Correct Solution Of The Right Angle Lever Paradox Of Special Relativity

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1.0 Introduction

This paper presents the first mathematically rigorous correct solution to the famous paradox of "The Right Angle Lever" in Einstein's special theory of relativity. This problem, first presented in 1909, remained unresolved until a paper by Hsiao-bai Ai appeared in the Journal Physics Essays[1]. That paper presented an essentially correct solution to the right angle lever paradox, which had eluded physicists for 90 years. However, that solution, as presented, lacks mathematical rigor and a theoretical basis for its conclusions. It appears therefore to be the result of a careful choice of mathematical formulas, which give the desired result. In this paper, these defects are addressed. The reason that the solutions obtained are correct is explained based on a mathematically rigorous correct theory of relativity.

The author of this paper, during a detailed analysis of Dr Ai's paper, discovered the reasons for the mistake in the traditional treatment and the one presented by Hsiao-Bai Ai. This paper sets out to present a rigorously correct solution by a new and different method. The results obtained are identical to the previous solution by Dr Ai and others. But, the solution given here has the advantage that it explains the source of the paradox as a mistake in mathematical procedure that not only resolves the lever paradox, but clarifies many of the other paradoxes in special relativity that have remained unsolved for nearly 100 years.

1.1 Purpose

The purpose of this paper is to briefly explain the paradox of the right angle lever, its history, and a new method of solution. The new solution is the first completely rigorous and correct solution to this problem. The method employs a new theory and procedure for the solution of problems involving the transformation of physical quantities between reference frames in the special theory of relativity (STR).

The new theory is based on the discovery of mathematical mistakes in the accepted traditional theory which falsifies it conclusions, methods, and theoretical basis. In previous papers, it was demonstrated that the theory is false and presented a new mathematical theory which corrects the mistakes. Here the purpose is to show that the mathematical errors in the traditional theory are responsible for the paradoxes of relativity and how these can be removed by using the correct theory. This paper gives an example of how the correct theory is used to remove a famous unresolved paradox in relativity known as the paradox of the right angle lever.

The new method brings to the forefront a mathematical concept called evaluation, which has always been used in STR, but has not been rigorously examined and correctly applied before. The essence of the solution of the right angle lever paradox is the application of a mathematically correct procedure for evaluation. An analysis of the method used by Hsiao-Bai Ai, indicated that he was implicitly using the concept of evaluation, but unfortunately he applied it incorrectly. This paper corrects the procedure and as a result achieves a simpler and more elegant solution.

1.2 Background

The paradox of the right angle lever has its origin in a paper by Lewis and Tolman[2], which appeared in 1909. They did not present a paradox, but used the example of a right angle lever, with equal forces applied at the ends, to demonstrate that, in order to maintain equilibrium when the lever was in a moving reference frame, the transformed longitudinal force was decreased by the same ratio as the Fitzgerald-Lorentz contraction of the lever arm in the direction of motion. This amounted to saying that the force in the direction of motion was contracted in magnitude relative to its rest frame value.

Unfortunately later analysis of the transformation of force gave a completely different conclusion. Von Laue, in the first textbook on relativity[3], showed that the forces did not transform in order to preserve an equilibrium of torque between reference frames. He obtained a residual torque in the moving reference frame, and the paradox of the lever was born.

The essence of the paradox is the following. The problem is posed with the right angle lever at rest in reference frame S. The lever is oriented so that one arm is parallel to a direction of motion, usually the x axis. The other arm is parallel to the y axis. The vertex of the right angle lever is assumed to be located at the origin of the coordinates, so that when the forces are applied at the ends of the arms, they have coordinates x and -y. Equal forces are applied to the ends of the equal arms so that a rotational torque about the origin results. The forces and lengths are taken to be the same for both arms, hence the torque adds to a sum total of zero in the rest frame. The lever is now viewed from an observer in a relatively moving frame S'. When the Lorentz transformations of length and force are worked out, the result is not an equilibrium, i.e. sum equal to zero, in the moving frame. This results in a paradox. Why does the equilibrium in the rest frame transform into nonequilibrium in the moving frame? This result has been viewed as physically absurd, hence the paradox of the right angle lever.

Von Laue used the modern transformation laws for force and confirmed that the transformed torque in the moving frame was not equal to zero as in the rest frame. Von Laue noticed that by taking the derivative of the transformed torque there appeared to be a flow of energy into the system in the moving frame. This concept is known as the Von Laue "energy current". The hypothesis which has become the accepted explanation is that the energy current flows perpendicular to the velocity. "The angular momentum of the system is indeed actually being increased by a flow of energy into it at exactly the rate demanded by this turning moment." Hence the result is that the energy current balances the torque in the moving reference frame.

The two analyses used different methods. The Lewis and Tolman approach was to deduce the force based on the assumption that there was no turning moment in the transformed rest frame, while the Von Laue analysis deduced the actual existence of a torque from the theory of the transformation of force. This difference in viewpoints is the basis for the controversy over the correct solution to this problem. The paradox was that the physical assumption, no net torque in the transformed reference frame, used by Lewis and Tolman is inconsistent with the results of the theory of transformation. The traditional solution was to accept the Von Laue result, because no error could be found in the mathematical derivation.

This explanation was adopted and remained unquestioned until 1965 when Arzelies disputed the explanation. There followed a spirited controversy which failed to adequately resolve the paradox. Since then there were many different attempted solutions and explanations.

In his solution to the paradox, Hsiao-Bai AI showed that there were four different ways to dispel the paradox, but none of them corresponded with the traditional solution. This prompted Dr. Ai to suggest that the problem was the way length was defined in special relativity. In his solutions, the paradox was dispelled by a different method of defining the evaluation of the Lorentz transformations, but there was no rigorous inquiry into the theory of this

1.3 Approach

The analysis presented here addresses the lever paradox from the viewpoint that the solution presented by Hsiao-Bai Ai is basically correct. He presents four different ways to dispel the paradox, in his demonstration of the solution. Ai's approach is basically the same as the approach used previously by advocates of a method called the asynchronous method of solution. This approach attributes the paradoxical result to incorrect definitions of length which result in errors in the calculations. This paper proposes to show that the problem is not the transformation of distance, which has been the claim by advocates of the true transformations and the asynchronous method, but the fault lies in an incorrect calculation of the transformation of force. This fault is corrected by a correct evaluation procedure, which indicates that the traditional transformations of force used in the lever paradox are incorrect. When the new transformation equations are applied as derived in this paper, the paradox disappears, since the calculated torque is the same in all reference frames and is equal to zero. hence there is no turning moment and no "energy current" is needed to explain the result. The results are also the same as obtained by Hsiao-Bai AI, by his method.

To adequately demonstrate why the traditional conclusions regarding the transformation of forces in STR are incorrect, the concept of evaluation will be developed and applied to the theory of the transformation of physical quantities. The result shows that particular care is needed in carefully defining the transformation problem. The analysis of this problem will be traced back to Einstein's original papers, which fail to apply adequate rigor to the mathematics of transformation. Once the idea of evaluation and the correct procedure for evaluation is carefully laid out, the procedure will be applied to the transformation of force. Then it is demonstrated that a mistake in the evaluation procedure, gives an incorrect result in the calculation of force.

It is clear that most readers will refuse to accept the thesis that Einstein's theory of relativity is wrong and that a new theory is needed. Therefore, much of this presentation will be to acquaint the reader with facts that refute this view. The method used will be to take the reader through the history of the author's own attempts to understand the problems and issues, which arise from the paradoxes of relativity. The main issue to be confronted is this. If the theory is correct in its present form, then why do paradoxes exist? But the idea that there are paradoxes is itself a false viewpoint. These so called paradoxes are contradictions which falsify the theory, and not minor difficulties in interpretation as apologists for the theory maintain. Once this basic fact is accepted as valid and correct, then it will be possible for physical theory to advance in a forward direction.

1.4 Hsiao-bi Ai's Straight Forward Method

This is the name Hsiao-bai Ai gives to his method. He presents four different solutions for the lever paradox problem. This is the first mystery that must be solved. Why are four solutions required? This is not made clear. From the traditional view, there should be only two solutions. One for an observer at rest in frame S, or Σ in Ai's notation, and one for an observer at rest in S', or Σ' in Ai's notation. These correspond to the usual rest or stationary and moving frames. Since there are only two Lorentz transforms, it appears we only need to have two solutions and not four. This is an important discovery and contribution to the final solution, for which Dr. Ai deserves much credit. The reason there are four solutions only becomes rigorously justified mathematically in the correct theory developed by the present author.

Hsiao-bi Ai's method is based on the work of others which he acknowledges. These authors are H. Arzeles, G. Cavalleri, and G. Spinelli. But these authors also acknowledge the work of F. Rohrlich. Investigation of the literature shows that papers written on the lever paradox are almost as numerous as the twins paradox. Many of the proposed alternative solutions are also correct solutions which eliminate the paradox. But these were always rejected by the physics community, because of the entrenched belief that Einstein's theory is correct. The rejection of these correct solutions will be examined.

As pointed out by Dr. Ai, these rejected solutions to the paradox involved the assertion that the transformation of length resulted in a dilation and not a contraction as claimed by Einstein. However, when the dilation of length is used to calculate the total torque in the moving frame, the result was zero, a consistent solution and not a contradiction, as obtained when Einstein's solutions are used. So the question arises, why should Einstein's theory be asserted as true, when it leads to contradictions, while theories which do not produce contradictory results are rejected? This kind of physics seems irrational and unscientific. Why should we believe it?

2.0 The Mathematical Concept of Evaluation

The title of this section uses the word evaluation to refer to a mathematical procedure frequently used in traditional relativity without giving it a name. The author uses this word to designate the particular technical procedure in the mathematics of relativity in order to obtain a solution. The failure to explicitly define this concept is the root cause of the problem in relativity. In his fundamental papers, Einstein deliberately uses an obscure, vague and undefined method to produce solutions. Later textbook writers were also careless and vague about evaluation. The result is that no one really understood what he was doing. They just copied what Einstein did without really knowing why. This problem became entrenched over time, because little attention was given to the problem. It was simply assumed that Einstein knew what he was doing, even though no one really understood why. Thus, the first step to correct this, must be to clarify what evaluation is, and what the correct method must be. To do this we will first examine the procedure as used by Einstein.

The purpose of this section is to discuss the critical notion of evaluation as it is used in the transformation of spatial and temporal intervals in Einstein's STR. Evaluation is an important part of the methods used in special relativity which lead to the famous conclusions that space contracts and time dilates. These conclusions can not be obtained without application of the mathematical procedure termed evaluation. This procedure is routinely applied without appreciation of its importance. Here it is shown that the determination of the correct procedure for evaluation is critical to the conclusions obtained in STR.

Evaluation defines the standard of comparison. What does this mean? When we transform space or time from frame A to frame B we need a reference of comparison in order to say that they are the same or if they are different. Obviously this standard is the standard of measure used in the frame where the measurement is to be performed. But this is where things start to go wrong. In STR, we begin by asserting that the standards of measure are the same in frames A and B by hypothesis or assumption when we define the problem. Now when in STR we discover that space contracts and time expands, we must attribute this to a phenomena occurring in the opposite reference frame. Special relativity claims that something metaphysical happens to time and space in the opposite rest frame where the observer makes his measurements. Time is dilated and space contracts. Unfortunately the meaning of this conclusion is fundamentally unclear. It has led to paradoxes and confusion in its interpretation that has not been adequately resolved.

The mathematical procedure which allows Einstein to assert that the transformed result in a reference frame is different from that of a standard of comparison is evaluation. Evaluation defines the reference frame of comparison. The usual procedure in STR is to transform from a rest frame into a relatively moving reference frame with the moving reference frame now interpreted as playing the role of rest frame. This does not automatically result from the application of a Lorentz transform. Initially, we will see that the procedure for evaluation appears to be that which establishes the moving frame as a new rest frame for purpose of comparison. Since this idea is unfamiliar, we will show how it is accomplished by using the examples presented by Einstein in his foundational papers for STR. However, the reader is forewarned that this initial interpretation will turn out to be false.

2.1 Einstein’s Use Of Evaluation

Here the thesis is presented that Einstein’s careless use of evaluation and his lack of care in explaining the conditions of evaluation results in most of the paradoxes and inconsistencies in STR

Lets examine the derivation of length contraction and time dilation in the 1905 paper. Einstein uses the Lorentz transform equations. The proof of length contraction proceeds by a backwards method. This is to define quantities in the moving frame first and then ask, what is viewed in the stationary frame. Einstein defines a sphere in the moving frame and then imposes an evaluation condition upon the Lorentz transform equations in order to solve for the equation in the stationary system. The evaluation condition imposed is that time in the stationary frame is t=0. This is a specific evaluation condition. Evaluation occurs for time t=0 in the stationary frame. It establishes that the standard of length for comparison is defined in terms of distance defined in the stationary frame of reference.

Time dilation is approached completely differently. A clock is defined at rest in the moving system. The Lorentz transform for time is then used to go backwards from the moving frame into the stationary frame as before for space. But now instead of specifying that the time measure be performed at x=0 in the stationary system where a reference clock is located, Einstein applies a completely different evaluation condition. He specifies that x=vt, and substitutes this into the Lorentz transform equation. It s important to realize that if he had merely set x=0 the resulting method would have given a result consistent with length, a Lorentz contraction of time. He would have correctly concluded that time contracts as well as space. But, he did not do this. Instead he concluded that time in the moving frame contracts relative to the stationary frame. This has been incorrectly called time dilation because of the confusion involved in its interpretation, which arose from Einstein's 1907 paper. Here is the main mistake in the theory.

Now in Einstein's 1907 paper, the transformation of space is made more rigorous. But, the evaluation condition is unclear. Instead of specifying that t=0 in frame S, as he did in 1905, he says "the following relations hold...relative to the reference system S at all times t of S..." This is a puzzle. What does he mean here? I think it is a mistake, but it is confusing because it implies that the measurement performed in S is independent of time of the measurement. This may be true, but the point is that the evaluation occurs at a specific instant of time when measurements are all performed simultaneously in S. This mistake is corrected in 1910 where he says that "At any instant t of the system S we will have the following relations.."

This amounts to the following. Given the Lorentz transform $x'=\beta(x-vt)$, set t=0 and obtain the result that; $x'=\beta x$. Now inverting this result to obtain that the space coordinate in S as a function of the coordinate in S' we have the result that $x=\beta^{-1}x'$.

A mystery is why Einstein does not apply the same procedure for time. The equivalent evaluation condition is x=0. The Lorentz transform is $t'=\beta(t-vx/c^2)$. Setting x=0 gives, $t'=\beta t$, which is inverted or reversed as before to obtain the result that; $t=\beta^{-1}t'$. A result which is in the same form as the result for space. The main question is why Einstein does not do this. In the 1907 paper he hides the procedure whereby he obtains the result that $t=\beta t'$. A result that he interprets to indicate that moving clocks run slow.

It seems clear that he has again used the evaluation condition x=vt. The procedure is as follows. Given $t'=\beta(t-vx/c^2)$, substitute x=vt and obtain the result: $t'=\beta(t-x^2t/c^2)=\beta t(1-v^2/c^2)=\beta^{-1}t$. But Einstein does not stop here, he inverts this result as for space to obtain that $t=\beta t'$. The last step of inversion seems justified by the need to express the time as transformed from the moving frame into the stationary frame. This is the result that justifies that moving clocks run slow. But we notice that it depends on the unjustified assertion that x=vt. This is not the same condition as imposed on space where t=0 was used. We saw above that using x=0 gives a result consistent with the result for space contraction, so the result that moving clocks run slow is derived because Einstein asserted the condition x=vt. But there does not seem to be a physical justification for this evaluation condition. This condition says that the clock used to compare time in S with time in S' obtained from the moving clock is moving in frame S with the velocity v along the x axis. This doesn't seem to make sense so it must be wrong.

In Einstein's 1910 paper, the divergence between the methods of evaluation for time and space finally becomes crystal clear. In this paper the method used in 1907 for space is almost exactly repeated with the clarification that the Lorentz transform is evaluated at the instant of time t=0 in S. The main revisions occur in the derivation and evaluation of time dilation. Einstein shifts to a simpler method. He makes two significant changes. He uses the inverse Lorentz transform in place of the Lorentz transform, and he discards the condition x=vt and replaces it with the condition that time is evaluated at x'=0. (Notice however that these are really equivalent conditions.) Hence instead of setting the condition x=0 in the Lorentz transform equation for time, he now uses the condition x'=0 in conjunction with the inverse Lorentz transform for time, i.e. the equation: $t=\beta(t'+vx/c^2)$. This reduces to $t=\beta t'$, the result which proves that moving clocks run slow when the evaluation condition x'=0 is specified. But this merely says that the clock in S' is located at x'=0, a condition that seems unnecessary. What is not specified is the location of the comparison clock in S, the clock that is used to determine the difference, if any, between the time marked by the clock in S' and the reference clock in S at rest at x=0.

We see that evaluation determines the result which we obtain for the transformation of either time or space. Whether the moving clock runs slow or runs fast is not determined by its motion, but by the evaluation conditions applied to the Lorentz transform equations. Hence, we see that there is no compelling reason to accept the result that moving clocks run slow is the correct prediction, because the theory also predicts that moving clocks run fast. Hence unless a physical reason can be established to determine why the conditions of evaluation, specified by Einstein are the correct ones, then we can not say which prediction of the theory is the correct one which is to be compared with experiment.

If the reader is bewildered by the preceding analysis, he is no worse off than students of relativity for the last 100 years. The problem is exceedingly and needlessly complicated. A lot of confusion and difficulty, not to mention controversy, could have been avoided by a careful and rigorous analysis of evaluation in Einstein's fundamental papers. But this never happened. The main point that the reader should consider is the fact that there is no physical necessity in the method used. Einstein simply picked an evaluation procedure which yielded a result consistent with the Larmor-Lorentz theory.

Different evaluations however are as justified as the ones chosen. Thus, no physical reasoning is applied at all. This lack of physical reasoning in the choice of evaluation condition, is a serious methodological problem. It makes it appear that the theory is correct not by physical necessity, but by accident.

The explanation for the examination of four different cases in the solution to the lever paradox by Hsiao Bi Ai can now be given. The reason is that there are two Lorentz transforms and two different ways to evaluate each of them, i.e. evaluation in S and evaluation in S. Hence there are four different evaluation solutions. So there are four different ways to define solutions to the lever paradox.

2.2 True Versus Apparent Transformations

In the 1960's F. Rohrlich introduced the notion of true and apparent transformations into special relativity[4]. When these concepts are examined in the light of evaluation, it is clear that these define different conditions of evaluation.

Unfortunately, Rohrlich's definitions are based on an inadequate understanding of the concept of evaluation. He expresses the idea of an apparent transformation as follows: "...the Lorentz contraction is only indirectly related to Lorentz transformations and is by itself not the Lorentz transform of a length. We shall refer to these transformations of subjective measurements as apparent transformations." This statement is flawed because it is not clear, as well as controversial. The argument regarding the correct or true nature of the transformations of distance or length is an old one. The question has been, are these space contractions real, or merely artifacts of measurement. The traditional view has been that they are real. Rohrlich now asserts that these transformations which result in contractions are merely apparent. This is controversial. The argument goes back as far as the very birth of the theory. It surfaced again in the late 1930's and again in the 1960's.

Rohrlich proceeds to express the nature of a true transformation by an example. He calculates the volume of a cube. He obtains that the volume is dilated, not contracted and calls this the true transformation. Thus we can simplify the definitions as follows: apparent transformations are given by the equation of contraction, for example $x'=\beta^{-1}x$, and true transformations are given by the equation of dilation, for example $x'=\beta x$.

True transformations can be obtained using the Lorentz transform as follows: Define transform equation as $x'=\beta(x-vt)$. Now evaluate at time t=0 in frame S to obtain: $x'=\beta x$. Apparent transformations use the evaluation condition $t=vx/c^2$. In this case we have the result that: $x'=\beta(x-v^2x/c^2)=\beta x(1-v^2/c^2)=\beta^{-1}x$.

To add to the confusion, we can derive these same results by a different method as follows. Instead of using the Lorentz transform, we can use the inverse Lorentz transform and then invert the result. The true transformation is obtained using: $x=\beta(x'+vt')$. But now we see that we cannot insert the condition $t=0$ as before. Instead of doing this, the procedure is to apply the equivalent condition $t'=-vx'/c^2$, which amounts to the same thing. Therefore, $x=\beta(x'-v^2x'/c^2)=\beta x'(1-v^2/c^2)=\beta^{-1}x'$. Inverting this gives the true transformation as above; $x'=\beta x$. Notice that the substitution of $t'=-vx'/c^2$ is used in place of the requirement that $t=0$.

The apparent transformation is obtained with the specification that $t'=0$ in place of $t=vx/c^2$ (which was used previously) and obtaining $x=\beta x'$. This result is now inverted to obtain the apparent transformation $x'=\beta^{-1}x$ as given above. Notice that $t'=0$ is used in place of the condition $t=vx/c^2$. Since the same result is obtained as before, these conditions must be equivalent. The difference depends upon which Lorentz transform is used.

We now see the nature of a transformation, either true or apparent, is determined by the evaluation condition that is imposed. The same condition imposed gives the same result irrespective of whether the Lorentz or inverse Lorentz transformation equation is used. What is critical is that the same or equivalent evaluation condition be imposed in both cases.

2.3 Synchronous and Asynchronous Transformation Methods

A viewpoint different from that regarding the true and apparent transformations is given by Cavalleri and Salgarelli. They say regarding the discrepancies and disputes involving the correct transformation formulas in special relativity "The reason of the discrepancies is that no author considered that there are two possible different formulations and that, in each treatment, particular definitions of physical quantities are more natural or even consequent. Comparing the quantities, defined by one of the two formulations, relevant to two different observers, a kind of transformation (true or apparent) follows. We will examine the two treatments. The first one is the usual synchronous formulation, ...the second formulation proposed here is called asynchronous since the laboratory observer must add additive quantities asynchronously if the quantities are relevant to different positions."[5]

This is an example of a movement, that began in the 1960s and continued into the following decade, which attempted to unify the concepts of relativity and eliminate the paradoxes and replace them with rigorous deductive conclusions. The new approach was founded on the recognition that there were different ways to deduce relativistic transformations. As we saw above, these different methods depend on the method used. But, essentially they depend upon the evaluation.

The idea that was used in place of an explicit notion of evaluation was the notion of synchronous and asynchronous methods of solution. These were explained as different ways to define the transformation of physical quantities. The asynchronous method which resulted in the true transformations was advocated as a way to eliminate the discrepancies and paradoxes that had plagued relativity for many years.[6,7,8]

The new terminology can be understood as follows. The synchronous formulation results in the apparent transformations defined by Rohrlich, and the asynchronous formulation results in true transformations. The argument was that by using the asynchronous method which resulted in true transformations, the discrepancies in relativity would be cleaned up. Unfortunately this effort did not receive this blessing of the traditional relativity establishment. It raised the old arguments that were thought to be settled. The establishment view was, and still is, that the paradoxes are not real but only seemingly contradictions. In their view, there were no discrepancies to correct. The introduction of true transformations threatened to topple the edifice of traditional relativity because it asserted that Lorentz contraction, one of Einstein's fundamental results, was only an apparent result and not a true transformation of space in relativity. So we can see why these ideas were not generally accepted and that today's textbooks do not reflect these ideas.

In this section, it will be shown that the terminology of synchronous and asynchronous formulations is not a fundamental change in the methods of relativity, but an attempt to clarify the conditions of solution of the transformation equations. The two methods, just as in the case of true and apparent transformations are really just different ways to specify the conditions of evaluation. Hence we will see that we can assert that synchronous transformation corresponds to the condition that measurement is performed at the same time in the opposite reference frame, while the asynchronous condition applies to different times.

To make these ideas more concrete, we will say that the synchronous method requires that the physical quantities be evaluated in the co domain of the transformation, while the asynchronous method requires that all quantities be evaluated in the domain of the transformation. Here the domain and co domain are defined in terms of the frames of definition and measurement. When we transform from frame S to frame S', we define the quantities to be transformed in S and call this the domain of the transformation. The results of the transformation are to be measured in the opposite frame S', which is called the co domain. Defining the quantities this way places some mathematical rigor on the solution. For example, when we have clearly defined the domain and co domain for the transformation problem, it is clear whether or not a final step of inversion is necessary. Consider the above examples. If the co domain coordinate or quantity appears on the right hand side and the domain on the left, then we know that a inversion step is necessary in order to have the transformation express the co domain result as a function of the domain quantities.

It is now clear that the asynchronous method results in true transformations. Consider the first example given above for a true transformation from S into S'. Here the domain is S and the co domain is S'. The evaluation condition is t=0, which is a condition specified in the domain S. Therefore, the result is a true transformation because evaluation is in the domain S. The asynchronous method corresponds to the apparent transformations. The apparent transformation in the second example results because evaluation is defined to be in S'. This is not immediately apparent until we realize that the condition $t=vx/c^2$, corresponds to t'=0, implying evaluation in frame S'.

3.0 Evaluation Solutions For Relativity

The mathematical method used here is to first solve the system of Lorentz and inverse Lorentz equations for space and time simultaneously using a specified condition of evaluation. Here the term evaluation is used in the same sense as it is used when a polynomial equation is solved for its roots by setting the equation to equal zero and solving for the indeterminates. The procedure used here is similar. A selected variable is set to zero, and the resulting solutions are obtained. Solutions are obtained by setting one of the following four variables equal to zero, and then solving for the remaining three. The following variables are set equal to zero and the resulting solutions obtained by evaluation: x=x'=t=t'=0, each taken in turn.

The Lorentz transformation equations in a simplified form are assumed as follows:
$x'=\beta(x-vt)$ $t'=\beta(t-vx/c^2)$ $x=\beta(x'+vt')$ $t=\beta(t'+vx/c^2)$ $\beta=(1-v^2/c^2)^{-1/2}$
Here there are four equations which express the simultaneous solutions for the transformation of coordinates. These equations are defined in the usual way in terms of two relatively moving reference frames S and S'. Where the origin of frame S' is in motion with velocity v in the positive x direction of S.

Notice that β is greater than unity when v is greater than zero, and that β^{-1} is less than unity when v is greater than zero. An equation of the form $t'=\beta t$ results in a dilation of the variable t' with respect to t because t' is greater than t The equation $t=\beta^{-1}t'$ results in a contraction of the variable t with respect to t' because t is less than t'. The definition of β implies that it is always equal to or greater than unity, and can never be less than unity.

The coordinate frames S and S' are assumed to be orthogonal coordinate systems with the requirement that time is defined such that t=t'=0 occurs when the origins coincide; i.e. x=x'=y=y'=z=z'=0 at t=t'=0. The axes for the x, y, and z directions are assumed to be parallel, and the y and z coordinates are assumed to be identical and coincide when the origins coincide at t=t'=0. The purpose of the solutions is to determine the relations governing the transformation of the x and t coordinates according to the Lorentz transform equations.

4.1 Results for x=0 (Specification of an evaluation in space)

To consider the role of evaluation in space, we determine the simultaneous solution of the four equations when we specify the condition that x=0. The results are as follows:
Equation 1: $x'=\beta(x-vt)=-\beta vt$
Equation 2: $t'=\beta(t-vx/c^2)=\beta t$
Equation 3: $x=\beta(x'+vt')^{=}0$, Therefore $x'=-vt'$
Equation 4: $t=\beta(t'+vx'/c^2)=\beta t'(1-v^2/c^2)=\beta^{-1}t'$

Notice that equation 4 is the inverse of equation 2, so they are the same solution. Equation 4 is solved by substitution with the result from equation 3. Therefore, from equations 2 and 4 we have the following solution for the condition x=0: $t'=\beta t$. The solutions for equations 1 and 3 give the results $x'=-\beta vt=-vt'$, from which we conclude that $t'=\beta t$. A result which is the same as obtained from equation 2 which is the primary result for the condition x=0.

4.2 Results for x'=0 (Specification of an evaluation in space)

To consider the role of evaluation in space, we determine the simultaneous solution of the four equations when we specify the condition that x'=0. The results are obtained as follows:
Equation 5: $x'=\beta(x-vt)=0$, Hence $x=vt$
Equation 6: $t'=\beta(t-vx/c^2)=\beta t(1-v^2/c^2)=\beta^{-1}t$
Equation 7: $x=\beta(x'+vt')=\beta vt'$
Equation 8: $t=\beta(t'+vx'/c^2)=\beta t'$.

Notice that equation 6 is the inverse of equation 8, so they are the same solution. Equation 6 is solved by substitution with the result from equation 5. Therefore, from equations 6 and 8 we have the following solution for the condition x'=0: $t=\beta t'$. The solutions for equations 5 and 7 give the results $x=vt=\beta vt'$, from which we conclude that $t=\beta t'$. A result which is the same as obtained from equation 8 which is the primary result for the condition x'=0.

4.3 Results for t=0 (Specification of an evaluation in time)

To complete the analysis of evaluation, we now consider the role of evaluation in time. We determine the simultaneous solution of the four equations when we specify the condition that t=0. The results are as follows:

Equation 9: $x'=\beta(x-vt)=\beta x$
Equation 10: $t'=\beta(t-vx/c^{2})= -\beta vx/c^{2}$
Equation 11: $x=\beta(x'+vt')^{=}\beta x'(1-v^{2/}c^{2})=\beta^{-1}x'$
Equation 12: $t=\beta(t'+vx'/c^{2})=0$, therefore $t'=-vx'/c^{2.}$

Notice that equation 11 is the inverse of equation 9, so they are the same solution. Equation 11 is solved by substitution with the result from equation 12. From equations 9 and 11, we have the following solution for the condition that $t=0$: $x'=\beta x$. The solutions for equations 10 and 12 give the results $t'=-\beta vx/c^{2}=-vx'/c^{2}$ from which we conclude that $x'=\beta x$. A result which is the same as obtained from equation 9 which is the primary result for the condition $t=0$.

4.4 Results for t'=0 (Specification of an evaluation in time)

To consider the role of evaluation with the opposite condition, we determine the simultaneous solution of the four equations when we specify the condition that $t'=0$. The results are as follows:
Equation 13: $x'=\beta(x-vt)=\beta x(1-v^{2/}c^{2})=\beta^{-1}x$
Equation 14: $t'=\beta(t-vx/c^{2})=0$, therefore $t= vx/c^{2}$.
Equation 15: $x=\beta(x'+vt')^{=}\beta x'$
Equation 16: $t=\beta(t'+vx'/c^{2})= \beta vx'/c^{2}$.

Notice that equation 13 is the inverse of equation 15, so they are the same solution. Equation 13 is solved by substitution with the result from equation 14. From equations 13 and 15 we have the following solution for the condition that $t'=0$: $x=\beta x'$. The solutions for equations 14 and 16 give the results $t=\beta vx'/c^{2}=vx/c^{2}$ from which we conclude that $x=\beta x'$. A result which is the same as obtained from equation 15 which is the primary result for the condition $t'=0$.

4.5 Comments on Above Results

The solutions presented as Equations 2 and 8 obtained with the evaluations $x=0$, and $x'=0$ correspond to the usually accepted reciprocally related equations which demonstrate time dilation for moving clocks in the special theory of relativity. These results lead to the commonly used description that moving clocks run slow. Equation 5 was used by Einstein in 1905 to deduce the result in equation 6 as time dilation. In 1910 Einstein revised his method and obtained the equation for time dilation as equation 8. Later authors cited equation 8 as the equation for time dilation. During the 1930s, some textbooks cited equation 2, and since then either equation 2 or 8, and sometimes both, have been used as the definition of time dilation. However, most textbooks continue to specify equation 8 as time dilation. Sometimes the inverse solutions given by equations 4 and 6 are referred to as equations for time dilation, but they are not generally or widely accepted as defining this concept. Notice that Einstein deduces time dilation using evaluation in frame S' by setting x'=0.

The solutions for t=0 and t'=0, given in equations 9 and 15, do not lead to the usually accepted reciprocally related solutions for FitzGerald-Lorentz contraction in the special theory of relativity. Instead, the inverse solutions given in equations 11 and 13 are given as the traditional solutions for the Lorentz-FitzGerald contraction. This leads to the commonly used description that moving objects physically contract in the direction of motion. Einstein obtained the result given by equation 11 in his 1905 paper. The primary solutions given by equations 9 and 15, do not appear as solutions in the special theory of relativity. Notice that Einstein deduces the FitzGerald -Lorentz contraction using evaluation in frame S by setting t=0. An evaluation procedure that is opposite to the evaluation used for the derivation of time dilation. In other words, time is evaluated in frame S' while space is evaluated in frame S. These results lead to the commonly used description in special relativity that in a moving frame of reference space contracts and time dilates.

The method used here provides the traditional results in a very simple and efficient manner. But notice that there are additional equations which the traditional theory does not obtain. These results show that solutions obtained by the traditional methods are not complete. Equations 1, 7, 12, and 16 appear here for the first time. In addition, this is the first time that a fully complete solution set of all possible solutions has been obtained.

To summarize, when an evaluation condition is imposed upon the system of Lorentz transformation equations, two simultaneous solutions result. One solution, which we call the primary result, and its inverse, which we call the secondary result. The primary results appear in equations 2, 8, 9, and 15, with the secondary results given by equations 4, 6, 11, and 13. The primary results always give a dilation of the transformed variable while the secondary result is always a contraction. The primary result is distinguished from the secondary because it is also obtained as a solution of the remaining two equations which are redundant with the result of the primary solution. The primary equation is considered the solution because the secondary solutions are inversely related to the primary.

The following primary solutions are therefore interpreted as the solutions for the four evaluation conditions. They are as follows:
For evaluation x=0, the solution is equation 2: $x'=\beta x$.
For evaluation x'=0, the solution is equation 8: $x=\beta x'$.
For evaluation t=0, the solution is equation 9: $t'=\beta t$.
For evaluation t'=0, the solution is equation 15: $t=\beta t'$.

Notice that none of these solutions is a contraction, they are all dilations of coordinates.

4.6 Discussion Regarding the Interpretation and Meaning of The Results

Consider the case of evaluation with x=0. There are four resulting equations which have the following interpretation. Equation 3 gives the equation for the motion of the origin of the coordinate system S, the coordinate x=0, in terms of the time and space coordinates of S'. It represents the equation of motion obtained by an observer in S' in terms of his coordinates. Equation 1 gives the motion of the origin of S relative to space in S' in terms of time defined in system S. Thus, an observer in S can calculate his position in S' using this equation and a clock at rest in S. An observer in S' measures the motion of the origin of S in terms of his space coordinates using a clock at rest in S' that records time in terms of t'. From the point of view of the observer in S using time t, the coordinates measured by an observer in S' using time t' are dilated in terms of the time measured in frame S. This is the result indicated by equation 2, the primary solution. The secondary solution given by equation 4 specifies how measurements of time performed in frame S' are transformed back to the time standard of frame S.

Consider the case of evaluation with x'=0. There are four resulting equations which have the following interpretation. Equation 7 gives the equation for the motion of the origin of the coordinate system S', the coordinate x'=0, in terms of the time and space coordinates of S. It represents the equation of motion obtained by an observer in S in terms of his coordinates. Equation 5 gives the motion of the origin of S' relative to space in S in terms of time defined in system S'. Thus, an observer in S' can calculate his position in S using this equation and a clock at rest in S'. An observer in S measures the motion of the origin of S' in terms of his space coordinates using a clock at rest in S that records time in terms of t. From the point of view of the observer in S' using time t', the coordinates measured by an observer in S using time t are dilated in terms of the time measured in frame S'. This is the result indicated by equation 8, the primary solution. The secondary solution given by equation 6 specifies how measurements of time performed in frame S are transformed back to the time standard of frame S'.

When we consider the meaning of the system of equations for evaluation specified by a time, we are considering the concept dual to the meaning of the motion of the origin in space. This dual concept is the equation of synchronization of time. The equation gives the time lead or lag of a clock at a space coordinate relative to a clock located at the origin of space coordinates. The equation specifies the time measured at a space coordinate when the clock at the origin reads zero time.

Consider the case of evaluation with t=0. There are four resulting equations which have the following interpretation. Equation 12 gives the equation for the synchronization of the clocks in coordinate system S' in terms of the time and space coordinates of S'. It represents the equation of synchronization for an observer in S' in terms of his space coordinates. Equation 10 gives the equation of synchronization of time in S' relative to space coordinates defined in S. Thus, an observer in S can calculate his time lag relative to the clocks in S' using this equation and his location in S. From the point of view of the observer in S using distance x, the coordinates measured by an observer in S' using distance x' are dilated in terms of the distances measured in frame S. This is the result indicated by equation 9, the primary solution. The secondary solution, given by equation 11 and usually called the Lorentz-FitzGerald contraction, specifies how measurements of distance performed in frame S' are transformed back to the distance standard of frame S.

Consider the case of evaluation with t'=0. There are four resulting equations which have the following interpretation. Equation 14 gives the equation for the synchronization of the clocks in coordinate system S in terms of the time and space coordinates of S. It represents the equation of synchronization for an observer in S in terms of his space coordinates. Equation 16 gives the equation of synchronization of time in S relative to space coordinates defined in S'. Thus, an observer in S' can calculate his time lag relative to the clocks in S using this equation and his location in S'. From the point of view of the observer in S' using distance x', the coordinates measured by an observer in S using distance x are dilated in terms of the distances measured in frame S'. This is the result indicated by equation 15, the primary solution. The secondary solution, given by equation 13 and usually called the Lorentz-FitzGerald contraction, specifies how measurements of distance performed in frame S are transformed back to the distance standard of frame S'.

5.0 The First Contradiction Theorem and Corollaries

The purpose of this section is to establish by rigorous mathematical proof the claim that the traditional solution of the right angle lever paradox is mathematically false. To do this the following crucial theorems are established which form the basis of the proof. The essence of the theorems is to make rigorous the observations is the preceding section regarding the mistaken conclusions of Einstein's fundamental papers. The error is that using different evaluation conditions; i.e. taking one evaluation in S and then another in S' leads to a contradiction. This best example is Einstein's claim of length contraction, obtained by evaluation in frame S, can not be consistently combined with the claim of time dilation, obtained by evaluation in frame S'. Expressed differently, the two different solutions given in equations 11 and 8 can not both be used in a mathematical derivation without mathematical contradiction. Since both of these equations are used in the traditional solution of the right angle lever problem, the resulting transform of force is incorrect.

According to the traditional interpretation of Einstein's relativity, the 16 equations given in section 4.0 should represent all the solutions needed for the transformation of intervals of time and space. This interpretation assumes that all the solutions are true statements in the theory of relativity, because according to Einstein's relativity postulate, all reference frames are equivalent as rest frames. Hence the results obtained for evaluation in frame S in sections 4.1 and 4.3 are true, and the results obtained for evaluation in frame S' in sections 4.2 and 4.4 are also true. This follows from the principle of relativity and symmetry. The following theorem proves this is false.

5.1 First Contradiction Theorem

Theorem: If the same standard of measure is valid in frames S and S', then either the solutions with evaluation in S are true, or the solutions with evaluation in S' are true, but solutions resulting from both evaluations; i.e. S and S' together, taken as simultaneously true, are contradictory.

Proof: Suppose both solution sets in section 4.0 are true and represent valid solutions. Then, for every true statement of the form A=L(B) obtained using an evaluation in S (S'), there exists a corresponding true statement of the form B=L(A) obtained using an evaluation in S' (S). Here the function L denotes a Lorentz transform, or an inverse transform, evaluated in S (S'). Both of these statements cannot be true simultaneously, so there is a contradiction.

In these statements, the objects A and B have the same meaning and are the same symbols in both statements. The functional relation can be expressed either as greater than or less than depending on the actual function. For $A=\beta B$, since $\beta>1$, we have A>B. For $A=\beta^{-1}B$, since $\beta^{-1}<1$, we have A<B. The procedure for forming statements with the opposite evaluation is Einstein's procedure for forming the inverse Lorentz transform. Given any valid statement, the true statement for the opposite evaluation is formed by exchanging symbols; i.e. replace A with B and B with A. The statements formed in this way are contradictory to the original statements. Hence the two statements taken together form a contradictory pair. Therefore, if we claim that only statements resulting from evaluation in S (S') are true, there is no contradiction because we no longer claim statements using the opposite evaluation S' (S) are true. QED

To make this clear consider the following example using equation 2 obtained by evaluation in S. We have for the condition x=0: $t'=\beta t$. Hence t'>t. By the procedure of exchanging symbols we have the result given in equation 8 obtained by evaluation in S'. We have for the condition x'=0: $t=\beta t'$. Hence, t>t'or t'<t. These results both taken as true are contradictory since they imply that t' is both greater than and less than t simultaneously. So both statements can not be true, otherwise we have a contradiction.

Alternate Proof by Enumeration of Solutions: We enumerate the contradictory equations for transformation of time as equations 2 and 8. Section 4.5 indicates these are the primary solutions for time, the secondary solutions are equations 4 and 6. We show that equations 2 and 8 are contradictory as follows. Solving equation 2 we have $\beta=t'/t$ and solving equation 8 we have $\beta=t/t'$. Hence $t'/t=t/t'$ or $(t')^2=t^2$. Therefore $t=t'$, which contradicts both equations 2 and 8. By similar procedures contradictions are obtained for equations 4 and 6 the secondary solutions for time. For the primary solutions for transformation of space equations 9 and 15 there is a contradiction and for the secondary solutions equations 11 and 13 there is a contradiction. The reader can easily prove the contradiction using the method given above for equations 2 and 8.

5.2 First Corollary To First Contradiction Theorem

Corollary: Conclusions obtained by combining two different statements obtained for space and time using different evaluation conditions are false.

Proof: Suppose we have a true statement regarding transformation of space (time) obtained by evaluation in S (S'), and we have a true statement regarding transformation of time (space) obtained by evaluation in S' (S). Then by the first contradictory theorem, both cannot be true statements. Hence any conclusions obtained or statements made which rely upon the simultaneous truth of both statements are false. QED

5.3 Second Corollary To First Contradiction Theorem

Corollary: Einstein's special theory of relativity is false.

Proof: Einstein's theory asserts, as a true statement, that there exists a Lorentz contraction for objects at rest in a moving reference frame in accordance with equation 11, obtained using evaluation in frame S. Einstein's theory also asserts, as a true statement, that there exists a time dilation for clocks at rest in a moving reference frame in accordance with equation 8, obtained using evaluation in frame S'. Einstein's theory claims these two statements are true statements within the theory and relies upon these two conclusions and uses them to obtain further conclusions. Since these two statements were obtained using evaluation in frames S and S', by the first corollary given above, any theory based on the simultaneous truth of both of these statements is false. QED

5.4 Implication of Second Corollary

The result that two different evaluation conditions cannot be used simultaneously requires a revision of conclusions regarding the action of Lorentz transforms on space and time. Since we can not use different evaluations, then only the solutions of sections

4.1 and 4.3 or sections 4.2 and 4.4 can be used together. Therefore, the simultaneous truth of equations 8 and 11 as claimed by relativity is impossible. The discussion in section 4.5 shows that equations 2 and 9 or equations 8 and 15 must be the only two possible pairs of solutions in relativity. Notice that in both cases these are both dilations of time and space. The proof that the transformation of space must be a dilation is shown by the solution of equations 10 and 15 with the result $x'=\beta x$ (Equation 9), or equations 14 and 16 with the result $x=\beta x'$ (Equation 15).

The observant reader should have noticed that equations 9 and 15 are the results advocated as the true or asynchronous transformations of space. The first transforms from S into S' and the second transforms from S' into S. But by the first contradiction theorem we can not claim both to be simultaneously true statements. We can use one or the other, but not both at the same time as long as we assign the same meaning to the symbols in both equations.

The solution to the right angle lever paradox is as follows. In the traditional method of solution, the evaluation condition for transformation of force is different from the evaluation condition for transformation of space, or more correctly distance. The force transformation is evaluated in frame S' and the distance transformation is evaluated in frame S. Hence by the first corollary in section 5.2 in the traditional solution method the force transformation is false and must lead to a contradiction or paradox, which is exactly what happens. The correct solution is to always use the same evaluation conditions for both distance and force. This is essentially what was done in the proposed solutions which advocated the use of the true and asynchronous methods as well as the solutions given by Hsiao-bai Ai.

There are four different correct ways to solve the problem. We can transform from S into S' or from S' into S. Now for these two different transformations there are two different ways to define the condition of evaluation, either in S or S'. So there are four different possible solutions. As Hsiao-Bai Ai shows, there are four correct solutions which all show that if the net turning moment is zero in the domain of the transformation, then the net turning moment is also zero in the co domain of the transformation. Hence the traditional thesis that there exists an energy current is false.

5.5 Evaluation For The Transformations Of Force

This section presents the solutions for the transformation of force in a manner consistent with the approach used in section 4. We begin by writing the Lorentz transformations (From S to S') for force as given by Hsiao-bai Ai. These are:

Equation 17: $F'_{x'} = F_x - \lambda u_y v F_y/c^2 - \lambda u_z v F_z/c^2$

Equation 18: $F'_{y'} = \lambda\beta^{-1}F_{y'}$

Equation 19: $F'_{z'} = \lambda\beta^{-1}F_z$

Equation 17 transforms the forces defined in the S frame into the force in the x' direction in the S' frame. Equation 18 transforms the forces defined in the S frame into the force in the y' direction in the S' frame. Finally, equation 19 transforms the forces defined in the S frame into the force in the z' direction in the S' frame. Notice that the force transformed in equation 17 depends on the force in all three directions in S.

The inverse Lorentz transformations (From S' to S) are:

Equation 20: $F_x = F'_{x'} - \lambda' u'_{y'} v F'_{y'}/c^2 - \lambda' u'_{z'} v F'_{z'}/c^2$

Equation 21: $F_y = \lambda' \beta^{-1} F'_{y'}$

Equation 22: $F_z = \lambda' \beta^{-1} F'_{z'}$

Equation 20 transforms the forces defined in the S' frame into the force in the x' direction in the S frame. Equation 21 transforms the forces defined in the S' frame into the force in the y direction in the S frame. Finally, equation 19 transforms the forces defined in the S frame into the force in the z direction in the S' frame. Notice that the force transformed in equation 20 depends on the force in all three directions in S'.

In these equations we also have the definitions: $\lambda' = (1-u_x v/c^2)^{-1}$ and $\lambda = (1+u'_{x'} v/c^2)^{-1}$. Now the problem is to solve the transformations for the appropriate conditions of evaluation. First we notice in equations 17 and 20 that u_z and $u'_{z'}$ are zero because we specify that the right angle lever moves along the direction of the x and x' axes. Furthermore, we have that u_y and $u'_{y'}$ equal to zero as well, because there is no motion in the y or y' directions. From this we conclude that $F'_{x'} = F_x$ and $F_x = F'_{x'}$. Equations 19 and 22 are irrevalent because there is no force in the z or z' directions. hence the evaluation problem becomes that of evaluating the transformation of the forces in the y and y' directions.

The evaluation is determined by the evaluation condition imposed upon the parameters $\lambda = (1-u_x v/c^2)^{-1}$ and $\lambda' = (1+u'_{x'} v/c^2)^{-1}$. But, what are these conditions? this is the problem and its solution is the key to the elimination of the paradox. Evidently there are two conditions $u_x=0$ and $u'_{x'}=0$. Now since the reference frame S' has velocity v relative to S and conversely reference frame S has velocity -v relative to S', we have the following solutions for evaluation with $u_x=0$:

Equation 23: $\lambda = (1-u_x v/c^2)^{-1} = (1-0\, v/c^2)^{-1} = 1$, therefore $F'_{y'} = \lambda\beta^{-1} = \beta^{-1} F_y$

Equation 24: $\lambda' = (1+u'_{x'} v/c^2)^{-1} = (1-v^2/c^2)^{-1} = \beta^{2,}$ therefore $F_y = \lambda' \beta^{-1} F'_{y'} = \beta F'_{y'}$

Where equation 24 is solved by setting by the substitution $u'_{x'}=-v$ because an object at rest in S' has a velocity -v relative to S. These are the so;ution s for evaluation in frame S. We have the following solutions for evaluation with $u'_{x'}=0$:

Equation 25: $\lambda=(1-u_x v/c^2)^{-1} = (1-v^2/c^2)^{-1}=\beta^2$, therefore $F'_{y'}= \lambda\beta^{-1}F_y= \beta F_y$

Equation 26: $\lambda'=(1+u'_{x'}v/c^2)^{-1} = (1-0\ v/c^2)^{-1} =1$, therefore $F_y = \lambda'\beta^{-1}F'_{y'}=\beta^{-1}F'_y$

Where equation 25 is solved by setting by the substitution $u_x=v$ because an object at rest in S has a velocity v relative to S'. These are the solutions for evaluation in frame S'. Notice that equations 23 and 24 and 25 and 26 form inversely related pairs. The mistake in the traditional solution is revealed by the solution of equations 24 and 26.

In the traditional synchronous solution, equation 26 is taken to be the transformation of force and there is no realization that the result of equation 24 is needed. When equation 26 is combined with the transformation of length given by equation 11, the result is incorrect because it produces the paradox.

Notice the important result that there are four ways to express the transformation of the force just as there are four ways to express the transformation of space or distance given in sections 4.3 and 4.4 by equations 9, 11, 13, and 15. The problem now is to correctly match these with equations 23, 24, 25, and 26. to do this we specify the direction of the transformation and the evaluation condition.

In section 4.3 the solution of the Lorentz transform from S to S' with evaluation in S is equation 9 and the inverse Lorentz transform from S' to S evaluated in S is equation 11. These correspond to the Lorentz transform for force from S to S' evaluated in S given by equation 24 and the inverse Lorentz transform of force from S' to S evaluated in S given by equation 23.

In section 4.4 the solution of the Lorentz transform from S to S' with evaluation in S' is equation 13 and the inverse Lorentz transform from S' to S evaluated in S' is equation 15. These correspond to the Lorentz transform for force from S to S' evaluated in S' given by equation 26 and the inverse Lorentz transform of force from S' to S evaluated in S' given by equation 25.

The following four equation pairs are therefore to be matched when calculating the transformation of the turning moment: Equations 9 and 23 to transform from S to S' with evaluation in S, Equations 11 and 24 to transform from S' to S with evaluation in S. Equations 13 and 25 to transform from S to S' with evaluation in S', Equations 15 and 26 to transform from S' to S with evaluation in S'. According to the discussion of section 5.4, when we match both the transformation directions and the evaluation conditions there will not be a contradictory result giving a paradox as in the traditional solution.

The traditional solution matches equation 11 with equation 26. This produces an incorrect result because equation 11 is deduced via evaluation in S and equation 26 is deduced by evaluation in S'. Since the evaluation conditions are different, the result is paradoxical or involves a mathematical contradiction. The traditional synchronous theory also uses equations 13 and 23 to transform from S into S'. In both these cases the transformed distance is contracted, and the evaluations are in two different reference frames. Hence, the result leads to a paradox. Therefore, the synchronous solutions are incorrect, not because the principle of the synchronous method is incorrect, but because the force transformations which have been traditionally derived based upon it are incorrect. This point is not mentioned in Dr. Ai's paper and it is a significant result of the analysis presented here.

6.0 MATHEMATICAL METHOD OF SOLUTION

This section demonstrates that the proposed mathematical approach to the solution of the lever paradox is the correct one. The first section addresses the traditional approach found in the established textbooks. This is followed by the successful solution advocated by Hsiao-bai Ai. Finally, the solution proposed here will be shown to be the same as that proposed by Dr. Ai.

6.1 Traditional Method of Solution

The basis of the traditional method of solution is the method established by Einstein's foundational papers. This leads to the concept of Lorentz contraction. The basic approach to the lever problem consisted of the following steps:

- Define the problem with the lever at rest in frame S'
- Transform length and force from frame S' into S

The paradox concerns the transformation of the lever arm length in the direction of motion and the force perpendicular to the direction of motion. The paradox arises because when the torque defined by these two qualities, $L'_xF'_{y'}$ is transformed into the frame S the result L_xF_y does not equal the value of the torque in frame S'. This simply defines the source and cause of the paradox; the transformation of distance and force in special relativity. The paradox is removed when the torque defined in the rest frame equals the transformed torque in the opposite frame.

Traditionally for historical reasons, the frame S' is called the moving frame and S is the rest frame. The lever is considered to be at rest in the moving frame and hence is moving with velocity v when observed from the frame S. The traditional solution asserts a Lorentz contraction of the lever arm L_x in the frame S. The force is also transformed as a contraction so the result obtained is $L_xF_y=\beta^{-2}L'_xF'_y$. Hence the paradox, which arises because an applied torque on the opposite lever arm transforms differently according to $L_yF_x=L'_{y'}F'_{x'}$. hence the transformed torque in frame S is the same as in frame S'. Since the torque $L'_{y'}F'_{x'}$ is assumed to be equal to the torque $L'_xF'_y$, the net turning moment is zero in frame S' but not zero in frame S.

The thesis of this paper is that the paradox results from an incorrect transformation of force. To understand why this happens, we examine the standard transformation of force. The traditional result is given by $F_y = \lambda'\beta^{-1}F'_y$, where $\lambda'=(1-u'_xv/c^2)^{-1}$. In the traditional solution of the lever paradox, this is solved by setting $u'_x=0$ since this is the velocity of the lever in the S' frame of reference. The lever is at rest in this frame, so the velocity is zero. The resulting transformation of force in frame S is then given by : $F'_y=dP'_y/dt'=dP'_y/\beta\lambda^{-1}dt=F_y\,\beta^{-1}\lambda'dt$, because $\lambda'=(1-u'_xv/c^2)^{-1}=1$.

The paradox can be eliminated in one of two ways. One approach advocated during the 1960's is the use of the asynchronous method which changes the transformation of length from $L_x= \beta^{-1}L'_x$, which is a Lorentz contraction, into a dilation of length expressed as: $L_x= \beta L'_x$. The paradox is resolved because now the transformation of torque gives the result that: $L_xF_y=\beta L'_x\beta^{-1}F'_y=L'_xF'_y$. Hence the asynchronous method gives the result that the torques are equal in the two reference frames. The second approach is the method used by Hsiao-Bai Ai. However, his method is essentially an extension of the asynchronous method. The major change in Dr. Ai's solution is a different solution for the transformation of force for the synchronous solution.

The essence of Hsiao's method, which he calls the straight forward method, involves a different approach to the transformation of force. This is accomplished by a redefinition of the problem. Dr. Ai's method redefines the lever problem so that in the synchronous solution the transformation of force becomes $F_y= \beta F'_y$ instead of the traditional result $F_y= \beta^{-1}F'_y$. This is achieved by defining $u'_x=v$ instead of $u'_x=0$. Now $\lambda'=(1-v^2/c^2)^{-1}=\beta^{-1}$ as opposed to the traditional result that $\lambda'=(1-u'_xv/c^2)^{-1}=1$. This approach resolves the paradox, but is it the correct answer?

6.2 Traditional Solution Of The Paradox Creates Another Paradox

This section presents a solution to the right angle lever paradox which suggests that the traditional solution is in error and suggests a reason for the error. The solution creates another paradox. When we calculate the transformation of torque, where we assume the right angle lever is at rest in frame S, we obtain the result that the torque is not in equilibrium in the moving frame S'. Here it is shown that if we begin with the assumption that the lever is moving in frame S, instead of at rest, with all other conditions the same, then when we transform into the frame S' the lever will be at rest in this frame. The result is astonishing. When we do this, the transformation equations show that if we begin with the lever moving with velocity v in frame S with the forces giving an equilibrium of torque in frame S, then when we transform into the frame S' the forces remain in an equilibrium of torque. A result that contradicts the conclusion when the lever was assumed at rest in S. This argument refutes the traditional explanation because now there is no need to postulate a fictious energy current.

The calculation proceeds as follows. The forces in frame S are F_x and F_y ; assumed to be equal in magnitude. The lever arms are L_x and L_y ; also assumed to be equal. The lever arms transform as before such that $L'_x=\beta^{-1} L_x$ and $L'_y=L_y$. The transformation of the forces determines the change in the outcome. The difference in the assumptions changes the transformation of the force F_y. The difference is accounted for in the calculation of the factor λ. This is calculated as follows with $u_x= v$: $\lambda=(1-u_x v/c^2)^{-1} = (1-v^2/c^2)^{-1}=\beta^2$, therefore $F'_{y'} = \lambda\beta^{-1}F_y= \beta F_y$. Observant readers will recognize this as the same solution as given by equation 25. Hence we have the transformed torque as: $L_yF_x=L'_{y'}F'_{x'}$ and $L_xF_y=\beta^{-1}L'_x\beta F'_y=L'_xF'_y$. Because the net turning moment was zero in S we see that it is also zero in S'. Because $L_yF_x-L_xF_y=0$, we have $L'_{y'}F'_{x'}-L'_xF'_y =0$. In this case there is no need for the hypothesis of an energy current.

The paradox is as follows. The problem is defined so that the two definitions of the problem are the same or equivalent. The lever moving in S with velocity v is the same as being at rest in frame S' which moves with velocity v. However, there are two contradictory physical interpretations of the physical situation. So there must be an error. The possible source of the error is suggested by the fact that the only change is the specification of the velocity and how this is interpreted in the calculation of the transformation of force. Indeed this is the major change that Hasio-bai Ai makes in his straight forward method. He uses a different method to evaluate the functions λ and λ' in the force transformations.

6.3 Hsiao-bai Ai's Solution

This section examines the four solutions presented by Dr. Ai and demonstrates that these are the same as deduced in section . The main problem encountered is to translate the terminology used by Dr Ai into the language used in this paper. the result makes clear that Dr. Ai's solution is equivalent to the rigorous method developed here.

Case 1. Dr. Ai defines the problem this way: "To define physical quantities in Σ', to derive their transformations in Σ'. In this case, the meaning of deriving their transformations in Σ' is to denote the quantities in Σ in terms of the physical quantities in Σ'." Translated into the terminology of this paper, case 1 is the transformation of physical quantities defined in reference frame S' into quantities defined in S, with evaluation specified in S'. Referring to section 5.5 above this corresponds to the pair of equations 15 and 26.

Expressed with mathematical rigor, the transformation domain is defined by specifying the definition of physical quantities and then the transformation to the co domain is specified as being a function of the domain quantities. Evaluation is specified by the statement "To define physical quantities in Σ'." Here Greek letters are used to define S and S'. This is followed by the confusing definition of the inverse Lorentz transformation from S' to S using the statement "to derive their transformations in Σ'." Here we see that the domain of transformation is defined as frame S', which follows from the first statement. The second statement says that the co domain is expressed in terms of the domain quantities. Thus we see that this specifies a transformation from S' as domain to S as co domain, because the quantities in S are functions of quantities defined in S'. This being the specification of an inverse Lorentz transformation.

Case 2. Dr, Ai defines the problem this way: "To define physical quantities in Σ', to derive their transformations in Σ. In this case to denote the quantities in Σ' in terms of the physical quantities in Σ. Case 2 is the transformation from S to S' with evaluation specified in S'. Referring to section 5.5 above this corresponds to the pair of equations 13 and 25.

Expressed with mathematical rigor, we see that the domain is defined as being S', but that the transformation is defined oppositely as from the co domain to the domain. Therefore this is the transformation inverse to the one defined in case 1. Here inverse is defined in terms of the domain as specified in S'. So we see that evaluation defines the domain of the transformation, and by inference the co domain of the inverse transformation. This becomes clear only when we apply the mathematically rigorous terminology as given here. It contradicts the traditional terminology which specifies the transformation from S' to S with S' as domain as the inverse Lorentz transformation. But strictly speaking this is incorrect because the inverse is by definition the transformation from co domain to domain.

Case 3. Dr. Ai defines the problem this way: "To define physical quantities in Σ, to derive their transformations in Σ. In this case, dt=0 and u=0, that is λ=1." Here it is assumed that the reader has understood the two previous cases, and that by inference he is now defining physical quantities in S instead of S'. This is made clear by the specific evaluation conditions which he gives. Thus case 3, specifies the transformation from S to S' with evaluation in S. Referring to section 5.5 above this corresponds to the pair of equations 9 and 23.

Expressed with mathematical rigor, the domain is specified as S and the Lorentz transformation is specified so that the co domain quantities are expressed as a function of the domain quantities.

Case 4. Dr. Ai defines the problem this way: "To define physical quantities in Σ, to derive their transformations in Σ'. In this case, dt=0 and u=0, that is $\lambda=1$." Here we see that the transformation is to be derived or specified in terms of the co domain quantities so it is the inverse transformation from S' to S with evaluation specified in S. It is the transformation inverse to case 3. Referring to section 5.5 above this corresponds to the pair of equations 11 and 24.

Expressed with mathematical rigor, the domain is again defined as frame S, but his time the transformation is specified in terms of the co domain quantities, so it is the inverse transformation from S' to S, because the domain quantities are defined as a function of the co domain quantities.

6.4 Correct Resolution Of Lever Paradox

Analysis of the methods of Hsiao-Bai Ai shows that this method is essentially correct as long as we use the procedure and definitions laid down. Problems arise when a reconciliation is attempted between the traditional method and the new method. An examination of this issue shows that the fault lies in the traditional method of solution. Here the error is explained.

The solution to the lever paradox lies in the correct transformation of force and not distance as advocated by others. The fault lies in the derivation of the transformation of force equations and the method used to evaluate these equations. The basic method for transformation of force is described here as simply as possible. The procedure is as follows. We define force as a change in momentum per unit time, hence $F_y=dP_y/dt$ and $F'_y=dP'_y/dt'$. Traditional theory defines the Lorentz transformation of momentum as follows: $dP'_y=dP_y$. The inverse transformation is: $dP_y=dP'_y$. The trouble lies in the traditional approach to the transformation of the differentials for time: $dt'=\beta(dt-vdx/c_2)$ and $dt=\beta(dt'+vdx'/c^2)$. These are transformed into the following equations and reduced by introducing the qualities $u_x=dx/dt$ and $u'_x=dx'/dt'$ as follows: $dt'=\beta dt(1-dxv/dtc^2)=\beta dt(1-u_xv/c^2)=\beta\lambda^{-1}dt$ and $dt=\beta dt'(1+dx'v/dt'c^2)=\beta dt'(1+u'_xv/c^2)=\beta\lambda'^{-1}dt'$. These are substituted into the expressions for the transformation of force : $F_y=dP_y/dt =dP'_y/\beta\lambda'^{-1}dt'=F'_y\beta^{--1}\lambda'$ and $F'_y=dP'_y/dt'=dP'_y/\beta\lambda^{-1}dt=F_y\ \beta^{-1}\lambda'dt'$. These equations are the source of the paradox which results from their use.

The correct transformations are obtained through the evaluation of the transformation of time in the following manner. The procedure is to write the transformation equations for time and applying the usual procedure for the evaluation of time between reference frames, before writing down the transformations equations for force. The procedure then becomes to evaluate the time transformations first, then substitute into the transformations of force rather than the reverse procedure.

The procedure for the traditional problem of the lever is as follows. To calculate the transformation of force we use the inverse Lorentz transformation of time. This is given above as : $t=\beta(t'+vx'/c^2)$. Evaluated with the condition x=0, the substitution, $x'=-vt$, is performed into the equation for time: $t=\beta(t'+vx'/c^2)=\beta(t'-v^2t'/c^2)=\beta^{-1}t'$. Therefore the transformation for the time differential is $dt=\beta^{-1}dt'$. Now when we substitute this result for the differential time into the definition of force we obtain the following result: $F_y=dP_y/dt=dP'_y/\beta^{-1}dt'=\beta F'_y$. Notice that the traditional result is $F'_y=dP'_y/dt'=dP_y/\beta\lambda^{-1}dt=F_y$ $\beta^{-1}\lambda'dt$ evaluated with $u'_x=0$, we obtain $F_y=\beta^{-1}F'_y$.

The transformation of torque using the revised evaluation gives the result:
$L_xF_y=\beta^{-1}L'_x\beta F'_y=L'_xF'_y$. This result eliminates the paradox. Now we have that the torque transforms as follows: $L_xF_y=L'_xF'_y$. This means that there is no imbalance in the transformation of torques and the paradox is removed. The problem was an incorrect equation for the transformation of force.

To demonstrate that the result is correct, the following demonstration shows that it is bijective. To do this we transform from S into S'. Now in this case the transformation of the lever arm is $L'_x=\beta L_x$. The transformation of the time differential is $dt'=\beta dt$. Here evaluation is defined in frame S when we transform into S'. The transformation of distance is as follows: $x'=\beta(x-vt/c^{2)}$. We evaluate in S, where t=0. Thus by substitution we have that $x'=\beta x$, or that $L'_x=\beta L_x$. Time is transformed as follows: $t'=\beta(t-vx/c^2)$ with evaluation in S , x=0 and we have that $t'=\beta t$. Hence the differential times transform as: $dt'=\beta dt$. The force transformation is:
$F'_y=dP'_y/dt'=dP_y/\beta dt=\beta^{-1}F_y$. This result is now transformed by substitution so that the equation for transformation becomes :$L'_xF'_y=\beta L_x\beta^{-1}F_y=L_xF_y$. This result shows that the torque transforms with the same result going from S into S' as it does going from S' into S. This result is expected as long as we use the bijective transformations for both distance and force.

6.5 Dr. Ai's Solution Is Incomplete

The resolution of the paradoxes in relativistic statics as given by Dr. Ai is mathematically correct, but the author believes it is incomplete and possibly misleading. The straight forward method, as demonstrated herein, is consistent with the correct mathematically rigorous solution of the paradox of the right angle lever and its application to problems of relativistic leads to the correct solutions. However, there are two significant problems which the author has attempted to correct with this paper.

The first is the lack of a mathematically rigorous method of proof to demonstrate that the straight forward method is necessarily the correct one. That flaw has been corrected in this paper by the use of a rigorous mathematical theory of evaluation. This theory demonstrates that the straight forward method is based upon the correct principles of solution. Furthermore, the principles of the method have been clarified and it is made more understandable how they are be applied to the solution of the problem.

The second flaw is that Dr. Ai does not make it clear that the mistake lies in the transformation of force and instead asserts that the problem lies in the transformation of length. Hence, he supports the claims of the proponents of the asynchronous method of solution. Here is it shown that the error lies in the incorrect transformation of force, when the synchronous method of solution is applied. (There was no mistake when the asynchronous method was used so it seemed that this was the correct method of solution.) As shown by Dr. Ai and confirmed here, the asynchronous and synchronous solutions both correctly demonstrate that the lever is in equilibrium in both frames when the correct force transformations are used. Hence the controversy over which of the two methods is the correct one, synchronous or asynchronous, is rendered moot, since both methods produce the same result, when the correct transformations of force are used.

7.0 Analysis, Commentary, and Conclusions

Here a nonmathematical analysis of the problem will be given that addresses the physical issues and the controversy in the literature. As discussed in the introduction, the explanation offered by Von Laue was accepted until Arzelies objected and offered a different solution, which was basically the true transformation or asynchronous method. These solutions showed that there was no need for an energy current, because the net turning moment was zero in both frames. Controversy ensued and the result was that the physics community never accepted this approach and the correct solutions never became incorporated into textbooks.

Following Arzeles, an Italian group attempted to reform the physics of relativistic statics. They produced a series of papers which advocated the asynchronous method of solution. Others followed with the result that a large number of papers were produced which advocated a new approach to relativistic physics. however, the new approach ran up against the entrenched beliefs of the physics community and never became accepted physics. The reason seems to be that the calculation of force in the asynchronous transformation method violated the intrinsic idea that measurements in the moving frame should be performed simultaneously. Hence the physics community adhered to the synchronous method which as we have demonstrated here gives the wrong calculation of the force transformation.

The reason for the failure must be the entrenched belief that everything in the special theory of relativity is correct, and fully and completely verified by experiment. The experts however knew this was not the case and continued to work on the discrepancies and recalcitrant difficulties, always believing that minor changes could remove the problems. When the offered solution contradicted the established ideas, the resistance offered by the entrenched traditionalists blocked any progress towards solution. Hence the problems and difficulties remain today as they have been for 100 years.

There is a peculiar ironic aspect of this problem. The idea of a tensor is based on the concept that physical quantities transform invariantly. This idea is in contradiction to the basic concept of special relativity which insists that physical quantities transform covariently. Hence, when the experts used tensor methods to analyze the problems of the lever they concluded that there was no energy current and that the asynchronous solution was correct. But of course, using the traditional methods as herein, the result was just the opposite. To an outside observer, the situation is peculiar. The tensor methods were unable to resolve the difficulties, because the physical interpretation could always be misinterpreted to claim the method used was false. Because the methods used were obscure, the arguments were allowed to continue and the conclusions ignored.

This seems to be the current approach to problems in special relativity. When Hsiao-Bai Ai published his paper, there was no reaction at all. There are several possible reasons. First, the current belief that the traditional theory as taught in textbooks is absolutely correct coupled with the idea, now fully entrenched and predominant, that anyone who disputes the truth of the currently accepted views is a crackpot. This is coupled with the view, expressed by many, that anyone who disagrees with the traditional conclusions of special relativity is automatically incorrect, because the theory is absolutely the truth. Hence, there is no need to investigate the issues raised by Dr. Ai. The second reason is more specific and relates to the author's reaction. The method as given is so obscure, and confusing that it can not be understood at all. Hence, there was no reason to refute it or react to it at all.

The final reason seems to be a reluctance of physicists to discuss the paradoxes and theoretical problems of special relativity. The cause of this appears to be the futile results of previous efforts. These failed for essentially two reasons. First, critics were unable to really get close to the root causes and difficulties. The method of evaluation, as discussed here, was completely misunderstood. Hence there was no way to rigorously evaluate the mathematical conclusions of the traditional theory. It was only with the advent of an understanding of the evaluation problem, motivated by a desire to understand Dr. Ai's method that a clear and rigorous understanding of evaluation has emerged. Unfortunately, that understanding contradicts the cherished ideas of the correctness of Einstein's foundational papers on special relativity. Simply put, a rigorous approach to evaluation shows that the idea of Lorentz contraction is invalid. Furthermore, the traditional derivation of time dilation is also wrong. This is the cause of all the problems and paradoxes of special relativity.

This brings us back to the real difficulty, which is that the acceptance of the traditional view and interpretation of special relativity is sacrosanct. Therefore, it is impossible to correct its problems and difficulties. As a result, absurd physics will continue to be taught as gospel in the textbooks. An excellent example is Von Laue's energy current. This has clearly been refuted by experts but belief in it dies hard.

After a long and arduous study of Dr. Ai's method, a clear theory of evaluation in special relativity has emerged. This gives a rigorous way to obtain solutions to the transformation problem. However, when the rigorous method is used, it reveals that the traditional solutions are incorrect. Furthermore, a new and different understanding of the transformation problem has been obtained that gives insight to resolving the paradoxes. It is unfortunate that this has disproved some of the most cherished beliefs held to be true by the traditional textbooks. However, this can not be avoided without disregarding the rigor of the mathematics.

8.0 References

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The literature is extensive. Refer to references in the cited papers for more extensive citations.