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Sagnac effect in GPS

*Investigating the motion of the opposite beams in a rotating closed loop and using an interferometer, Sagnac compared the distances passed by the beams and was sure that he had proved the existence of the ether. Despite the fact that later the ether hypothesis was recognized as erroneous, the Sagnac effect is now widely used in fiber-optic laser gyroscopes. This effect arises and is taken into account in the GPS system, where the source moves on the satellite and the signal receiver, together with the Earth and the atmosphere, rotates relative to the inertial frame Ignoring the Sagnac effect will lead to a positioning error of the order of several tens of meters [1,2]. Relativists assert that the Sagnac effect is a relativistic effect. Since 1921, when Langevin gave the first relativistic explanation of the Sagnac effect, many explanations based on both special and general relativity have been proposed [5]. Nevertheless, the effect still has no generally accepted explanation and more conflicting opinions are expressed about it than about any other relativistic effect. Some authors say that the Sagnac effect is incompatible with the special theory of relativity, since the speeds of beams change in a rotating frame, others assume that the speed of the light does not change if to take into account the time dilation in the rotating frame [6]. In 2004, an experiment was carried out with a rotating fiber-optic gyroscope, which, according to the authors, is a generalization of the Sagnac effect on translational motion, and the Sagnac effect itself is only a special case of their discovery [7]. It is also argued that the fiber-optic conveyor must respond to the rotation of the Earth and can even be used for a decisive experiment to test the relativistic postulate of the constancy of the speed of light. In 2012, in the article "**Analysis of the Phase Difference in a Fiber-Optical Conveyor**" we showed that the authors mistakenly explained their widely advertised experiment and that this experiment is not a "generalization" of the Sagnac effect and will not react to the rotation of the Earth [8].*

In this article we show that the Sagnac effect is not relativistic, and we explain it on the basis of purely classical concepts.

In our previous paper [9], using Fermat's principle, we explained the Sagnac effect on the basis of classical concepts, making the following two assumptions:

- in a rotating contour, photons move between reflections from mirrors rectilinear relative to the inertial frame,
- the point of the space from which photons are currently emitted, remains stationary during the time while the first front of photons moves to the next mirror, that is the light source is considered to be stationary relative to the inertial frame.

These two conditions let us to explain the Sagnac effect without using the ether hypothesis or relativistic effects and to conclude that, relative to the rotating contour, the beams follow curvilinear trajectories and pass with the same speed C / n different distances:

the beam traveling in the direction opposite to rotation travels a shorter path relative to the contour than the beam traveling in the direction of rotation.

The same conditions allowed us to explain the Sagnac effect in fiber-optic gyroscopes, where the Sagnac effect is significantly enhanced due to multiple rounds of the contour and the paths of the beams are also compared along the lines of interference [10].

In the GPS system, the Sagnac effect is explained much more simply than in the Sagnac interferometer or in the laser gyroscope, where two opposing beams, repeatedly reflecting, travel along a closed contour and passed distances are compared along the lines of interference. In the GPS system, the signal follows an open path without intermediate reflections. And the most important difference is

that only one signal can be viewed without comparing it with the second. The latter became possible thanks to the use of modern ultra-precise atomic clocks in the GPS system: instead of interferometric comparison of two beams, it became possible to directly measure the time it takes for one beam to travel from a satellite to a receiver on a rotating Earth.

Another difference is that the signal source does not rotate with the receiver, but moves at a constant speed along an orbit that is stationary relative to the inertial frame.

The principle of constructing the trajectory and determining the distance traveled by the signal remains the same: relative to inertial frame, the photons with the speed C / n move along a straight line connecting the point at which the source was at the moment of emission with the point at which the receiver is at the moment of arrival of the signal, and the distance traveled is determined by the length of this straight line. Relative to the rotating frame, the photons move along a curved trajectory and travel the same distance. The time it takes for the signal to arrive at the receiver is determined by dividing the distance by the speed of movement C / n.

Assumptions made when analyzing signal traectories

Before analyzing the trajectories of a signal from a satellite to a receiver on Earth, let us consider in more detail the above conditions on the straightness of the motion of photons relative to the inertial frame and the immobility of the point at which they were emitted.

Photons in a vacuum move from a source S to a receiver R rectilinearly with a speed C. If some transparent medium (for example, a glass plate or a gas cloud) moves perpendicular to the line SR with a speed V, photons are re-emitted by this medium and move relative to it with a speed C / n. But before the speed of the photons becomes equal to C / n, the following happens.

A photon approaches a moving glass plate with a speed C and meets the first re-emitting atom at point K (Fig. 1). But since the plate moves with the speed V, the photon meets the re-emitting atom with a speed equal to the vector sum $\overline{C} + \overline{V}$ that directed, as shown by the dotted line, along the line S1-K. An atom absorbs a photon and, after a certain delay, emits it with a slightly higher frequency in

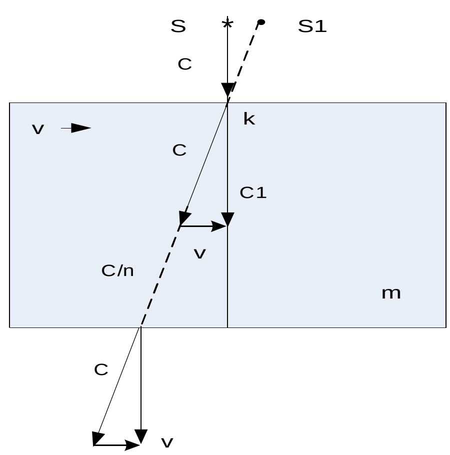

Fig.1

the same direction with a speed equal to C. In this direction, the photon goes to the next atom, is re-emitted by it and with an average speed C / n moves further in the glass.

The observer moving with the glass sees that the light comes to him not from the real direction to the source S, but from the apparent direction to the source S1, that is, he sees that the light comes at an angle of aberration (which Bradley discovered when observing the stars).

The last re-emitting atom of a moving glass plate emits a photon with a speed C. But since the plate moves with a speed V, the photon speed decreases to a speed equal to the vector difference $\overline{C}-\overline{V}$ and is directed exactly towards the receiver. With such speed, the photon moves until meets some atom a real rarefied medium and re-emitted by it(and the frequency, accordingly, decreases to its previous value).

In this whole process we are only interested in the fact that after the interaction of photons with a moving medium, the direction of their motion does not change.

Figure 2 shows a simple example of how the second condition is used - the stationary of the point from which the photons of the first wavefront are emitted.

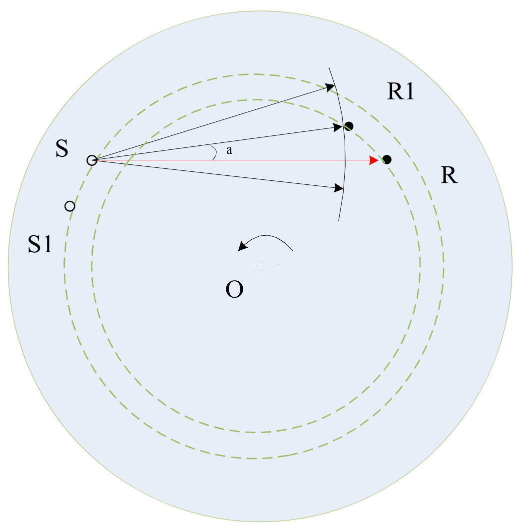

Fig.2

If the source S and the receiver R are stationary relative to the atmosphere, the signal travels along the direct line S-R and arrives at the receiver in SR / C time. The receiver sees photons emitted exactly in its direction.

How will the signal transit time change if the source and receiver move, for example, in orbits around the Earth? During the time while the signaltravels to the receiver, the source and receiver shift from the places where they were at the moment of signal emission.

Since the receiver moves at a different speed, the distance to the source and the moment of signal arrival obviously depend on its movement. The signal meets the receiver at the point at which the receiver arrives during the signal movement.

But it is also obvious that the movement of the source does not in any way affect the moment the signal arrives at the receiver: after the source sends a signal at the moment, the photons of the first wavefront go with a speed C in the direction of the receiver and neither their speed, nor their direction of the motions do not depend on what happens to the source after the moment t = 0. The photons of the first front move exactly as they would go from a stationary source. Thus, the origin of the satellite signal trajectory is at point S. And what is especially important for a correct understanding of the

Sagnac effect, the receiver receives not those photons that the source emitted in the direction of the receiver, but those that the receiver emitted at a certain angle to the direction in which the receiver is at the moment t = 0. Therefore, at the moment of signal emission, the receiver sees that the signal does not go directly to it, but at an angle "a" to the direction where the satellite with the signal source is at the moment of emission (Fig. 2).

Thus, the problem is reduced to determining the distance between the point in space from which the photons were emitted and the point at which the receiver arrives for the time while the photons of the first wavefront travel to it.

Analysis of signal trajectories in the GPS system

The orbits of GPS satellites move with the Earth in an orbit around the Sun, maintaining orientation relative to the inertial frame regardless of the Earth's rotation. The satellites simultaneously send signals that, depending on the location of the receiver relative to the satellites, do not arrive at the same time, which makes it possible to determine the coordinates of the receiver relative to the known coordinates of the satellites. Due to the fact that the receiver rotates with the Earth, the Sagnac effect occurs and the moments of arrival of signals to the receiver change, which leads to the appearance of additional positioning errors

The following analysis of the Sagnac effect using the example of the GPS system shows that this effect, like the gravitational frequency shift, velocity frequency shift and other "relativistic" effects, has a simple classical explanation.

In Figure 3 on a conditional scale, it is shown the propagation of GPS signals from two satellites: satellite Sw located to the west of the receiver R and satellite Se located at the same distance to the east of the receiver R. The signal paths are not related and can be considered separately. We consider two trajectories at the same time only for the purpose to greate clarity. The propagation of signals from the moment the signal is emitted by the satellite until the moment the signal arrives at the receiver is analyzed relative to the inertial frame.

If we imagine that the Earth does not rotate, the signals from the satellites should go with the speed C / n along the straight lines Sw-R and Se-R and the distance (Sw-1) = (Se-1) should pass for the same time t0. In this case, the trajectories of the photons should obviously be straight lines not only relative to the inertial system, but also relative to the Earth.

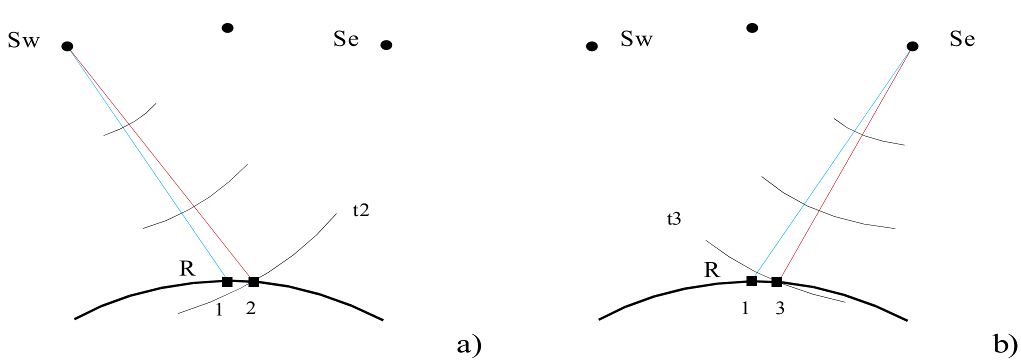

Fig.3

In a real situation, at the moment of signal emission, the satellites Sw and Se are at the same distance from the receiver R. During the time while the signal goes to the receiver, the Earth turns by a certain angle and the receiver from the initial position 1 shifts to the east - to position 2 for the satellite Sw (Fig. 3, a) and to position 3 for the Se satellite (Fig. 3b).

In the inertial frame, the photons go out from the point at which the satellite was at the moment of emission, and arrive not at point 1, where the receiver was at that moment, but at a new point (respectively 2 or 3). Since the photons always move in a straight line relative to the inertial frame, the trajectories of Sw-2 and Se-3 turn out to be straight lines. The Sw-2 distance turns out to be greater than the Sw-1 distance, and the Se-3 distance turns out to be less than the Se-1 distance.

Thus, due to the Sagnac effect:
the signal from the satellite Sw, located to the west of the receiver R, passes a greater distance and therefore comes to the receiver later,
the signal from the satellite Se, located to the east of the receiver R, passes a shorter distance and therefore comes to the receiver faster,
and in both cases, positioning errors occur in the east direction.

However, relative to the rotating frame in which the receiver R is located, the trajectories turn out to be curved. This can be explained as follows.

Imagine again that the Earth does not rotate and the signal is sent from the satellite towards the receiver in the form of a very narrow laser beam. The signal arrives at the receiver R only if the laser is directed exactly along the Sw-1 line (Fig. 3, a).

In a real situation, due to the rotation of the Earth, the receiver R is displaced from point 1 and therefore such beam will obviously not come to the receiver R, but will hit a point to the west of it. In order for the signal to arrive exactly at the receiver R, the laser must be directed not along the Sw-1 line, but along the Sw-2 line, that is, with some anticipation by deflecting the laser from the true direction at an angle 1-Sw-2. In this case, the receiver will see that at first time the beam goes not directly to it, but is deflected to the east, and then changes direction and comes to it.

The beam directed at an angle to the true direction arrives at point 2, which from the point of view of an observer on Earth means that the beam does not travel in a straight line, but along some curve.

Note:
It is possible to get rid of the "observer's point of view" accepted by relativists if we return to the real physics of the process. In fact, the satellite sends in the direction to the receiver not a narrow beam (as in the case of a very narrow laser beam, which must be turned by a certain angle to hit the receiver), but a continuous front, in which all photons have the same phase, but each of them moves relative to the inertial frame along its own rectilinear trajectory, emerging from the point from which this front was emitted. When the receiver is in a non-rotating frame, those photons that turned out to be emitted strictly in its direction (Sw-1 or Se-1) come to it. But in the case of a rotating frame, not these photons come to the receiver, but others emitted in the other direction (Sw-2 in Fig. 1, a or Se-3 in Fig. 1, b). And these "other" photons move relative to the inertial frame in the same rectilinear, but different trajectories.

By plotting the signal trajectories on a scale in Visio-7, we have determined the distances Sw-2 and Se-3 passed by the signals (Fig. 4). The angle at which the receiver sees the satellite is chosen arbitrarily and equals (as in Fig. 1) 30 degrees (from the vertical).

If we do not take into account the Sagnac effect, the signals should follow the traectoties Sw -1 and Se -1. At an orbital altitude of 20 180 km, the Sw and Se distances are equal to 20 860 km.

The signalsl pass through most part of these distances in a highly rarefied atmosphere at a speed almost equal to C = 299792.458 km / s. In the dense layers of the atmosphere (in the last 50-100 km, where the refractive index increases to 1.0003), the speed of light decreases to approximately 299700 km / s. A change in velocity in dense layers of the atmosphere does not fundamentally affect the nature of the calculations below. Therefore, to simplify the analysis, we assume that the signal travels all the way at a speed of C = 299 792 458 m / s.

Signal from satellite Sw

Figure 4 shows the signal paths in Visio 7 at a scale of 10 km / 1 division.
That is, the Sw-1 trajectory (length 20,860 km) corresponds to a 2086-division line segment.

If we imagine that the Earth does not rotate and therefore the Sagnac effect is absent, the signal from the satellite Sw to the receiver R should arrive in time

$$t_0(S_W\text{-}1) = 20860 / 299792.458 = 0.069\ 581\ 470\ 258\ 334 \text{ сек}$$

(0.069 581 470 258 334 517 541 465 302 639 47)

But since the Earth actually rotates, the signal goes along the trajectory Sw-2 and after a time interval t (Sw-2) comes to the receiver R, which at that moment is at point 2.

The magnitude of the receiver displacement L1-2 is determined by the duration of the time interval t0 (Sw-2), during which the signal comes from the satellite Sw to point 2, and by the speed V with which the receiver moves: $V\, t_0(S_W\text{-}2)$.

The speed of the receiver from point 1 of the inertial frame to point 2 is determined by the latitude at which the receiver is located. At the equator, this speed is 460 m / s. As an example, we consider the case when the receiver is located at a latitude where its velocity V is 300 m / s.

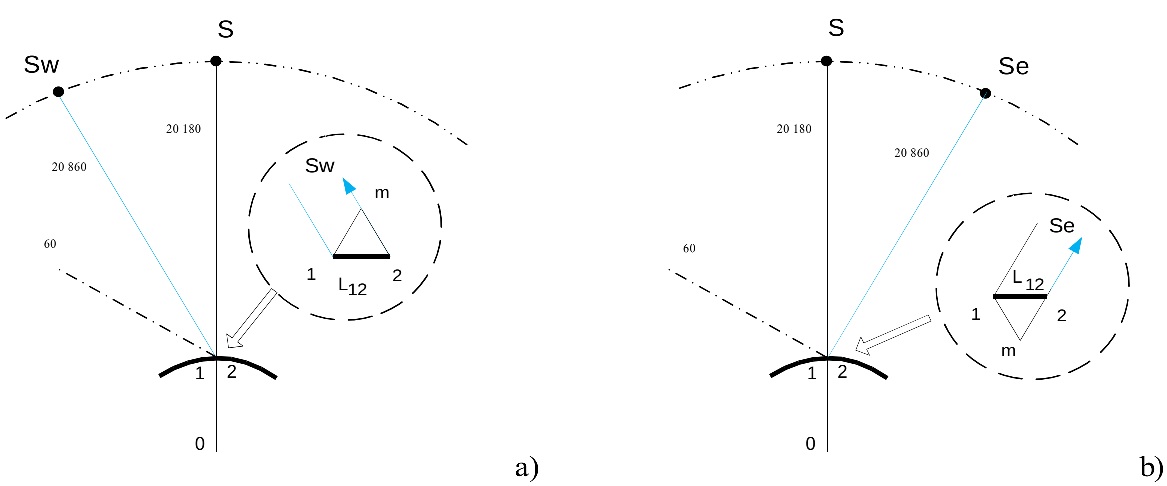

Fig.4

Instead of the time t0 (Sw-2) with a negligible error, a very close known time t0 (Sw-1) can be used. During such interval of time, the receiver moving at a speed of V = 300 m / s displaces eastward by a distance $L_{1\text{-}2} = V\, t_0(Sw\text{-}1) = 20.87$ м (0. 020 874 441 077 500 355 262 439 590 791 841 км)

In triangle 1-m-2, shown in Fig. 4, a on an enlarged scale, segment 1-2 is practically a straight line and line 1-m is perpendicular to the trajectory Sw-1. Segment 2-m determines the difference in distances t0 (Sw-2) - t0 (Sw-1). The length of the 2-m segment in this case turns out to be practically equal to the displacement L1-2 (segment 2-2): L2-m = 0.02087 km

The signal travels this distance of 20.87 m in a time 0. 000 000 069 сек

(0. 000 000 069 629 640 507 835 441 485 454 55)

Thus, the signal from the **Sw satellite** follows the Sw-2 trajectory, the length of which turns out to be 20, 874 m longer and is equal to 20 860.020 874 km,
comes to the receiver in a time of 0. 069 581 539 887 975 sec, (0.069 581 539 887 975 025 376 906 788 094 02)
more by 0. 000 000 069 629 640 sec (0. 000 000 069 629 640 507 835 441 485 454 55)
and the positioning error turns out to be equal to L1-2 = **20.87** m

For satellite Se,
located **to the west** of the receiver and visible by the receiver at the same angle of 30 degrees, the positioning error is determined according to Fig. 4, b and is practically the same in magnitude. The signal follows a trajectory, the length of which turns out to be less and is equal to 20 859.97912 km, and arrives at the receiver in a time of 0.069 581 400 630 165 sec.

For a satellite located above the receiver, the distance to the receiver is 20 180 km and the signal arrives in a time of 0. 067 313 234 410 987 sec. The positioning error turns out to be less and equal to **20, 19** m

Positioning error increases significantly for satellites seen at high angles. So, for example, for a satellite visible at an angle of **60** degrees, the distance to the receiver is 22 730 km, the signal arrives in 0.077 486 314 530 721 seconds and the positioning error increases to **58.15** m

Thus, **due to the Sagnac effect**, for satellites located to the west of the receiver, the length of the trajectory increases and the signal arrives later, and for satellites located to the east of the receiver, the length of the trajectory decreases and the signal arrives earlier, but in both cases the positioning errors are directed in one direction: in both cases, if no correction for this effect is introduced, the coordinate of the receiver, determined from the satellites signals, is shifted several tens of meters to the east. This error turns out to be the greater, the closer to the equator the receiver is located and the greater the angle at which the receiver sees the GPS satellite. Unfortunately, we could not compare the positioning errors we calculated with the actually observed ones, since for some reason in the published articles they only say that these errors “reach tens of meters”, but no specific figures are given anywhere. Perhaps because of a misunderstanding of the essence of the Sagnac effect and only its "relativistic" explanation?

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