P7. General Sagnac Effect and Invalid Relativistic Explanation

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Abstract

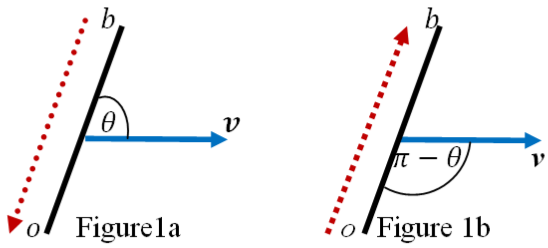
We show that the experimentally verified formula of Wang et el [1] underlying Sagnac interference is an immediate result of the theory of universal space and time (TUST). The general Sagnac effect which includes rotational and translational types is discussed. We demonstrate that a certain form of Sagnac's effect occurs under rotation even if the optical circuit is not closed, and under translation provided the circuit is not closed and its enclosing segment is not perpendicular to its velocity [2]. We finally show that the Sagnac effect cannot be explained within the framework of the special relativity theory.

1.. Introduction

We start by a review of basic results pertaining to the optical length of a rod discussed in P5 [3], which is needed to study a more general type of Sagnac interference. Next, we show that the Wang formula which quantifies the general Sagnac effect is an immediate consequence of the scaling transformation of second type (STII) in the theory of universal space and time.

Suppose that a rod ob of geometric length L is translating in the universal frame S at a constant velocity $\boldsymbol{v}$ that makes an angle θ with the vector $\boldsymbol{ob}$. We found in P5 [3] that the optical length of the light trip $(b \to o)$ (Figure 1a) is given by either side of the scaling transformation of the second type STII:

$$\gamma c t_{bo} = E(\beta, \theta) L \tag{1.1}$$



In both figures the rode ob is translating in S at a constant velocity $\boldsymbol{v}$. The red arrow points in the direction of a light pulse that starts from one end (an emitter) and ends at the other (receiver). The angle between the radius vector of the emitter relative to the receiver and the velocity $\boldsymbol{v}$ is θ in Fig. 1a and $\pi - \theta$ in Fig.1b.

where $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$,

$$E(\beta, \theta) = \frac{-\beta \cos\theta + \sqrt{1-\beta^2 sin^2\theta}}{1-\beta^2} \tag{1.2}$$

is the Euclidean factor, and t_{bo} is the duration of the trip. Note that $\theta = \angle(\boldsymbol{ob}, \boldsymbol{v})$ is the angle between the velocity $\boldsymbol{v}$ and the radius vector $\boldsymbol{ob}$ of the light's source b relative to the receiver o.

The duration of the latter light's trip can also be written in the form

$$t_{bo} = \frac{T}{\Gamma(\beta,\theta)} = \frac{L}{c\Gamma(\beta,\theta)}. \tag{1.3}$$

where

$$\Gamma(\beta, \theta) = \frac{\beta \cos\theta + \sqrt{1-\beta^2 sin^2\theta}}{\sqrt{1-\beta^2}} = \sqrt{1-\beta^2} E(\beta, \pi - \theta) \tag{1.4}$$

is the scaling factor.

We have subscripted the duration t of the trip by (bo) to emphasize the light's trip's beginning b and end o. It is easily seen that the duration of the trip $(o \to b)$ is

$$t_{ob} = \frac{T}{\Gamma(\beta,\pi-\theta)} = \frac{L}{c\Gamma(\beta,\pi-\theta)} \tag{1.5}$$

where $\pi - \theta = \angle(\boldsymbol{bo},\boldsymbol{v})$ is the angle between the radius vector of the emitter o relative to the receiver b.

2. Deriving Wang Formula from the STII

We show here that the experimentally verified formula of Wang [1] is an immediate result of the scaling transformation of the second type.

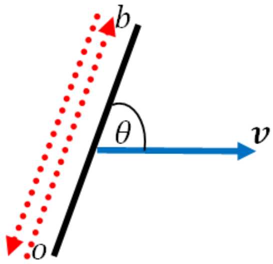


Figure 2. At the same time a pulse of light emanates from each end and travels to the other.

Suppose that at the same instant $t = 0$ a light's pulse emanates from each end of the rod ob and propagates towards the other end (Figure 2). By equations (1.5) and (1.3) the difference in the durations of the two trips $(o \to b)$ and $(b \to o)$ is

$$\Delta t(\theta) = t_{o\to b} - t_{b\to o} = \frac{L}{c}[\Gamma(\beta,\theta) - \Gamma(\beta,\pi-\theta)] = \frac{2L}{c}\beta\gamma cos\theta, \tag{2.1}$$

which is positive for $0 \le \theta < \pi/2$, negative for $\pi/2 < \theta \le \pi$, and vanishes for $\theta = \pi/2$. The difference $\Delta t(\theta)$ reduces to

$$\Delta t(0) = 2(L/c)\beta\gamma, \tag{2.2}$$

for $\theta = 0$, and to $\Delta t(\pi) = -\Delta t(0)$ for $\theta = \pi$. In both latter cases the pulse travelling opposite to the rod's velocity arrives at the other end before the pulse travelling along the velocity direction.

The difference (2.1) can be expressed in a form of inner product

$$\Delta t(\theta) = (2\gamma/c)\beta L cos\theta = (2\gamma/c^2)\boldsymbol{v}.\boldsymbol{ob} \tag{2.3}$$

The latter relation is valid even if the rod ob is of infinitesimal geometric length dl. Consider thus the situation in which light is guided to follow an arbitrary path (optical circuit) which itself is moving in S, and think of two light beams traveling in opposite directions along the path, and imagine that the path is divided into infinitesimal line elements of geometric lengths dl. The difference between the durations of two opposite light's beams traversing one such line element is

$$\Delta t(\theta) = \frac{2\gamma}{c^2}\boldsymbol{v}.\boldsymbol{dl} \tag{2.4}$$

The total difference Δt between the durations of two light's trips each starting from an end and following the given path till arriving at the other is obtained through integrating the latter expression over the whole path:

$$\Delta t = \frac{2}{c^2}\int \gamma\boldsymbol{v}.\boldsymbol{dl}. \tag{2.5}$$

The Wang formula for Sagnac interference

$$\Delta t = \frac{2}{c^2}\int \boldsymbol{v}.\boldsymbol{dl} \tag{2.6}$$

approximates equation (2.5) for low velocities. Indeed, for $v \ll c$ we have

$$\int \gamma\boldsymbol{v}.\boldsymbol{dl} \approx \int (1 + \frac{v^2}{2c^2})\boldsymbol{v}.\boldsymbol{dl} \approx \int \boldsymbol{v}.\boldsymbol{dl}, \tag{2.7}$$

which reduces (2.5) to Wang formula. However for high velocities Wang formula must be replaced by equation (2.5) which was derived from STII in the theory of universal space and time.

It is important to note however that equation (2.5) is valid as long as the line element $\boldsymbol{dl}$ is invariant under the path's motion, which implies that the optical circuit moves like a rigid body keeping the distances between all the circuit's points constant. Noting that the line element, or the metric, is invariant only under translation or rotation, or any transformation composed of them, we deduce that (2.5) is valid only when the circuit executes such motions, and thus is moving like a rigid body.

In a uniform rectilinear motion all points of the path moves at a constant vector velocity, and equation (2.5) takes the form

$$\Delta t = \frac{2\gamma}{c^2}\int \boldsymbol{v}.\boldsymbol{dl} \qquad (2.8)$$

Example: Electromagnetic Transmission Round the Equator

In [5] the TUST was employed to calculate the time delay (advance) for an electromagnetic wave heading eastward (westward) and making a full revolution round the Earth's equator. The TUST agrees fully with the common expression of Sagnac interference equation where both predict 207 ns advance (delay)[6]. By the TUST the time difference (2.8), in which we set $\gamma = 1$. is

$$\Delta t = \frac{2}{c^2}(\text{Earth spin's speed at the equator}) \times (\text{Earth cicumfrence})$$

$$= \frac{2s^2/km^2}{9.10^{10}} \times 0.4651 km/s \times 40075 km = 414.2\, ns.$$

3. Translational Sagnac's Effect.

It is demonstrated here that "a certain form" of Sagnac's effect is intimate to non-closed optical paths translating at a constant velocity relative to a universal frame *S* (i.e. in a uniform rectilinear motion in *S*). Indeed, if the optical circuit, shown in red (Figure 3), is translating at a constant velocity $\boldsymbol{v}$ in *S*, then equation (2.8) takes the form

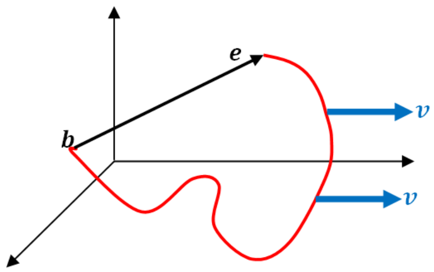


Figure 3. An open optical circuit translating at a constant velocity ***v.*** At the same instant of time a pulse of light originates from each end and travels to the other.

$$\Delta t = \frac{2\gamma}{c^2}\boldsymbol{v}.\int \boldsymbol{dl} = \frac{2\gamma}{c^2}\boldsymbol{v}.(\boldsymbol{R_e} - \boldsymbol{R_b}) = \frac{2\gamma}{c^2}\boldsymbol{v}.\boldsymbol{be}, \qquad (3.1)$$

where $\boldsymbol{R_b}$ and $\boldsymbol{R_e}$ denote the position vector of the beginning *b* of line integral (the circuit) and its end *e*. Note that

-The path need not be planar.

- The time difference Δt in (3.1) refers to (time recorded at *e* when receiving the light)- (time recorded at *b* when receiving the light). Thus if t_b and t_e denote the instants at which light emitted at $t = 0$ from both ends is received at *b* and *e* respectively then

$$\Delta t = t_e - t_b \qquad (3.2)$$

The quantity Δt in (3.1), which is positive for figure 3, will of course reverse sign if the beginning and end of the line integral were interchanged.

-The phrase “a certain form of Sgnac effect” we have used in the abstract and at beginning of this section refers to the difference in time arrival of the light’s beams that started simultaneously from the opposite ends. Only when the circuit’s two ends coincide one may observe fringe’s shift.

Equation (3.1) shows that

(a)The Sagnac effect is intimate to any constant translational motion of a non-closed optical circuit; it vanishes however when its closing vector $\boldsymbol{be}$ is perpendicular to its velocity $\boldsymbol{v}$.

(b) When it is closed, the beginning and end of the circuit are coincident, and $\Delta t = 0$ on the account of $\boldsymbol{be} = \boldsymbol{0}$ in (3.1). Therefore, there is no Sagnac effect associated with a closed circuit executing a uniform translational motion.

(c) Two optical circuits with respective ends (b, e) and (b', e') yields the same time difference Δt if $\boldsymbol{be} = \boldsymbol{b'e'}$**.** It follows that, under uniform translational motion, every circuit is equivalent to its closure, and all circuits that have equal vector closure are Sagnac equivalent,

4. Rotational Sagnac Effect

We mentioned in section 2 that equation (2.5) holds for any motion in which the circuit moves as a rigid body. We retrieve here the familiar expression that quantifies the rotational Sagnac effect [4],

$$\Delta t = \frac{4}{c^2}\boldsymbol{\omega}.\boldsymbol{A} \qquad (4.1)$$

which pertains to a polygonal circuit of area $\boldsymbol{A}$ rotating about an axis with a constant angular velocity $\boldsymbol{\omega}$. The relation which we shall derive using (2.5) applies however to any shape of a planar circuit including that which is not even closed.

Consider a rigid planar optical circuit of any shape that is rotating at a constant angular velocity $\boldsymbol{\omega}$ about an axis oo'. The axis of rotation, which is not necessarily perpendicular the circuit’s plane, intersects the latter at o (Figure 4). The velocity of a point $\boldsymbol{R}$ of the circuit is $\boldsymbol{u} = \boldsymbol{\omega} \times \boldsymbol{R}$, and hence

$$\boldsymbol{u}.\boldsymbol{dl} = (\boldsymbol{\omega} \times \boldsymbol{R}).\boldsymbol{dl} = \boldsymbol{\omega}.(\boldsymbol{R} \times \boldsymbol{dl}) = \boldsymbol{2\omega}.\boldsymbol{dA}, \qquad (4.2)$$

The quantity $\boldsymbol{R} \times \boldsymbol{dl}$ represents twice the area $\boldsymbol{dA}$ of the triangle with base $\boldsymbol{dl}$ and vertex at the point o. Substituting in (2.5) yields

$$\Delta t = \frac{2}{c^2}\int \gamma(u)\boldsymbol{u}.\boldsymbol{dl} = \frac{4}{c^2}\boldsymbol{\omega}.\int \gamma(u)\boldsymbol{dA} \qquad (4.3)$$

which is the expression of rotational Sagnac interference as implied by TUST.

Although the velocity magnitude is a function in the position $\boldsymbol{R}$ (and $\boldsymbol{\omega}$**),** $\gamma(u) = \gamma(|\boldsymbol{\omega} \times \boldsymbol{R}|)$, in some cases the factor $\gamma(u)$ can be pulled out of the integral; these include the following:

(a). All the circuit’s points are at equal distances from the center of rotation, and hence their velocities have the same magnitude.

(b) The dimensions of the circuit are much smaller that its distance from the axis of rotation. In this case one can neglect the variation in $\gamma(u)$ and deal with it as a constant function. The interferometer of Michelson and Gale experiment, consisting of 4 mirrors situated on Earth surface, and rotating with the earth about the (South-North pole) axis is an example.

(c) The velocities of the circuits components are small $\ll c$.

In all cases above we may make the approximation

$$\Delta t \approx \frac{4\gamma}{c^2}\boldsymbol{\omega}.\int \boldsymbol{dA} = \frac{4\gamma}{c^2}\boldsymbol{\omega}.\boldsymbol{A} \qquad (4.4)$$

or even set $\gamma = 1$.

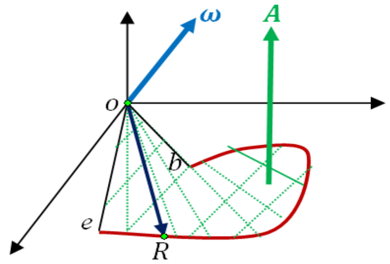


Figure 4. $\Delta t = (4\gamma/c^2)\boldsymbol{\omega}.\boldsymbol{A}$ where A is the area subtended by the circuit and the lines connecting its ends e and b with the center of rotation.

It is noted that the circuit considered need not be closed and A is the area subtended by the circuit and the lines ob and oe, where b and e are the beginning and end points of the circuit. When the circuit is closed A reduces to the circuit's area and the familiar relation (4.1) approximates (4.4) on neglecting terms of order $u^2/2c^2$ and higher.

5. Point Circuits

Examples of optical point circuits are the Sagnac interferometer which consists of a number of small mirrors M_i $(i = 1,2, \ldots, n)$, or a set of satellites revolving in one orbit, or a set of transmission stations on a great circle on Earth's surface.

By the STII the time duration of a light's trip along the side $(m_{i+1} \to m_i)$ of an n-polygonal circuit in S and in any other frame is

$$t_{i,i+1} = c^{-1}\gamma_i(-\boldsymbol{\beta_i}.\boldsymbol{R_{i,i+1}} + \sqrt{R_{i,i+1}^2 - |\boldsymbol{\beta_i} \times \boldsymbol{R_{i,i+1}}|^2}). \tag{5.1}$$

where $R_{i,i+1} = L$ is the geometric length of the side M_iM_{i+1} and $\gamma_i = \gamma(\beta_i)$.

If the polygon is regular, and the rotation is taking place about its center the difference between the durations of the trips $(m_i \to m_{i+1})$ and $(m_{i+1} \to m_i)$ is given by

$$\Delta t_i \equiv t_{i+1,i} - t_{i,i+1} = \frac{\gamma}{c}(\boldsymbol{\beta_i}.\boldsymbol{R_{i,i+1}} - \boldsymbol{\beta_{i+1}}.\boldsymbol{R_{i+1,i}}) = \frac{2\gamma(\beta)}{c}(\boldsymbol{\beta_i}.\boldsymbol{R_{i,i+1}}), \tag{5.2}$$

where $\beta = |\boldsymbol{\beta_i}|$ is the speed of any vertex (in the unit c). The latter expression is the familiar translational Sagnac effect as given by (2.3), and which corresponds to the rod making an angle $\theta = \pi/n$ with its velocity.

From (5.2) we may proceed to reach either the usual expression of the rotational Sagnac effect (4.4), or the Wang's formula. To reach the expression (4.4) of a circuit rotating at an angular velocity $\boldsymbol{\omega}$, we proceed from (5.2) as follows

$$\Delta t = \sum_{i=1}^{i=n} \Delta t_i = \sum_{i=1}^{i=n} \frac{2\boldsymbol{\beta_i}.\boldsymbol{R_{i,i+1}}}{c\sqrt{1-\beta^2}} = \frac{2}{c^2}\boldsymbol{\omega}.\sum_{i=1}^{i=n} \frac{\boldsymbol{OM_i} \times \boldsymbol{M_iM_{i+1}}}{\sqrt{1-\beta^2}} \tag{5.3}$$

where we identify $\boldsymbol{M_nM_{n+1}}$ by $\boldsymbol{M_nM_1}$. The latter relation yields the time difference between the durations of two light beams propagating in opposite directions along the ring interferometer:

$$\Delta t = 4\,\boldsymbol{\omega}.\boldsymbol{A}/c^2\sqrt{1-\beta^2}. \tag{5.4}$$

Another interesting expression for this time difference is the following:

$$\Delta t = \frac{2nLu\cos\frac{\pi}{n}}{c^2\sqrt{1-\beta^2}}, \tag{5.5}$$

where n is the number of the sides of the regular polygon and u is the speed of any vertex of the interferometer when rotating about its center. Indeed,

$$\Delta t = \sum_{i=1}^{i=n} \frac{2\boldsymbol{\beta_i}.\boldsymbol{R_{i,i+1}}}{c\sqrt{1-\beta^2}} = \frac{2}{c^2\sqrt{1-\beta^2}} \sum_{i=1}^{i=n} \boldsymbol{u_i}.\boldsymbol{M_iM_{i+1}}$$

$$= \frac{2}{c^2\sqrt{1-\beta^2}} \boldsymbol{u}.\boldsymbol{circum}, \qquad (5.6)$$

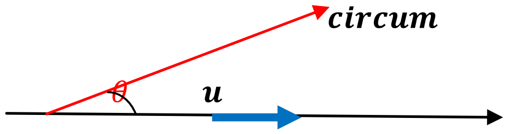


Figure 5. Geometric representation of (5.6)

where $\boldsymbol{circum}$ is a vector whose length is equal to the circumference nL of the polygon and $\boldsymbol{u}$ is a vector velocity that makes an angle $\theta = \pi/n$ with $\boldsymbol{circum}$ and whose magnitude $u = |\boldsymbol{u_i}|$ is equal to of the speed of any vertex when the interferometer rotates about its center (Figure 5).

6. Special Relativistic Explanation of Sagnac Effect is Invalid

We show here that the special relativistic explanation of Sagnac effect as given in Wikipedia (prior to July 12, 2020) is incorrect, and in the next section we demonstrate that the special relativity theory cannot explain this effect.

We quote from Wikipedia:

"**Relativistic derivation of Sagnac formula**[edit]

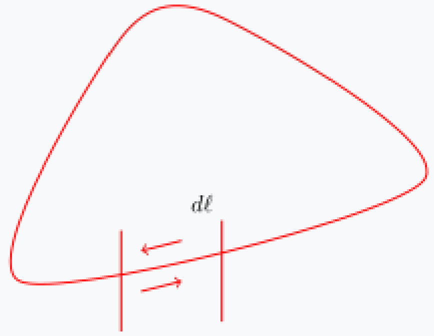


Fig. 4: A closed optical fiber moving arbitrarily in space without stretching.

Consider a ring interferometer where two counter-propagating light beams share a common optical path determined by a loop of an optical fiber, see Figure 4. The loop may have an arbitrary shape, and can move arbitrarily in space. The only restriction is that it is not allowed to stretch. (The case of a circular ring interferometer rotating about its center in free space is recovered by taking the index of refraction of the fiber to be 1.)

Consider a small segment of the fiber, whose length in its rest frame is dl'. The time intervals, $dt'_{\pm}$, it takes the left and right moving light rays to traverse the segment in the rest frame coincide and are given by

$$dt'_{\pm} = \frac{n}{c} dl'$$

Let $dl = |\boldsymbol{dx}|$ be the length of this small segment in the lab frame. By the relativistic length contraction formula, $dl' = \gamma dl \approx dl$ correct to first order in the velocity **v** of the segment. The time intervals $dt_{\pm}$for traversing the segment in the lab frame are given by Lorentz transformation as:

$$dt_{\pm} = \gamma(dt' \pm \frac{\boldsymbol{v}.\boldsymbol{dx'}}{\boldsymbol{c^2}}) \approx \frac{n}{c} dl \pm \frac{\boldsymbol{v}.\boldsymbol{dx}}{c^2}$$

correct to first order in the velocity **v**. In general, the two beams will visit a given segment at slightly different times, but, in the absence of stretching, the length *dl* is the same for both beams.

It follows that the time difference for completing a cycle for the two beams is

$\Delta T = \int (dt_+ - dt_-) = \frac{2}{c^2} \oint \boldsymbol{v}.\boldsymbol{dx}$ " - quotation finished.

The above argument which is sound in a relativistic sense contradicts experimental findings. According to special relativity the time intervals $dt'_\pm$ taken by the left and right moving light in the rest frame to traverse the given segment coincide: $dt'_\pm = dt'$. Therefore the durations of a full trip round the ring in opposite directions are equal:

$$\int dt'_+ = \int dt'_-$$

and the two beams arrive back at the point from which they started at the same instant. This means that the Sagnac effect is not observed in the rest frame, which is not true. Indeed, the fringes shift is a physical reality and the Sagnac effect is observed in the lab frame as well as in the rest frame. The Michelson and Gale experiment is a cogent proof that the Sagnac effect is observed in the rest frame. The location of the latter experiment on the surface of the rotating Earth's is the rest frame.

7. Sagnac Effect is not Explicable by Special Relativity

We abide throughout this section with special relativity theory and show that it is incapable of explaining Sagnac effect. Consider again the circuit drawn in the previous section and let s be the rest frame of the given segment (line element) drawn there. We name the segment's left and right ends by 1 and 2 respectively. The rest frame s is moving in the lab frame S at velocity $\mathbf{v}$ and can be considered inertial during the extremely short duration of the experiment.

Consider the following events in the lab frame S:

- The right moving light,
  passes by the left end (1) of the segment at $t = 0$: $(0, x_1, y_1, z_1) \equiv (0, \boldsymbol{r_1})$, and arrives at the right end (2) of the segment at dt_+: $(dt_+, x_2, y_2, z_2,) \equiv (dt_+, \boldsymbol{r_2})$.
- The left moving light
  passes by end (2) of the segment at $t = \epsilon$: $(\epsilon, X_2, Y_2, Z_2) \equiv (\epsilon, \boldsymbol{R_2})$, and arrives at end (1) at $t = \epsilon + dt_-$: $(\epsilon + dt_-, X_1, Y_1, Z_1) \equiv (\epsilon + dt_-, \boldsymbol{R_1})$.

Note that the left moving light may pass from the end 2 at an instant that differs slightly (by ϵ) from the instant at which the right moving light passes through end 1. However, this difference is so small and can be neglected (i.e. $\epsilon \approx 0$) with no bearing on subsequent results.

The 4-interval between the first two events is represented by the 4-vector $(ct_+, \boldsymbol{r_2} - \boldsymbol{r_1})$ in Minkowski space M_4. Similarly, the 4-interval between the last two events is the 4-vector $(ct_-, \boldsymbol{R_1} - \boldsymbol{R_2})$. The difference between the latter vectors is the 4-vector

$$(c(dt_+ - dt_-), (\boldsymbol{r_2} - \boldsymbol{r_1}) - (\boldsymbol{R_1} - \boldsymbol{R_2})) = (cdt, \boldsymbol{dr} - \boldsymbol{dR}), \tag{7.1}$$

where we set

$$dt_+ - dt_- = dt, \ \boldsymbol{dr} = \boldsymbol{r_2} - \boldsymbol{r_1}, \ \ \boldsymbol{dR} = \boldsymbol{R_1} - \boldsymbol{R_2}. \tag{7.2}$$

Let's take the inner product of the latter vector by the velocity 4-vector of the given circuit element, $\gamma(\mathrm{v})(c, \mathbf{v})$, and denote the result by I:

$$I = (cdt, \boldsymbol{dr} - \boldsymbol{dR}).\gamma(c, \mathbf{v}) = \gamma c^2 dt - \gamma \mathbf{v}.(\boldsymbol{dr} - \boldsymbol{dR}) \tag{7.3}$$

We remind the reader that the inner product of two 4-vectors

$$A = (a_0, a_1, a_2, a_3) \equiv (a_0, \boldsymbol{a}), \ B = (b_0, b_1, b_2, b_3) \equiv (b_0, \boldsymbol{b}) \tag{7.4}$$

in M_4 is defined by

$$A.B = a_0 b_0 - (a_1 b_1 +, a_2 b_2 + a_3 b_3) \equiv a_0 b_0 - \boldsymbol{a}.\boldsymbol{b} \tag{7.5}$$

An important property of the inner product is that: It is invariant under orthogonal transformations in M_4, and thus in particular under Lorentz transformation. In the rest frame s of the given segment, the 4-velocity of the segment is $(c, \mathbf{0})$, and the vector corresponding to (7.1) is

$$(c(dt'_+ - dt'_-), (r'_2 - r'_1) - (R'_1 - R'_2)) = (cdt', \boldsymbol{dr}' - \boldsymbol{dR}') \tag{7.6}$$

However, in the rest frame s, $dt' = 0$ because light takes the same duration to traverse the segment from either end to the other, and $\boldsymbol{dr}' = -\boldsymbol{dR}'$. Thus the vector (7.1) is transformed in s to $(0, 2\boldsymbol{dr}')$. Equation (7.3) and the invariance of the inner product yield

$$I = \gamma c^2 dt - \gamma \mathbf{v}.(\boldsymbol{dr} - \boldsymbol{dR}) = (c, \mathbf{0}).(0, 2\boldsymbol{dr}') = 0 \tag{7.7}$$

In the frame S, $\boldsymbol{dr}$ and $\boldsymbol{dR}$ are the vectors from one end of the segment to the other but at different instants of time. Each vector joins the location of the end when light enters the segment and the location of the other end when it leaves. Neglecting quantities of high order in smallness yields $(\boldsymbol{dr} = -\boldsymbol{dR})$, and hence

$$I \approx \gamma(c^2 dt - 2\mathbf{v}.\boldsymbol{dr}) = 0 \tag{7.8}$$

The total difference in time arrival of the two beams to their common starting point in the circuit is obtained through integrating over the circuit:

$$\Delta t = \frac{2}{c^2}\int \mathbf{v}.\boldsymbol{dr} \tag{7.9}$$

However, in the rest frame of each segment of the circuit there is no difference in time arrival of the two beams at the opposite ends. As a result, the two beams return to their starting point at the same time, and no displacement in interference fringes will be observed in a frame attached rigidly to the circuit. The latter result is at contradiction with experiment.

Conclusion

The Wang formula for Sagnac interference follows directly from the scaling transformation of the second type in the theory of universal space and time. In contrast, the special theory of relativity is utterly incapable of explaining Sagnac effect. It was also demonstrated that that a certain form of Sagnac's effect occurs under rotation even if the optical circuit is not closed, and under translation provided the circuit is not closed and its enclosing segment is not perpendicular to its velocity.

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