and (2) referring to different physical situations; these are inverse and must of course correspond to an exchange of the symbols t and $\tau$. This is exactly what expressions (1) and (2) express. There is no contradiction.” But Dingle did not claim that there was a contradiction, he claimed that there was an inconsistency. This being a polite way of saying that Einstein’s solution was wrong. Clearly Born understood this and that explains his confusing and unfair response.

There is a final ionic aspect of this story. In Born’s redefinition of the problem, Dingle’s solution (2) $t=\tau \sqrt{(1-v^2/c^2)}$ is supposed to be corrected to represent the problem that Dingle actually solved according to Born’s interpretation. But in the world of relativity things are not always easily accomplished (Because they are backwards from the way they should be.) Born's redefinition is incorrect. Born incorrectly believed that the solution to Dingle's problem was that the clock in K is fast when viewed from k. But the correct solution should be that that the rate of the clock in K, when viewed from system k is slow. So Born's redefinition of Dingle's problem is incorrect. This is why it appears that Born has deliberately redefined Dingle's problem to make Dingle's solution appear wrong. But it is Born's mistake.

It is evident that Born's refutation has not been viewed as satisfactory, because other writers have attempted to extend and clarify it. However, they all implicitly assume that Dingle's criticism is in the form of a contradiction, and therefore attempt to refute the contradiction. In Nature vol 16 Oct 14 1967 W.H. McCrea[13] echoes Born's' refutation with his own views. He says "In his 1962 paper Dingle...claimed to infer a contradiction. His assertion is false, because here he is not talking about the same thing, but two different things...More exactly, the equations concern distinct sets apart from the unique common event for which t=0 and t'=0..." This is a curious statement. Here McCrea is claiming that there can not be a contradiction for the two cases, because they have only one solution in common. Hence, the claim of a contradiction between two different and simultaneous solutions is impossible. But if we correctly interpret the system mathematically, the result is that the equations are inconsistent and have no common solution. So McCrea proves that there is an inconsistency.

The refutation by McCrea[13] is made again in a reinforced refutation included in a paper by I.J. Good[14], Physics Essays, Vol 4, No 4, 1991. Good essentially repeats the mistakes made by Born and McCrae. But he apparently felt that they were not sufficiently clear and precise because he repeats their arguments and includes some improvements. Good's basic improvement is to claim that Dingle makes a mistake in "coordinate geometry". This essentially boils down to the explanation given above. There can not be a contradiction, because there is only one solution, a trivial one -the origin, that is correct. Good says "Two different linear equations can both be true when they refer to two different straight lines."

The refutation by Good[14] basically is a sophisticated version of Born's approach. Good redefines the problem by stating "While a clock in system K registers time t, it 'appears', within the NCS of K, that a clock in the 'moving' system k registers time τ, where, as I mentioned earlier $\tau=t(1-v^2)^{1/2}$ (3). Dingle then writes down a parallel passage, deliberately following Einstein's passage almost verbatim, to prove that $t=\tau(1-v^2)^{1/2}$ (4). There is thus a seeming contradiction, since both observers (believing the special theory) agree that both Eq. (3) and (4) are true."

Good is mistaken, along with Born and McCrae, in asserting a contradiction. Dingle doesn't assert this at all. He claims there is an inconsistency, which must be resolved by determining which solution is incorrect; his or Einstein's. He does not claim that both versions are correct as misinterpreted by his critics. Good proceeds to elaborate upon the argument that there can not be a contradiction. But, this misses the point. The refutations of Dingle's arguments fail because they do not refute that his argument is wrong when compared with Einstein. They fail to address the issue, and hence miss the point entirely and become irrelevant.

We end up with another ironic situation. In their attempts to refute Dingle, the offered proofs reinforce the conclusion that there is an inconsistency. The two equations $t'=\beta^{-1}t$ and $t=\beta^{-1}t'$ are shown to have no common solution. Hence they are mathematically inconsistent. But by taking their inverses, we obtain $t=\beta t'$ and $t'=\beta t$. These equations must also not have a common solution. Hence the assertion that in the theory of relativity these equations represent reciprocal solutions must be false.

In a letter to Physics Today, January 1972, James Terrell[15] makes the following comment. "There have perennially been a few physicists who have refused for philosophical reasons to accept this easily derived result (time dilation). Herbert Dingle for one, carried on this controversy for decades until he finally realized that this prediction regarding clocks (or twins) did indeed follow from special relativity, at which point he decided that special relativity itself must be rejected[2]". Here the reference note refers to Dingle's article "Special Theory Of Relativity" discussed in this paper. Terrell's assertion seems peculiar in view of the actual content of Dingle's argument. That Terrell misrepresents Dingle's position is not unusual. It is typical of the tactics used by the opponents to Dingle's views. In reading these, alleged refutations, one is prone to wonder: Are these opponents really so thickheaded that they can't understand the basis of Dingle's argument, or are they deliberately trying to obfusticate the argument?

This appendix has demonstrated the typical approach of the defenders of Einstein's relativity. They fail to examine the possibility that Einstein makes a mistake. Dingle has consistently claimed an inconsistency in STR. This has not been approached as a simple resolution of the problem to determine who is right and who is wrong. This is because Einstein is wrong. That is why it has been necessary to approach Dingle's criticism as a contradiction and then show that there is no contradiction. This approach avoids the problem of demonstrating that Dingle's mathematics is incorrect. That issue is never addressed in any of the refutations. Hence, they are invalid. They must address why Dingle's mathematics is wrong, not confuse the issue by changing the argument into a different problem.

The main issue is not who is right and who is wrong. The issue is to determine what are the principles that must be used to answer this question. Dingle rejects answers that assert he is wrong. He accepts only answers which explain why the principles of the theory demonstrate that his answer is wrong.

## Appendix B: Einstein's Mistake

The purpose of this appendix is to explain Einstein's mistake in the 1905 paper. This is the main source of confusion and the reason the author argues that Einstein's proof is wrong. Simply explained. Einstein defines the problem in a confusing and ambiguous manner, which leads to error in definition and an incorrect mathematical procedure. As noted above, Born tries to correct the error by restating the problem. The error involves the use of the phrase "What is the rate of this clock, when viewed from the stationary system?" The meaning of this statement is unclear. It implies that the measurement involves a comparison of the time indicated on the moving clock with the stationary clock. Dingle specifically notes that this is unclear, because in his revised statement he adds (other) to clarify which clock is being referenced.

Einstein's procedure is to transform from the moving frame into the stationary frame. So a unit of time in k is compared with time in K. But the equation which Einstein obtains, does not do this. He obtains the equation which compares a unit of time marked in K with the measure of time obtained in k. This being the inverse of the equation which describes the required transformation from k into K.

But, we see that Einstein uses the phrase, "What is the rate...?" This seems to mean that he is asking for the inverse relation between the times of the two clocks. The mathematics which he presents does not transform the rates, but the time intervals, which are inversely related to rate. If we invert the equation presented by Einstein and compare the rates, we observe that the rate in k is less than the rate in K. So Einstein is correct. But Einstein did not present this as part of his proof. Hence it is incomplete, confusing, ambiguous, and unclear. This author takes the view that the correct proof should transform time intervals and show that time is dilated in frame K, or show that the rate, or frequency of the clock in k is slow relative to the reference clock in K. Einstein does not do this, it leads to confusion and Dingle correctly shows that there is an inconsistency in the theory by deriving the transformation of time interval from k to K, and showing that this leads to the conclusion that moving clocks run fast.

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