

A Wrong Argument in a Seminal Physics Paper

(An updated variant of the published version available [here](#))

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Abstract.—This article examines a wrong argument in a seminal physics paper published 114 years ago: Einstein’s paper on the electrodynamics of moving objects. Although the argument in question is not determinant for the main conclusions of the paper (thanks to the prevalence of Lorentz transformation also deduced in the same paper), it contains several basic and significant errors that have remained undetected to this day. Those errors are consequences of a misinterpretation of the Principle of Relativity, surprisingly the same misinterpretation one can find in some naïf critiques of special relativity. Considering the relevance of the paper and of its author, and the importance of the errors, there is no alternative but to make them public.

Keywords: relativity of simultaneity, Principle of Relativity, Einstein test for synchronous clocks, special relativity, history of physics.

1. INTRODUCTION

Errors are an invaluable source of experience and knowledge. But, obviously, errors have to be detected and corrected. And errors in seminal papers must be noticed to avoid their dissemination and to facilitate the understanding of the birth, reception, and development of the new ideas usually introduced in such seminal papers. In the case of the errors examined in this paper, it is striking that they have never been noticed because, even not being conclusive for the main objective of the paper, they are really evident and important, and illustrate Einstein’s wrong way of dealing with the Principle of Relativity.

It seems convenient to recall that from the title of the paper, from its introductory text [3, pp. 891-892], and from Einstein’s own words 48 years after its publication (letter to Carl Seelig dated in 1953 [1, p. 104]), the main objective of Einstein’s paper was not to lay the foundation of special relativity as a new theory of spacetime, but to apply Lorentz transformation to Maxwell’s equations of electromagnetism. This could have been the reason for which Einstein did not properly revised the §2 (On the Relativity of Lengths and Times) of his paper that contains the wrong argument.

In any case, much more important than the errors themselves is the fact that they have not been detected for more than 114 years, Einstein’s paper being one of the most read (?) and cited works in modern physics, and the errors being so basic and evident (as the reader will immediately see). This anomalous circumstance should merit the attention and concern of contemporary physicists. And the errors should be noticed according to their importance and to the influence of the paper and of its author.

In §2 of his paper, Einstein introduces equation (7) through the sentence “we find that (finden wir)” [3, p. 896], without giving details on where (7) comes from. It was by searching where (7) comes from that I discovered Einstein’s wrong argument. And there is no doubt it comes from a misinterpretation of the Principle of Relativity. As far as I know, such a misinterpretation is revealed for the first time in this work. Einstein’s paper is introduced in Section 2, and his argument on the relativity of lengths and times in Section 3. Then, in Section 4, Einstein’s errors are explained in detail. And Section 4 summarizes them and some of their consequences.

2. A SEMINAL PAPER

Einstein’s paper “Zur Elektrodynamik bewegter Körper” [3] is divided into two parts, the Kinematical Part and the Electrodynamical Part. In the Kinematical Part, and after a short discussion on the relativity of lengths and times that includes the statements of the two principles of the special relativity, Einstein deduces, according to both principles, Lorentz transformation (LT from now on), as well as a formula for the composition of velocities. The Electrodynamical Part applies these results to Maxwell’s equations of electromagnetism and to other electrodynamics questions. The erroneous argument examined here is in the §2 of the Kinematical Part, entitled “On the Relativity of Lengths and Times”. The English translation of the Einstein’s texts quoted in this work is in the public domain [5] and has been contrasted with French [4] and Spanish [2] translations as well as with the original paper in German language [3]. It is worth noting that in his Spanish translation of Einstein’s paper [2], Ruíz de Elvira disagrees and critiques some Einstein’s obscure arguments, particularly the

arguments in §2 on the relativity of lengths and times. I recognize his criticism also motivated this work.

3. RELATIVITY OF SIMULTANEITY

In §1 of his article, Einstein defines a criterion (EC hereafter) to test the synchronous state of two clocks placed at two points A and B [3, pp. 893-894] [equation number is mine]:

If at the point A of space there is a clock [...] If there is at the point B of space another clock in all respects resembling the one at A ... we establish *by definition* that the “time” required by light to travel from A to B equals the “time” it requires to travel from B to A . Let a ray of light start at the “ A -time” t_A from A towards B , let it at the “ B -time” t_B be reflected at B in the direction of A , and arrive again at A at the “ A -time” t'_A . In accordance with definition the two clocks synchronize if:

$$t_B - t_A = t'_A - t_B \quad (1)$$

In accordance with the above Einstein’s words, the time t_{AB} light travels when going from A to B equals the time t_{BA} it travels when going back from B to A . So, and according to the Second Principle of Relativity, the distance d_{AB} light travels from A to B equals the distance d_{BA} it travels when going back from B to A . This is what the following corollary makes explicit and proves:

EC corollary.-If $d_{AB} \neq d_{BA}$, EC is not conclusive.

Proof.-Let $d_{AB} \neq d_{BA}$, and assume the clock at A is ahead of the clock at B by a time $\Delta t \geq 0$ (a similar argument applies if the clock at B is ahead of the clock at A). According to the Second Principle of Relativity, it can be written:

$$d_{AB} \neq d_{BA} \Rightarrow t_{AB} \neq t_{BA} \quad (2)$$

$$t_B = t_A + t_{AB} - \Delta t; \quad t'_A = t_A + t_{AB} + t_{BA} \quad (3)$$

$$t_B - t_A = t_{AB} - \Delta t; \quad t'_A - t_B = t_{BA} + \Delta t \quad (4)$$

$$t_B - t_A = t'_A - t_B \Leftrightarrow t_{AB} - \Delta t = t_{BA} + \Delta t \quad (5)$$

$$t_{AB} - \Delta t = t_{BA} + \Delta t \Leftrightarrow t_{AB} - t_{BA} = 2\Delta t \quad (6)$$

Therefore, if $d_{AB} \neq d_{BA}$ and $\Delta t = 0$, both clocks are synchronous, but EC gives them as asynchronous because of (2) and (4): $t_B - t_A = t_{AB} \neq t_{BA} = t'_A - t_B$. On the other hand, if $d_{AB} \neq d_{BA}$ and $\Delta t > 0$, both clocks are asynchronous, but EC gives them as synchronous if $t_{AB} - t_{BA} = 2\Delta t$ (5)-(6). In consequence, if $d_{AB} \neq d_{BA}$, EC is not conclusive. \square

In §2 of his paper, and before deducing LT, Einstein tries to prove the synchronous state of two clocks is not the same for all observers [3, pp. 896-897] (emphasis and paragraphs and equation numbering is mine):

P1. Let there be given a stationary rigid rod; and let its length be l as measured by a measuring-rod which is also stationary. We now imagine the axis of the rod lying along the X axis of the stationary system of coordinates, and that a uniform motion of parallel translation with velocity v along the X axis in the direction of increasing x is then imparted to the rod. [...]

P2. We imagine further that at the two ends A and B of the rod, clocks are placed which synchronize with the clocks of the stationary system, that is to say that their indications correspond at any instant to the “time of the stationary system” at the places where they happen to be. These clocks are therefore “synchronous in the stationary system.”

P3. We imagine further that *with each clock there is a moving observer*, and that *these observers apply to both clocks* the criterion established in §1 for the synchronization of two clocks. Let a ray of light depart from A at the time¹ t_A , let it be reflected at B at the time t_B , and reach A again at the time t'_A . Taking into consideration the principle of the constancy of the velocity of light $[V]$, we find that:

$$t_B - t_A = \frac{r_{AB}}{V - v}; \quad t'_A - t_B = \frac{r_{AB}}{V + v} \quad (7)$$

¹“Time” here denotes “time of the stationary system” and also “position of hands of the moving clock situated at the place under discussion.” (Einstein)

where r_{AB} denotes the *length of the moving rod -measured in the stationary system*. Observers moving with the moving rod would thus find that the two clocks were not synchronous, while observers in the stationary system would declare the clocks to be synchronous.

4. DISCUSSION

Since “*with each clock [placed at the end of the rod] there is a moving observer*” and “*these [moving] observers apply [EC] to both clocks*” (P3), both clocks must be at rest relative to the moving observers and in their immediate proximity, so that they must also move with the same uniform velocity v relative to the stationary system as the moving observers and the moving rod while the moving observers apply EC to both clocks. This (obviousness) will be confirmed next by (12)-(20), and then by Einstein’s (7). For this reason, I will use the following nomenclature (in addition to Einstein’s): the stationary observers and their inertial rest frame will be referred to as s-observers and F_s respectively; the moving observers will be referred to as m-observers; A_c and B_c will denote the moving clocks placed at the respective ends A and B of the moving rod; the rod, the clocks A_c , B_c and m-observers move at the same uniform velocity v relative to F_s in the direction of the increasing x of the X axis of F_s , which is also the direction from A to B ; the inertial rest frame of the rod, the clocks A_c , B_c and m-observers will be denoted by F_m ; d_{AB} , t_{AB} will denote respectively the distance and time light travels from A to B ; d_{BA} , t_{BA} will denote respectively the distance and time light travels from B to A ; and c will denote Einstein’s speed of light V .

If m-observers refer their EC measurements to their rest frame F_m , they will find that $d_{AB} = d_{BA} = l$, and then $t_{AB} = l/c = t_{BA}$. Consequently:

$$t_B = t_A + t_{AB}; \quad t'_A = t_B + t_{BA} \quad (8)$$

$$t_B - t_A = t_{AB}; \quad t'_A - t_B = t_{BA} \quad (9)$$

$$t_B - t_A = t'_A - t_B \quad (10)$$

and both clocks would be synchronous, which is not what Einstein claims. What Einstein claims is just the contrary: for m-observers, and according to (7), the clocks A_c and B_c are not synchronous. And Einstein’s (7) can only come from:

$$t_A < t_B < t'_A \quad (11)$$

$$d_{AB} = r_{AB} + v(t_B - t_A) \quad (12)$$

$$d_{BA} = r_{AB} - v(t'_A - t_B) \quad (13)$$

$$t_B - t_A = d_{AB}/c = (r_{AB} + v(t_B - t_A))/c \quad (14)$$

$$t'_A - t_B = d_{BA}/c = (r_{AB} - v(t'_A - t_B))/c \quad (15)$$

$$c(t_B - t_A) = r_{AB} + v(t_B - t_A) \quad (16)$$

$$c(t'_A - t_B) = r_{AB} - v(t'_A - t_B) \quad (17)$$

$$(t_B - t_A)(c - v) = r_{AB} \quad (18)$$

$$(t'_A - t_B)(c + v) = r_{AB} \quad (19)$$

$$t_B - t_A = \frac{r_{AB}}{c - v}; \quad t'_A - t_B = \frac{r_{AB}}{c + v} \quad (20)$$

In consequence, and unusual as it may be, m-observers do not refer their EC measurements to their proper frame F_m but to F_s , either as a moving frame or as a stationary frame. In fact, both alternatives, and only them, lead to Einstein’s (7) through (11)-(20). But these alternatives imply that, according to (11)-(13), light travels a distance d_{AB} (12) when going from A_c to B_c greater than the distance d_{BA} (13) it travels when going back from B_c to A_c . In these conditions, EC does not work because of EC corollary.

So, Einstein cannot claim that, according to the EC carried out by m-observers, A_c and B_c are not synchronous for m-observers because of (7), given that (7) comes from (11)-(20), which implies $d_{AB} \neq d_{BA}$ (11)-(13), and EC is not conclusive if $d_{AB} \neq d_{BA}$ (EC corollary). Alternatively, starting from (7), i.e. starting from (20), going back to (17)-(16), and taking into account that t_A , t_B and t'_A are F_s times of F_s ’ synchronous clocks (see Einstein’s footnote), $c(t_B - t_A)$ and $c(t'_A - t_B)$ are respectively the distances $r_{AB} + v(t_B - t_A)$ and $r_{AB} - v(t'_A - t_B)$ that light travels in F_s when going

from the moving A_c to the moving B_c , and then when going back from the moving B_c to moving A_c , which confirms that, according to Einstein, m-observers refer their EC measurements to F_s .

In consequence, F_s would be a sort of universal frame of reference to which not only s-observers but also m-observers refer their corresponding measurements. So, and being all measurements referred to F_s , it must be concluded that, according to Einstein's argument P1-P3, the clocks A_c and B_c are (s-observers), and are not (m-observers), synchronous in the same frame F_s . And the same would apply to any other measurement carried out relative to F_s by s-observers and by m-observers. These contradictory results are the consequences of a misinterpretation of the Principle of Relativity. Indeed, the Principle of Relativity of special relativity requires that each observer refers all his measurements to his rest frame, otherwise contradictory results, as the above one, are inevitable.

In the epigraph 3 of his paper, Einstein deduces LT, from which the relativity of simultaneity immediately follows, making it unnecessary the above erroneous argument P1-P3 on the relativity of time. According to LT, if two clocks separated by a proper distance d are synchronous in their proper inertial frame, from another inertial frame relative to which the clocks move at a uniform velocity v in the direction in which they are separated by the proper distance d , the chasing clock is observed ahead of the other by a time dv/c^2 . Maybe Einstein was not aware of this genuine relativity of simultaneity linked to LT because, apart from the wrong argument P1-P3, he only comments the loss of synchronism between two clocks as a consequence of time dilation, also linked to LT [3, p. 904].

5. CONCLUSIONS

The above discussion on Einstein's argument P1-P3 leads to the following conclusions:

- (1) P1-P3 contains several basic and significant errors.
- (2) Einstein's conclusion (7) on m-observers implies that they carry out EC in disagreement with EC definition.
- (3) Einstein cannot claim that (7) implies that A_c and B_c are not synchronous for m-observers, because (7) comes from a wrong EC ($d_{AB} \neq d_{BA}$), from which no conclusion can be drawn (EC corollary).
- (4) According to Einstein, the clocks A_c and B_c are (s-observers), and are not (m-observers), synchronous in the *same frame* F_s .
- (5) P1-P3 is in conflict with the Principle of Relativity. It is, in fact, the same rude error we frequently found in some naive critiques of special relativity. It is shocking that the same author that makes such a rude error can correctly deduce LT by his own independent reasoning.
- (6) P1-P3 suggests that in 1905, Einstein's experience in relativity (including classical relativity) could be less developed than expected.
- (7) It is reasonable to assume that Einstein did not realize the way the relativity of simultaneity is linked to LT.
- (8) Since it is impossible to properly understand a theory with undetected disagreements and contradictions, Einstein's wrong argument P1-P3 has surely contributed to make the theory of special relativity, at least initially, more difficult than it actually is.
- (9) I suppose Einstein's paper has been read by thousands of scientifically educated readers all around the world, in spite of which its errors have remained unnoticed for 114 years.
- (10) The above conclusion 9 points to a deficit of criticism and to an excess of reverence to certain authors and theories in contemporary science.

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