P6. The Sagnac's Interference, and Michelson and Gale Experiment

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Since its experimental verification in 1913 by the French physicist Georges Sagnac, various explanations of the Sagnac's effect were published [1-4], In this work we employ the STII to explain the rotational type of this effect, as well as, the Michelson and Gale experiment. The general type of Sagnac effect is presented in [5].

1. The Sagnac Interferometer

The Sagnac's interferometer consists of n small plane mirrors $m_1, m_2, \dots ,m_n$ occupying the vertices of a regular polygon $m_1 m_2 \dots m_n m_1$ with a center o, radius a, side L, and a source of light capable to send simultaneously two light's beams in opposite directions along the polygonal loop. We enumerate the mirrors to increase counter clockwise which conventionally is the positive sense of rotation, and we take the normals to the diagonals to point counter clockwise, i.e. rotation-wise.

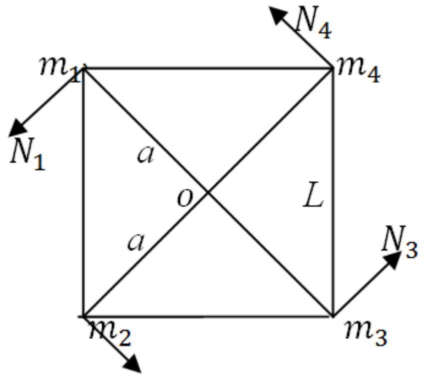


Fig 1. A diagram of a square Sagnac interferometer

If two pulses of light set out at the same time from the same point m_1 and make two closed trips in opposite directions along the rim of the polygon then they return to $m_1$at the same time if the interferometer is at rest in S. If the interferometer is rotating, the two pulses do not arrive back at m_1 simultaneously, for, *the pulse moving counter rotation arrives earlier than the pulse moving rotation-wise.*

A frame attached to the terrestrial laboratory with respect to which the interferometer is rotating can be considered during the short period taken by the experiment a timed inertial frame S of fixed stars. We assume that $\omega \gg 2\pi\, radian/day$, i.e. the angular velocity of the interferometer is much greater than that of the earth about its axis. In the frame S the velocity $\boldsymbol{u_i}$ of the mirror m_i when at $M_i \in S$ is perpendicular to the vector oM_i, and hence

$$\angle(\boldsymbol{M_i M_{i+1}}, \boldsymbol{u_i}) \equiv \theta = \frac{\pi}{n}, \quad \angle(\boldsymbol{M_{i+1} M_i}, \boldsymbol{u_{i+1}}) = \pi - \theta, \qquad (1.1)$$

where M_i and M_{i+1} are the positions of the mirrors m_i and m_{i+1} in S at the instant of light emission. If the system is stationary, light which is reflected at a mirror will take a duration $T = L/c$ to reach either adjacent mirror. We showed in section 3 in P5 of this series of articles. that the duration of the trip $(m_{i+1} at\ M_{i+1} \to m_i)$ is the same whether the observer m_i is moving at his actual velocity $\boldsymbol{u_i}$ or at a velocity $\boldsymbol{u}_i'$ which is the mirror image of $\boldsymbol{u_i}$with respect to the line $m_i m_{i+1}$. Since $\boldsymbol{u}_i' \parallel \boldsymbol{u_{i+1}}$, the equations of STII (2.3) and (2.4) in P5 can be applied.

2. Explaining Sagnac Effect

We rewrite STII in a convenient form to use. Let M_i and M_{i+1} be the laboratory, or S-observers, that are conjugate to m_i and m_{i+1} respectively at the instant light is reflected at m_{i+1} towards m_i, and set $\boldsymbol{R_{i,i+1}} = \boldsymbol{M_i M_{i+1}}$. The duration $t_{i+1,i}$ of the counter rotating pulse belonging to the trip $(m_{i+1}\ at\ M_{i+1} \to m_i)$ is given by:

$$ct_{i+1,i} = \gamma_i(-\boldsymbol{\beta_i}.\boldsymbol{R_{i,i+1}} + \sqrt{R_{i,i+1}{}^2 - |\boldsymbol{\beta}_i \times \boldsymbol{R_{i,i+1}}|^2}), \qquad (2.1)$$

where $\boldsymbol{\beta}_i = \boldsymbol{u}_i/c$ and $R_{i,i+1} = |\boldsymbol{M_i M_{i+1}}| = L$. Because $max\beta_i \ll 1$ we neglect terms in second order in $\boldsymbol{\beta}_i$ in the right hand-side, and approximate the transformation by

$$ct_{i+1,i} \approx -\boldsymbol{\beta}_i.\boldsymbol{R_{i,i+1}} + R_{i,i+1} = L - \boldsymbol{\beta}_i.\boldsymbol{R_{i,i+1}}, \qquad (2.2)$$

which is the time length of the trip $(m_{i+1}\ at\ M_{i+1} \to m_i)$. Similarly, the time length of the co-rotating trip $(m_i\ at\ M_i \to m_{i+1})$ is approximated by

$$ct_{i,i+1} \approx L - \boldsymbol{\beta}_{i+1}.\boldsymbol{R_{i+1,i}}. \qquad (2.3)$$

Figure 2 illustrates the latter equation for a square interferometer.

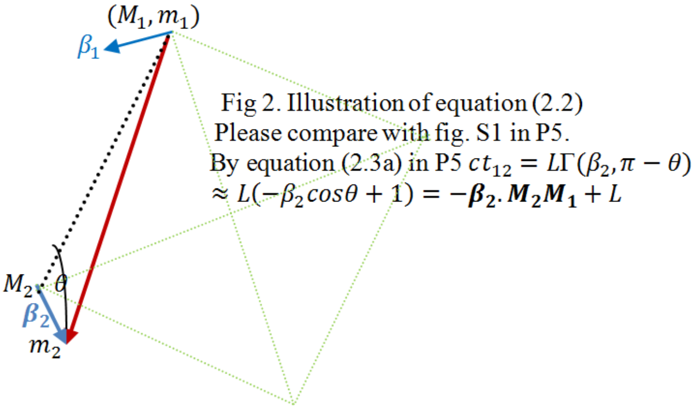


Fig 2. Illustration of equation (2.2)
Please compare with fig. S1 in P5.
By equation (2.3a) in P5 $ct_{12} = L\Gamma(\beta_2, \pi - \theta)$
$\approx L(-\beta_2 cos\theta + 1) = -\boldsymbol{\beta_2}.\boldsymbol{M_2 M_1} + L$

Rotation About the Polygon's Center

When the polygon rotates about its center o, the velocity of the mirror m_i is

$$\boldsymbol{\beta}_i = \frac{1}{c}\boldsymbol{\omega} \times \boldsymbol{oM_i}, \qquad (2.4)$$

and by (2.2) the duration of the co-rotation trip $(m_i\ at\ M_i \to m_{i+1})$ is

$$ct_{i,i+1} = L - \boldsymbol{\beta}_{i+1}.\boldsymbol{M_{i+1}M_i} = L - \frac{\boldsymbol{\omega}}{c}.\boldsymbol{oM_{i+1}} \times \boldsymbol{oM_i}. \qquad (2.5)$$

Similarly, the duration of counter-rotation trip $(m_{i+1}\ at\ M_{i+1} \to m_i)$ is

$$ct_{i+1,i} = L - \frac{\boldsymbol{\omega}}{c} \times \boldsymbol{oM_i}.\boldsymbol{M_i M_{i+1}} = L - \frac{\boldsymbol{\omega}}{c}.\boldsymbol{oM_i} \times (\boldsymbol{oM_{i+1}} - \boldsymbol{oM_i})$$
$$= L - \frac{\boldsymbol{\omega}}{c}.\boldsymbol{oM_i} \times \boldsymbol{oM_{i+1}}. \qquad (2.6)$$

The difference Δt_iin durations of the two trips for one side of the polygon is

$$\Delta t_i \equiv (t_{i,i+1} - t_{i+1,i}) = \frac{2\boldsymbol{\omega}}{c^2}.\boldsymbol{oM_i} \times \boldsymbol{oM_{i+1}}. \qquad (2.7)$$

The total difference in the durations of the two trips is

$$\Delta t = \sum_{i=1}^{i=n} \Delta t_i = \frac{2}{c^2}\boldsymbol{\omega}.\sum_{i=1}^{i=n} \boldsymbol{oM_i} \times \boldsymbol{oM_{i+1}} = \frac{4}{c^2}\boldsymbol{\omega}.\boldsymbol{A}, \qquad (2.8)$$

where we identify M_{n+1} by M_1, and $\boldsymbol{A}$ is the vector area of the polygon directed in the positive sense.

The same time difference (1.9) will be recorded if the interferometer is put on a turntable which rotates about a point O that is distinct from the center o of the polygon [4].

3. The Michelson and Gale Experiment

This experiment is a version of an interferometer placed on a turntable which is the Earth in this case. Here S is the frame of fixed stars and s is a frame attached to Earth. The $720m \times 320m$ rectangular interferometer is set up horizontally on earth's surface. The rotation vector $\boldsymbol{\omega}$ of the earth is in the direction of the (south pole→north pole) axis, and its magnitude is 2π radian per sidereal day. The area vector $\boldsymbol{A}$ makes an angle α with the rotation vector $\boldsymbol{\omega}$; moreover, α ($\pi - \alpha$) is the angle between the equatorial plane and the horizontal interferometer's plane in the northern (southern) hemisphere. The Sagnac effect is given by

$$\Delta t = (4/c^2)\, \boldsymbol{\omega}.\boldsymbol{A}, \tag{3.1}$$

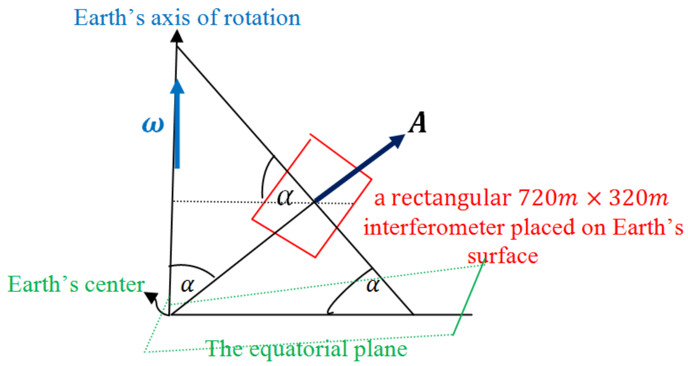
where $\boldsymbol{A}$ is the vector area of the interferometer. The inner product

$$\boldsymbol{\omega}.\boldsymbol{A} = \omega A\ cos\alpha = \omega\ proj_{\boldsymbol{\omega}}\boldsymbol{A}$$

has the magnitude

$$|\boldsymbol{\omega}.\boldsymbol{A}| = \omega \times \text{ projection of the area } A \text{ on the equatorial plane.}$$

The latter results constitute an explanation of the Michelson and Gale experiment.



The relation (3.1) shows that the Sagnac's effect (i.e. time difference, or fringe shift)

(i)Assumes maximal values at the north and south poles: $\Delta t = \pm(4/c^2)\, \omega.A$, with the plus (minus) sign corresponding to north (south) pole.

(ii) Decreases in absolute value when moving towards the equator till vanishing at it.

(iii) Not influenced by the orientation of the interferometer in its horizontal site, as well as, by the site's longitude.

(iv) Is nil wherever the interferometer is placed parallel to the earth's axis of rotation.

(v) Has the same maximum magnitude measured at a pole if the interferometer is placed vertically at the equatorial plane.

(vi) The fact (iv) promotes a method to measure the latitude of the location at which the interferometer is placed. Indeed, rotating the norm of the turntable (and hence the turntable) southwards (northward) in the northern (southern) hemisphere by its the latitude angle diminish the Sagnac effect. Alternatively, and if in order to attain the maximal Sagnac effect ($\Delta t = 4\omega.A/c^2$) we need to rotate the norm of the turntable (and hence the turntable itself) by an angle α towards the north (south) in the northern (southern) hemisphere, then $\frac{1}{2}\pi - \alpha$ is the latitude of the location of the interferometer.

References

[1] Gianni Pascoli, The Sagnac effect and its interpretation by Paul Langevin https://doi.org/10.1016/j.crhy.2017.10.010

[2] Math Pages, The sagnac Effect. https://www.mathpages.com/rr/s2-07/2-07.htm

[3]Doug Marett 2012, The Sagnac Effect: Does it Contradict Relativity? http://www.conspiracyoflight.com/SagnacRel/SagnacandRel.html

[4] Viazminsky C.P., The Sagnac effect and the Michelson and Gale experiment, *gs Journal*. July. 7, 2009

[5] Viazminsky C.P., The General Sagnac Effect, *gs Journal* , November 1, 2009.