# Optical Length of a Moving Rod

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## 1. Universal Synchronization

Inertial frames in the theory of universal space and time (TUST) are equivalent sites for one physical world [1,2]. Only one arbitrary inertial frame, say *S*, may be taken as universal. This means that: (i) *S* is considered a stationary frame with respect to which the motion of all other frames is referred, and (ii) *S* is endowed with a global time which also prevails in any other inertial frame.

Property (i) implies in particular that the direction of a light's trip in a moving frame *s* is derived from the trip's characteristics in the universal frame *S*. Property (ii) is expounded here in different wording. A universal frame *S* is a synchronous frame which we envisage as having a system of timers that run at synchrony with each other. The time read by these timers is the time in *S* as well as in all other frames. Regardless of its state of motion, an observer that is contiguous to a point $B \in S$ at some instant *t*, as read by the clock at *B*, adopts also the same value *t* as the instant at which he passes by *B*. There is only one time, namely the time of the chosen universal frame *S*, which prevails all over the physical world and in any other inertial frame; it may be called the universal time or just time. The latter explanatory statements concerning "one time" are utterly redundant in Newtonian mechanics, simply because they are considered intuitively true and self-evident. The concept of absolute simultaneity in Newtonian mechanics was taken for granted as a reflection of the absolute nature of time, and it was substantiated practically through appealing to universal timers [3].

In his theory on special relativity (1905), Einstein introduced the concept of relative simultaneity by which simultaneous events in a reference frame are usually not so in another [4]. Also, time is relative and does not flow equably as it was envisaged in non-relativistic physics. In fact, moving clocks undergo retardation resulting in stationary clocks recording greater time intervals than moving ones.

Objections concerning relativistic stipulations about the constancy of light's velocity and the synchronization process, together with the associated consequences of time dilation and length contraction persist [5,6]. Utilizing the absoluteness of geometric length in TUST we derive in current paper the optical length of a rod in terms of its geometric length and velocity. Equivalently, we derive a second type of the scaling transformation [7] by which the Michelson and Gale experiment, the Sagnac effect, and the general Sagnac effect are explained [8].

## 2 . Optical Length of a Moving Rod

The geometric length $L$ of a rod, when stationary in *S*, is the distance between its ends. This can be measured by a calibrated ruler (unit of length) as defined in [7], or by the duration of a light trip which begins at one end and terminates at the other; no matter which end the light trip starts from. Both ways for measurements amount to determine the number $N$ of the specific wavelengths $\lambda_{0l}$ contained in $L$. One can always adopt shorter wavelengths if more accuracy is needed. The equivalence of all inertial frames implies that the geometric length of a rod is invariant, which means that it has the same length L when measured in any frame in which the rod is stationary. If a rod of geometric length $L$ in *S*, or a copy of it, is placed in another inertial frame s, it will still have the same geometric length $L$ in $s$. In both frames the same count $N$ of the wavelength $\lambda_{0l}$ we talked about in [7] will be obtained for the length of the given rod. Thus the phrase "a rod is of geometric length (or just length) $L$" is fully meaningful by itself and refer to the outcome of its length measurement in any frame in which it is stationary.

The problem which we discuss here is the relation between the optical length of a rod which is moving in an inertial frame $S$ and its geometric length. It is worth to mention that *the optical length of the moving rod, which depends of course on the rod's velocity and the angle it makes with the rod, depends also on the rod's end from which the pulse of light starts and travel to the other end*. I.e. a rod that is at rest in $s$ has in general two optical lengths in $S$.

Suppose that a rod $bo$ of geometric length $L$ is at rest in the inertial frame $s$ which is moving at a velocity $u\boldsymbol{i}$ in $S$, and let $A(t') \in S$ and $B(t') \in S$ be the locations of its ends in the universal frame $S$ at an instant $t'$. At $t' = 0$ which corresponds to the rod occupying the position $BO$ in $S$, a pulse of light is sent from one end, say $b \in s$, to the other end $o \in s$. The pulse hits the other end $o$ when the latter is at a position $o' \in S$. In the frame $S$, $|\boldsymbol{A}(t')\boldsymbol{B}(t')| = L$ for any $t'$, and in particular $|\boldsymbol{BO}| = L = cT$, where $T$ denotes the geometric time length of the rod when at rest in $S$.

Since the end $o$ is moving in $S$ we set provisionally $Oo' = \gamma ut$ and $Bo' = \gamma ct$, where $t$ is the time at which the pulse hits $o$. By figure 1S we get

$$\gamma t\sqrt{1-\beta^2} = \frac{T}{\Gamma(\beta,\theta)} \qquad (2.1)$$

where $\beta = u/c$.

If $s$ is the universal frame then the rod $\boldsymbol{BO}$ which is moving at velocity $-u\boldsymbol{i}$ in $s$ coincides at $t' = 0$ with the rod $\boldsymbol{bo}$ in $s$, and a pulse of light that emanates from the end $B$ heading to $O \in S$, arrives at an instant $T$ that corresponds to $O$ occupying a position $O' \in s$. By figure 1s we get

$$\gamma T\sqrt{1-\beta^2} = \frac{t}{\Gamma(\beta,\pi-\theta)} = t\Gamma(\beta,\theta) \qquad (2.2)$$

Equations (2.1) and (2.2) yield $\gamma^2(1-\beta^2) = 1$, or $\gamma = 1/\sqrt{1-\beta^2}$, and thus the time $t_{bo} \equiv t$ taken by the pulse emitted from $b$ to reach $o$ is

$$t_{bo} = \frac{T}{\Gamma(\beta,\theta)} = \frac{L}{c\Gamma(\beta,\theta)}. \qquad (2.3a)$$

We have subscripted $t$ by ($bo$) to emphasize on the beginning and end of the light's trip. It is easily seen that

$$t_{ob} = \frac{T}{\Gamma(\beta,\pi-\theta)} = \frac{L}{c\Gamma(\beta,\pi-\theta)} \qquad (2.3b)$$

The durations $t_{bo}$ and $t_{ob}$ of the trips $(b \to o)$ and $(o \to b)$ can be expressed respectively as

$$\frac{ct_{bo}}{\sqrt{1-\beta^2}} = E(\beta,\theta)L \qquad (2.4a)$$

and

$$\frac{ct_{ob}}{\sqrt{1-\beta^2}} = E(\beta,\pi-\theta)L, \qquad (2.4b)$$

where

$$E(\beta,\theta) = \frac{-\beta\cos\theta + \sqrt{1-\beta^2 sin^2\theta}}{1-\beta^2} \qquad (2.5)$$

is the Euclidean factor. Equations (2.3) and (2.4) are referred to as the scaling transformations of second type (STII).

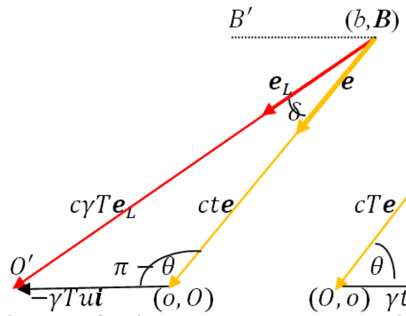

Fig.1$s$.The view of $s$ when stationary of the true trip ($\boldsymbol{B}$ $at$ $b \to O'$)

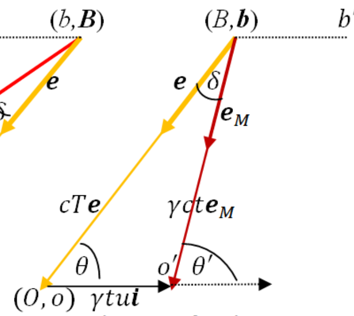

Fig.1$S$. The view of $S$ when stationary of the true trip ($\boldsymbol{b}$ $at$ $B \to o'$)

In the relations (2.3a) or (2.4a), $\theta = \angle(\boldsymbol{OB}, OX)$ is in $S$ the angle between the initial radius vector of the source's position and the velocity of $s$ (or $o$) in $S$.

The transformation (2.3a) *determines the duration* $t_{bo}$ *of the trip* ($\boldsymbol{b} \to o'$), (in $S$ *and in* $s$), *in terms of the geometric length* $T$ *of a hypothetical trip* ($B \to O$), *which is already known in S*. We call $t_{bo}$ *the universal* (*or optical*) *duration* of the light trip ($\boldsymbol{b} \to o'$). Parallel statements hold for the light trip ($o \to b$) and the relations (2.3b) and (2.4b).

If $s$ is the universal frame then its global time prevails in any other inertial frame and the period of any light trip can be determined in terms of its geometric length in $s$. In this case $L = ct_{bo}$ and the transformation (2.3a) determines the duration $T_{BO}$ of the true ($B \to O'$). Thus we should set in (2.3a) $t_{bo} = L/c$,

$$t_{bo} \equiv \frac{L}{c} = \frac{T_{BO}}{\Gamma(\beta,\theta)} \tag{2.6}$$

which yield

$$T_{BO} = \frac{L}{c\Gamma(\beta,\pi-\theta)}. \tag{2.7a}$$

Similarly, the duration $T_{OB}$ of the trip $O \to B$ ($s$ is stationary) is

$$T_{OB} = \frac{L}{c\Gamma(\beta,\theta)} \tag{2.7b}$$

It must be kept in mind that the direction of the true trip ($b \to o$) in $s$, when stationary, is the same as the direction of the hypothetical trip ($B \to O$) in $S$, when stationary.

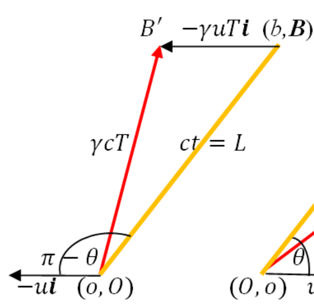

Fig.2$s$. The view of $s$ when stationary the true trip ($\boldsymbol{O}$ $at$ $o \to B'$)

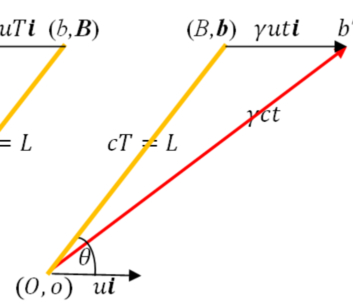

Fig.2$S$. The view of $S$ when stationary of the true trip ($\boldsymbol{o}$ $at$ $O \to b'$)

The case in which the trip $(b \to o)$ occurs in the direction of the rod's velocity (**Fig.3**) corresponds to $\theta = \pi$ in (2.3a) and yields

$$t_{bo} = \frac{T}{\Gamma(\beta,\pi)} = T\frac{1+\beta}{\sqrt{1-\beta^2}} = \frac{L}{c}\sqrt{\frac{1+\beta}{1-\beta}} \tag{2.8a}$$

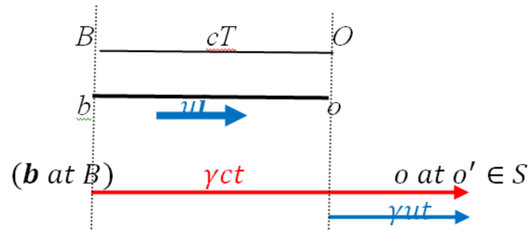


Fig 3. A rod *bo* of geometric length $L = cT$ and moving in $S$ at velocity $u\boldsymbol{i}$.

The duration of the trip $(b\ at\ B \to o' \in S)$ is $t = \frac{T}{\Gamma(\beta,\pi)} = \sqrt{\frac{1+\beta}{1-\beta}}T$.

And its optical length is $\gamma ct = \frac{cT}{1-\beta}$.

Similarly, and on setting $\theta = 0$ (Fig.4) in (2.3a) we obtain

$$t_{ob} = \frac{T}{\Gamma(\beta,0)} = T\frac{1-\beta}{\sqrt{1-\beta^2}} = \frac{L}{c}\sqrt{\frac{1-\beta}{1+\beta}} \tag{2.8b}$$

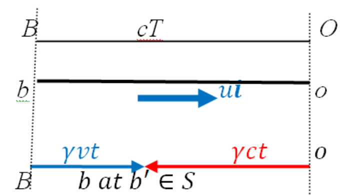


Fig.4. A rod $ob$ is of geometric length $L = cT$ and moving in $S$ at velocity $u\boldsymbol{i}$.

The duration of the trip $(o\ at\ O \to b' \in S)$ is $t = \frac{T}{\Gamma(\beta,0)} = \sqrt{\frac{1-\beta}{1+\beta}}T$.

And its optical length is $\gamma ct = \frac{cT}{1+\beta}$.

If s is the universal frame the rod $BO$ moves at velocity $-\beta\boldsymbol{i}$ relative to $s$, and since $bo$ is at rest in $s$ and of geometric length $t_{bo} = L/c$, the duration of the trip $(\boldsymbol{B} \to O)$ which is identified in s as $(B\ at\ b\ \to O\ at\ O' \in s)$is obtained on setting $\theta = \pi$ in (2.7a)

$$T_{BO} = \frac{L}{c\Gamma(\beta,0)} = t_{ob}. \tag{2.9a}$$

Similarly

$$T_{OB} = \frac{L}{c\Gamma(\beta,\pi)} = t_{bo}. \tag{2.9b}$$

## 3. A Basic Symmetry of the STII.

The STII exhibits an obvious symmetry [6]. Indeed, the rotation of the triangle $BOo'$ about $Bo'$ spans two circular cones in $S$, shown in green and blue in Figure 5. Both cones have the same axis of symmetry $Bo'$, but different vertices, $B$ and $o'$ respectively. Let $s'$ be an inertial frame translating relative to $S$ at a constant velocity $\boldsymbol{u_p}$ that is parallel to any side of the blue cone and equal in magnitude to $u$. Assume at an instant $t = 0$, corresponding to $(a \in s' at\ A \in S\ and\ p \in s' at\ B \in S)$, a pulse of light is emitted from the source $p$ towards the receiver $a$. It is easy to see that the pulse arrives at $a$ when $a$ is at $o' \in S$. In other wards, the duration $t$ of the trip $(p \in s' at\ B \in S \to a \in s')$ is the same as the duration of the trip $(b \in s\ at\ B \in S \to o \in s)$. This implies that all pulses emitted from sources at $B \in S$ that have velocities making the same angle $\theta'$ with $Bo'$ (and hence the same angle $\theta$ with the cone) and having the same magnitude $u$ take the same duration $t$ to arrive at $o' \in S$.

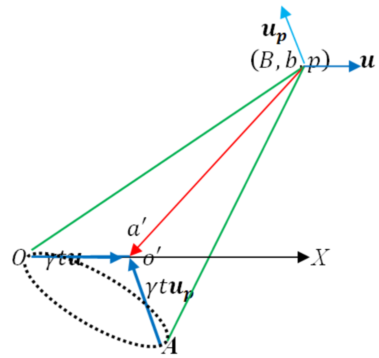


Fig. 5. Pulses from sources at $B$ that have velocities resulting from rotating the velocity $\boldsymbol{u}$ of the source $b$ about $Bo'$ take the same time to arrive at $o'$.

## 4. Return Trips

Assume that when the moving rod $ob$ coincides with the segment $OB$ in the universal frame $S$, say at $t = 0$, a pulse of light emanates from one end, arrives at the other end, and is reflected back to the first end.

The return trip $(b \to o \to b)$ is seen in $S$ as $(B \to o' \to b")$ which is shown in red (Figure 6). The duration of this return trip is

$$t_{ret} = t_{b\to o} + t_{o\to b} = \frac{T}{\Gamma(\beta,\theta)} + \frac{T}{\Gamma(\beta,\pi-\theta)} = 2\frac{\sqrt{1-\beta^2 sin^2\theta}}{\sqrt{1-\beta^2}}T \qquad (4.1)$$

The return trip $(o \to b \to o)$, which is observed in $S$ as $(O \to b' \to o")$, shown in yellow in Figure 6, takes the same duration $t_{ret}$ given by equation (4.1). Thus, the two latter return trips end at the same time provided they started at the same time, i.e. when $(o\ at\ O$ and $b\ at\ B)$.

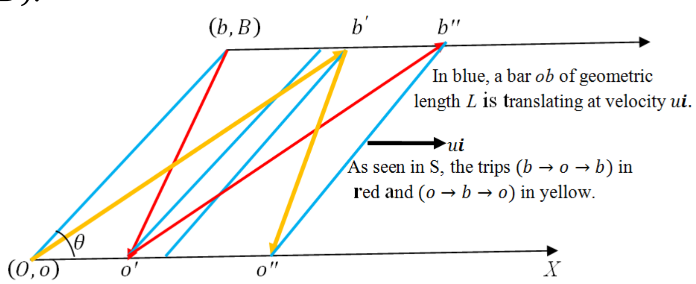


Fig.6. Return trips from both ends have the same length

If the rod is perpendicular to its velocity vector in $S$, then setting $\theta = \frac{1}{2}\pi$ in (4.1) yields the duration of either return trip given by

$$t_{ret} = 2T. \qquad (4.2)$$

If the rod is aligned along its velocity vector, i.e. $\theta = 0\ or\ \theta = \pi$, then

$$t_{ret} = \frac{2T}{\sqrt{1-\beta^2}}. \quad (4.3)$$

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