JVR

Journal of Vectorial Relativity
JVR **3** (2008) **1** 1-13

First Solutions to Gravitation and Orbital Precession under Vectorial Relativity

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ABSTRACT: In previous work, it was established the correct equation that governs gravitational effect of a massive body, considered it with a fixed and constant mass M, on a planet of mass $m = M_0 \Big/ \left(1 - v^2/c^2\right)^{3/2}$, moving at velocity v and momentum p, became: $\frac{d^2 r}{dt^2} - r.\omega^2 + \frac{dm}{m.dt} = -\mathscr{G}$, where the value of the gravitational field $\mathscr{G}$ exerted by the massive body on the planet's variable mass was found to be: $\mathscr{G} = \dfrac{\frac{2.G.M}{r^2}.\frac{v}{V_0} - v.\frac{dv}{dr}.\left(\frac{p_0}{p} - \frac{p}{p_0}\right)}{\frac{p_0}{p} + \frac{p}{p_0}}$, denoting by p_0 the constant value of the linear momentum for the planet attracted by the massive body, at its nearest point (r_0). One of the reasons of the success of Einstein's General Theory of Relativity (GTR) was that it allowed to calculate planet's precession (rotation of the elliptical path axis with time, i.e.: Mercury Precession). The fact that its occurrence has been experimentally observed is not accounted by classic Kepler's or Newton Laws because it is only applicable to constant masses. This work shows that planet's precession is a direct consequence of considering variable planet's mass inside accepted physical laws in our known three-dimensional space. According to us, this work positively confirms new definitions of mass and Energy obtained under Vectorial Relativity.

KEYWORDS: Universal Gravitation, Kepler Laws, Vectorial Lorentz Transformations, Orbital Precession, Mercury's Precession, STR, GTR.

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I. INTRODUCTION

This work is a continuation of previous ones on Vectorial Relativity referred to: Mass and Energy [1], Gravitation [2] and Precession [3]. In the present work it is derived an exact equation of a planet

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motion about a very massive body (a star or Sun) and a very approximate expression of angle of precession as an effect of the variation of the planet's mass, with its velocity. In Section II, in order to put clear the used procedure for achieving such derivation we repeat that referred to the classic vision appeared in [2]. Section III discusses how the consideration of variable mass inside Vectorial Relativity Theory allows predicting the effect of precession observed in planets motion. The derivation of the expression for calculating the perihelion advance of Mercury is presented. At the end of this work are presented some conclusions.

II. *CLASSIC EQUATION AND SOLUTION FOR PLANET'S MOTION.*

For constant masses it was presented in [2] how Kepler's Laws can be directly deduced from Newton's Universal Law of Gravitation [2], let's resume it here. Letting $\mathbf{r} = r.\mathbf{U_r}$, the following expressions of velocity and acceleration can be obtained:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt}.\mathbf{U_r} + r.\frac{d\theta}{dt}.\mathbf{U}_\theta \quad \text{and} \quad \mathbf{a} = \frac{d\mathbf{r}}{dt} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right].\mathbf{U_r} + \left[r.\frac{d^2\theta}{dt^2} + 2.\frac{dr}{dt}\frac{d\theta}{dt}\right].\mathbf{U}_\theta \qquad (1)$$

By applying the definition of Force and Newton's Universal Gravitation Law to two attracting bodies of constant masses M and m, we can write that:

$$\frac{d\mathbf{p}}{dt} = m.\mathbf{a} = -F.\mathbf{U_r} = -\frac{G.M.m}{r^2}.\mathbf{U_r} \qquad \Rightarrow \qquad \mathbf{a} = -\frac{G.M}{r^2}.\mathbf{U_r} \qquad (2)$$

Substituting the obtained expression (1) of acceleration in (2) we arrive at:

$$\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right].\mathbf{U_r} + \left[r.\frac{d^2\theta}{dt^2} + 2.\frac{dr}{dt}\frac{d\theta}{dt}\right].\mathbf{U}_\theta = -\frac{G.M}{r^2}\mathbf{U_r} \qquad (3)$$

This vectorial equation creates the following two scalar equations:

$$\begin{aligned} &1) \quad r.\frac{d^2\theta}{dt^2} + 2.\frac{dr}{dt}\frac{d\theta}{dt} = 0 \\ &2) \quad \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -\frac{G.M}{r^2} \end{aligned} \qquad (4)$$

By remembering that angular velocity is defined as $\omega = \frac{d\theta}{dt}$, first equation generates Angular Momentum Conservation Law:

$$r.\frac{d^2\theta}{dt^2} + 2.\frac{dr}{dt}\frac{d\theta}{dt} = 0 \Rightarrow \frac{\frac{d^2\theta}{dt^2}}{\frac{d\theta}{dt}} + 2.\frac{\frac{dr}{dt}}{r} = \frac{\frac{d\omega}{dt}}{\omega} + 2.\frac{\frac{dr}{dt}}{r} = 0 \Rightarrow \frac{d\omega}{\omega} + 2.\frac{dr}{r} = 0 \Rightarrow Log\frac{\omega}{\omega_0} = Log\frac{r_0^2}{r^2}$$

$$\omega.r^2 = \omega_0.r_0^2 = \text{Constant} \quad \Rightarrow \quad \frac{d\theta}{dt}.r^2 = \text{Constant}, \quad \text{for } \omega_0,\ r_0 \text{ measured at perihelion} \tag{5}$$

Development of the second scalar equation allows obtaining the following expression:

$$q^2 = \left(\frac{dr}{dt}\right)^2 = r_0^4.\omega_0^2.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - 2.G.M.\left(\frac{1}{r_0} - \frac{1}{r}\right) = r_0^2.v_0^2.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - 2.G.M.\left(\frac{1}{r_0} - \frac{1}{r}\right) \tag{6}$$

Doing $\frac{d\theta}{dt}.r^2 = \frac{d\theta}{dr}.\frac{dr}{dt}.r^2 = \frac{d\theta}{dr}.q.r^2 = \mathbf{K} \quad \Rightarrow \quad d\theta = \frac{K.dr}{q.r^2}$ (7)

Defining $u = \frac{1}{r} \Rightarrow du = -\frac{dr}{r^2}$, and substituting q by its equivalent expression, we obtain:

$$d\theta = \frac{-du}{\sqrt{\left(u_0^2 - u^2\right) - \frac{2.G.M}{r_0^2.v_0^2}.\left(u_0 - u\right)}} = \frac{-du}{\sqrt{\left(u_0 - h\right)^2 - \left(u - h\right)^2}} \quad \text{for} \quad h = \frac{G.M}{r_0^2.v_0^2}$$

By integrating both sides of this equation we get the final solution for angle:

$$\theta = arc\cos\left(\frac{\frac{1}{r} - h}{\frac{1}{r_0} - h}\right) \quad \Leftrightarrow \quad r = \frac{\frac{1}{h}}{1 + \left(\frac{1}{r_0.h} - 1\right).\cos\theta} \quad \Leftrightarrow \quad r = \frac{1/h}{1 + e.\cos\theta} \tag{8}$$

Equation (8) represents a conic with eccentricity $e = \frac{1}{r_0.h} - 1$, with one focus at origin. Applied to planets, for $\theta = 0 \Rightarrow r = r_0$, is the value of radius at perihelion.

However, it is necessary to introduce the following important comment:

Because equation (8) represents a perfect conic, the phenomenon of precession (rotation of the conic plane axis, respect to the center of the conic) is absent in this solution. The non-rediction of this phenomenon in classic analysis, in author's opinion, is given by the non-consideration of the relativistic dependence of mass on its speed and on the speed of light.

III. ***RELATIVISTIC* (*VECTORIAL*) *EQUATION FOR PLANET'S MOTION.***

Let's try to follow a similar procedure for obtaining a general expression of the equation for a planet's motion around a massive body, similar to that of [2].

$$\frac{d\mathbf{p}}{dt} = -F.\mathbf{U}_r \quad \Rightarrow \quad \frac{dm.\mathbf{v}}{dt} = m.\frac{d\mathbf{v}}{dt} + \mathbf{v}.\frac{dm}{dt} = -F.\mathbf{U}_r$$

$$\mathbf{r}=r.\mathbf{U}_r \quad \Rightarrow \quad \mathbf{v}=\frac{dr}{dt}.\mathbf{U}_r+r.\omega.\mathbf{U}_\theta \quad \Rightarrow \quad \frac{d\mathbf{v}}{dt}=\left[\frac{d^2r}{dt^2}-r.\left(\frac{d\theta}{dt}\right)^2\right].\mathbf{U}_r+\left[r.\frac{d^2\theta}{dt^2}+2.\frac{dr}{dt}.\frac{d\theta}{dt}\right].\mathbf{U}_\theta$$

Substituting properly,

$$\left[m.\frac{d^2r}{dt^2}-r.m.\left(\frac{d\theta}{dt}\right)^2+\frac{dm}{dt}.\frac{dr}{dt}\right].\mathbf{U}_r+\left[r.m.\frac{d^2\theta}{dt^2}+2.m.\frac{dr}{dt}.\frac{d\theta}{dt}+r.\frac{dm}{dt}.\frac{d\theta}{dt}\right].\mathbf{U}_\theta=-F.\mathbf{U}_r$$

By preserving the definition of the gravitational field as quotient between Force and mass:

$$\frac{d\mathbf{p}}{dt}=-F.\mathbf{U}_\mathbf{r} \;\Rightarrow\; \frac{d\mathbf{p}.d\mathbf{r}}{dt}=-F.d\mathbf{r}.\mathbf{U}_\mathbf{r} \;\Rightarrow\; d\mathbf{p}.\mathbf{v}=d(m.\mathbf{v}).\mathbf{v}=m.\mathbf{v}.d\mathbf{v}+\mathbf{v}.\mathbf{v}.dm$$

$$\frac{dp}{p}=-\frac{F}{p.v}.dr=-\frac{\mathcal{G}}{v^2}.dr, \text{ for, } F=\mathcal{G}.m; \text{ or, } \mathcal{G}=\frac{F}{m}; \; m.v.dv+v^2.dm=dp.v=-F.dr \tag{9}$$

$$\frac{dm}{m.dt}=\frac{dp}{p.dt}-\frac{dv}{v.dt}$$

Where $\mathcal{G}$, denotes the gravitational field. The second condition applied to a mass moving around another one originates an equation similar to that of following equation:

$$\frac{d^2r}{dt^2}-r.\left(\frac{d\theta}{dt}\right)^2+\frac{dm}{m.dt}.\frac{dr}{dt}=\frac{d^2r}{dt^2}-\omega^2.r+\left(\frac{1}{p}\frac{dp}{dt}-\frac{1}{v}\frac{dv}{dt}\right)\frac{dr}{dt}=-\frac{F}{m}=-\mathcal{G}$$

An **exact** expression obtained for this case is [2]:

$$q^2=\frac{K^2}{m^2}.\left(\frac{1}{r_0^2}-\frac{1}{r^2}\right)+\frac{K^2}{r_0^2}.\left(\frac{1}{m_0^2}-\frac{1}{m^2}\right)-\left(V_0^2-v^2\right) \tag{10}$$

The assumed expression, that ensures conservation of angular momentum becomes:

$$q^2=\frac{K^2}{m^2}.\left(\frac{1}{r_0^2}-\frac{1}{r^2}\right)-2.G.M.\left(\frac{1}{r_0}-\frac{1}{r}\right).\frac{m_0}{m} \tag{11}$$

Assumption in this case implies the equality of these two terms:

$$-2.G.M.\left(\frac{1}{r_0}-\frac{1}{r}\right).\frac{m_0}{m}=\frac{K^2}{r_0^2}.\left(\frac{1}{m_0^2}-\frac{1}{m^2}\right)-\left(V_0^2-v^2\right) \tag{12}$$

Thus, through this process the following expressions were obtained [2]:

$$\mathcal{G} = \frac{\frac{2.G.M}{r^2}.\frac{v}{V_0} - v.\frac{dv}{dr}.\left(\frac{p_0}{p} - \frac{p}{p_0}\right)}{\left(\frac{p}{p_0} + \frac{p_0}{p}\right)} \qquad \text{(Gravitational Field)} \tag{13}$$

$$m = m_0.\left[\sqrt{\left(\frac{G.M}{v^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right)^2 + \frac{V_0^2}{v^2}} - \frac{G.M}{v^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right]$$

$$m_0 = m.\left[\sqrt{\left(\frac{G.M}{V_0^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right)^2 + \frac{v^2}{V_0^2}} + \frac{G.M}{V_0^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right] \qquad \text{(Mass)} \tag{14}$$

$$p = p_0.\left[\sqrt{\left(\frac{G.M}{v.V_0}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right)^2 + 1} - \frac{G.M}{v.V_0}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right]$$

$$p_0 = p.\left[\sqrt{\left(\frac{G.M}{v.V_0}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right)^2 + 1} + \frac{G.M}{v.V_0}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right] \qquad \text{(Momentum)} \tag{15}$$

$$r = \frac{r_0}{1 - \frac{r_0.v.V_0}{2.G.M}\left(\frac{p_0}{p} - \frac{p}{p_0}\right)} \qquad \text{(Generic Radius)} \tag{16}$$

$$q = \sqrt{\frac{K^2}{m^2}.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - 2.G.M.\left(\frac{1}{r_0} - \frac{1}{r}\right).\frac{m_0}{m}} \qquad \text{(Radial velocity)} \tag{17}$$

$$v^2 = V_0^2.\frac{m_0^2}{m^2} - 2.G.M.\left(\frac{1}{r_0} - \frac{1}{r}\right).\frac{m_0}{m} \qquad \text{(Tangential velocity)} \tag{18}$$

A) CONSISTENCY OF PREVIOUS ASSUMPTION FOR PLANETS BY CHECKING OBTAINED VALUE FOR LINEAR MOMENTUM.

Additionally to those controls done in [2] let's check that obtained expression for p meets the general relationship: $\frac{dp}{p.dr} = -\frac{\mathcal{G}}{c^2}$.

The expression of linear momentum, can be put as:

$$p = p_0.\left[\sqrt{\left(\frac{G.M}{v.V_0}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right)^2+1}-\frac{G.M}{v.V_0}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right] = p_0.\left[\sqrt{a^2+1}-a\right] \quad \text{for} \quad a=\frac{G.M}{v.V_0}.\left(\frac{1}{r_0}-\frac{1}{r}\right) \qquad (19)$$

Operating on this relationship:

$$\frac{dp}{p.dr} = \frac{\dfrac{p_0.d\left(\sqrt{a^2+1}-a\right)}{dr}}{p_0.\left(\sqrt{a^2+1}-a\right)} = \frac{\dfrac{d\left(\sqrt{a^2+1}-a\right)}{dr}}{\left(\sqrt{a^2+1}-a\right)} = \frac{\dfrac{a.\frac{da}{dr}}{\sqrt{a^2+1}}-\dfrac{da}{dr}}{\left(\sqrt{a^2+1}-a\right)} = \frac{\dfrac{a}{\sqrt{a^2+1}}-1}{\left(\sqrt{a^2+1}-a\right)}.\frac{da}{dr} = -\frac{1}{\sqrt{a^2+1}}.\frac{da}{dr}$$

$$\frac{da}{dr} = \frac{d}{dr}\left[\frac{G.M}{v.V_0}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right] = \frac{G.M}{v.V_0}.\frac{1}{r^2}-\frac{G.M}{V_0.v^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right).\frac{dv}{dr}$$

Substituting, $\frac{1}{2}.\left(\frac{p_0}{p}+\frac{p}{p_0}\right) = \sqrt{\frac{G.M}{v.V_0}.\left(\frac{1}{r_0}-\frac{1}{r}\right)^2+1}$, and $\frac{1}{2}.\left(\frac{p_0}{p}-\frac{p}{p_0}\right) = \frac{G.M}{v.V_0}.\left(\frac{1}{r_0}-\frac{1}{r}\right)$

$$\frac{dp}{p.dr} = -\frac{1}{\sqrt{a^2+1}}.\frac{da}{dr} = -\frac{\dfrac{G.M}{v.V_0}.\dfrac{1}{r^2}-\dfrac{1}{2}.\left(\dfrac{p_0}{p}-\dfrac{p}{p_0}\right).\dfrac{dv}{v.dr}}{\dfrac{1}{2}.\left(\dfrac{p_0}{p}+\dfrac{p}{p_0}\right)}. = -\frac{1}{v^2}\frac{\dfrac{G.M}{r^2}.\dfrac{v}{V_0}-\dfrac{1}{2}.\left(\dfrac{p_0}{p}-\dfrac{p}{p_0}\right).v.\dfrac{dv}{dr}}{\dfrac{1}{2}.\left(\dfrac{p_0}{p}+\dfrac{p}{p_0}\right)} = -\frac{\mathcal{G}}{v^2}$$ It is obtained effectively and consistently that: $\frac{dp}{p.dr} = -\frac{\mathcal{G}}{v^2}$

B) EXACT EQUATION FOR PLANET'S MOTION AROUND A MASSIVE BODY.

On the other hand, as we will see the following conversions can be used perfectly and adequately on variable mass for obtaining suitable equations of motion [5] [6] [7]:

$$\frac{d}{dt} = \frac{d\theta}{dt}\frac{d}{d\theta} = \frac{L}{m.r^2}\frac{d}{d\theta}; \qquad u=\frac{1}{r}; \qquad \frac{d\left(\frac{1}{r}\right)}{d\theta} = \frac{du}{d\theta} = -\frac{1}{r^2}\frac{dr}{d\theta}; \qquad (20)$$

In effect, in the equation, $\frac{d^2r}{dt^2}-r.\left(\frac{d\theta}{dt}\right)^2+\frac{1}{m}.\frac{dm}{dt}.\frac{dr}{dt} = -\mathrm{G}$, by operating on the first term, $\frac{d^2r}{dt^2}$:

$$\frac{d^2r}{dt^2} = \frac{d}{dt}\left[\frac{dr}{dt}\right] = \frac{d}{dt}\left[\frac{L}{m.r^2}\frac{dr}{d\theta}\right] = -.K.\frac{L}{m.r^2}\frac{d}{d\theta}\left[\frac{1}{m}\frac{du}{d\theta}\right] = -\frac{L}{m.r^2}\left[\left(\frac{d}{d\theta}\left(\frac{1}{m}\right)\right)\frac{du}{d\theta}+\frac{1}{m}\frac{d^2u}{d\theta^2}\right]$$

$$\frac{d^2r}{dt^2}=-\frac{L^2}{m}.u^2\left[\frac{du}{d\theta}\frac{(-1)}{m^2}\frac{dm}{d\theta}+\frac{1}{m}\frac{d^2u}{d\theta^2}\right]=\frac{L^2}{m^3}.u^2.\frac{du}{d\theta}\frac{dm}{d\theta}-\frac{L^2}{m^2}.u^2.\frac{d^2u}{d\theta^2}$$

By using this conversion and others in the equation of motion for planets:

$$\frac{d^2r}{dt^2}-r.\left(\frac{d\theta}{dt}\right)^2+\frac{1}{m}.\frac{dm}{dt}.\frac{dr}{dt}=\frac{L^2}{m^3}.u^2.\frac{du}{d\theta}\frac{dm}{d\theta}-\frac{L^2}{m^2}.u^2.\frac{d^2u}{d\theta^2}-\frac{L^2}{p^2}u^3+\frac{1}{m}.\frac{dm}{dt}.\frac{dr}{dt}=-\mathcal{G}$$

Using previous conversions on the last term, $\frac{1}{m}.\frac{dm}{dt}.\frac{dr}{dt}$, we get:

$$\frac{1}{m}.\frac{dm}{dt}.\frac{dr}{dt}=\frac{1}{m}.\left(\frac{L}{m}.u^2\frac{dm}{d\theta}\right).\left(\frac{L}{m}.u^2\frac{dr}{d\theta}\right)=\frac{1}{m}.\left(\frac{L}{m}.u^2\frac{dm}{d\theta}\right).\left(\frac{L}{m}.(-)\frac{du}{d\theta}\right)=-\frac{L^2}{m^3}.u^2.\frac{dm}{d\theta}.\frac{du}{d\theta}$$

Substituting and simplifying we have the **exact** equation of motion for PLANETS:

$$\frac{d^2r}{dt^2}-r.\left(\frac{d\theta}{dt}\right)^2+\frac{1}{m}.\frac{dm}{dt}.\frac{dr}{dt}=-\frac{L^2}{m^2}.u^2.\frac{d^2u}{d\theta^2}-\frac{L^2}{m^2}u^3=-\frac{L^2}{m^2}.u^2.\left(\frac{d^2u}{d\theta^2}+u\right)=-\mathcal{G}$$

$$\Rightarrow\quad u^2.\left(\frac{d^2u}{d\theta^2}+u\right)=\frac{m^2}{L^2}\frac{\frac{2.G.M}{r^2}.\frac{v}{V_0}-v.\frac{dv}{dr}.\left(\frac{p_0}{p}-\frac{p}{p_0}\right)}{\left(\frac{p}{p_0}+\frac{p_0}{p}\right)} \qquad (21)$$

C) A VERY APPROXIMATE SOLUTION FOR PLANET'S MOTION.

In order to obtain the solution of this exact equation we successfully used all the results and definitions obtained through the novel Vectorial Theory of Relativity [4]:

a) The new definition of relativistic mass, $m=\frac{\overset{0}{m}}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$, that corrects that given by Einstein's in 1905, $m=\frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}}$, where $\overset{0}{m}=M_0$ is the rest mass. The new definition is also a direct consequence of correcting erroneous assumptions inside Lorentz Transformations [4],

b) The New definition of Relativistic Kinetic Energy [1], $\Delta K=m.(2.v^2-c^2)-m.(2.v^2-c^2)$, which is consequence of the corrected definition of mass indicated in (a).

c) The consideration of gravitation as the effect of a central force, produced by a massive body, considered fixed, on varying moving masses depending on its velocities and the speed of light. From here a new definition of gravitational field was obtained and a new definition of tangential velocity [2]: $v^2 = V_0^2 \cdot \frac{m_0^2}{m^2} - 2.G.M.\left(\frac{1}{r_0} - \frac{1}{r}\right).\frac{m_0}{m}$

In order to have this equation more suitable to handle, let's calculate the gravitational force F as a result of operating over the kinetic energy. Remember that:

$$\frac{d\mathbf{p}}{dt} = -F.\mathbf{U_r} \quad \Rightarrow \quad \frac{d\mathbf{p}.d\mathbf{r}}{dt} = dE = -F.d\mathbf{r}.\mathbf{U_r} \quad \Rightarrow \quad dE = -F.dr \quad \Rightarrow \quad \frac{dE}{dr} = -F \tag{22}$$

And that [1]: $dm = 3.m.\frac{v.dv}{\left(c^2 - v^2\right)}$. Thus,

$$\frac{dE}{dr} = \frac{dE}{dv}\frac{dv}{dr} = \left[\frac{d}{dv}\left(m.\left(2.v^2 - c^2\right) - m_0.\left(2.V_0^2 - c^2\right)\right)\right].\frac{dv}{dr}$$

$$\frac{dE}{dv} = 4.m.v + \left(2.v^2 + c^2\right)\frac{dm}{dv} = 4.m.v + \left(2.v^2 + c^2\right)\frac{3.m.v}{c^2 - v^2} = \frac{4.c^2 - 4.v^2 + 6.v^2 - 3.c^2}{c^2 - v^2}.m.v = \frac{c^2 + 2.v^2}{c^2 - v^2}.m.v$$

$$\frac{dE}{dr} = \frac{dE}{dv}\frac{dv}{dr} = \frac{c^2 + 2.v^2}{c^2 - v^2}.m.v.\frac{dv}{dr}$$

Previous one is the expression of gravitational force: Dividing by mass, we obtain gravitational field:

$$\frac{c^2 + 2.v^2}{c^2 - v^2}.v.\frac{dv}{dr} = -\mathcal{G} = -\frac{F}{m} \tag{23}$$

Working on the generic tangential velocity: $v^2 = V_0^2 \cdot \frac{m_0^2}{m^2} - 2GM.\left(\frac{1}{r_0} - \frac{1}{r}\right).\frac{m_0}{m}$, where:

$$v^2 = V_0^2 \cdot \frac{\left(1 - \frac{v^2}{c^2}\right)^3}{\left(1 - \frac{V_0^2}{c^2}\right)^3} - 2GM.\left(\frac{1}{r_0} - \frac{1}{r}\right).\frac{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}{\left(1 - \frac{V_0^2}{c^2}\right)^{\frac{3}{2}}}$$

Making the following approximations (given that $v, V_0 << c$):

$$v^2 \cong V_0^2.\left(1 - 3\frac{v^2}{c^2}\right).\left(1 + 3\frac{V_0^2}{c^2}\right) - 2GM.\left(\frac{1}{r_0} - \frac{1}{r}\right).\left(1 - \frac{3}{2}\frac{v^2}{c^2}\right).\left(1 + \frac{3}{2}\frac{V_0^2}{c^2}\right)$$

$$v^2 \cong V_0^2.\left(1 + 3\frac{V_0^2}{c^2} - 3\frac{v^2}{c^2} - 9.\frac{V_0^2.v^2}{c^4}\right) - 2GM.\left(\frac{1}{r_0} - \frac{1}{r}\right)\left(1 + \frac{3}{2}\frac{V_0^2}{c^2} - \frac{3}{2}\frac{v^2}{c^2} - \frac{9}{4}.\frac{V_0^2.v^2}{c^4}\right)$$

Discarding last terms with c^4

$$v^2 \cong V_0^2.\left(1+3\frac{V_0^2}{c^2}-3\frac{v^2}{c^2}\right)-2GM.\left(\frac{1}{r_0}-\frac{1}{r}\right).\left(1+\frac{3}{2}\frac{V_0^2}{c^2}-\frac{3}{2}\frac{v^2}{c^2}\right)$$

And grouping, we have:

$$v^2.\left(1+3\frac{V_0^2}{c^2}-3.\frac{GM}{c^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right)=\left(V_0^2\right)+3\frac{V_0^4}{c^2}-2GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)-3\frac{V_0^2}{c^2}.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)$$

$$v^2=\frac{\left(V_0^2\right)+3\frac{V_0^4}{c^2}-2GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)-3\frac{V_0^2}{c^2}.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)}{1+3\frac{V_0^2}{c^2}-3.\frac{GM}{c^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)} \qquad (24)$$

Making new approximations:

$$v^2 \cong \left[V_0^2+3\frac{V_0^4}{c^2}-2GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)-3\frac{V_0^2}{c^2}.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right]\left[1-3\frac{V_0^2}{c^2}+3.\frac{GM}{c^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right] \qquad (25)$$

Taking derivatives relative to time and simplifying:

$$2.v.\frac{dv}{dr}=\left(-\frac{2GM}{r^2}-3.\frac{V_0^2}{c^2}.\frac{GM}{r^2}\right)\left(1-3\frac{V_0^2}{c^2}+3.\frac{GM}{c^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right)+$$
$$+\left[V_0^2+3.\frac{V_0^4}{c^2}-2.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)-3.\frac{V_0^2}{c^2}.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right].3.\frac{GM}{c^2.r^2}$$

$$2.v.\frac{dv}{dr}=\frac{-2.GM-3.GM.\frac{V_0^2}{c^2}+\frac{6GM.V_0^2}{c^2}+\frac{9.GM.V_0^4}{c^4}-\frac{6.G^2M^2}{c^2.r_0}-\frac{9.V_0^2.G^2M^2}{r_0.c^4}}{r^2}-\frac{6.\frac{G^2M^2}{c^2}+9.\frac{V_0^2}{c^2}.\frac{G^2M^2}{c^2}}{r^3}+$$
$$+\frac{3.GM.\frac{V_0^2}{c^2}+9.GM.\frac{V_0^4}{c^4}-6.\frac{G^2M^2}{r_0.c^2}-9.\frac{V_0^2}{c^4}.\frac{G^2M^2}{r_0}}{r^2}-\frac{6.\frac{G^2M^2}{c^2}+9.\frac{V_0^2}{c^2}.\frac{G^2M^2}{c^2}}{r^3}$$

$$2.v.\frac{dv}{dr}=\frac{-2.GM+\frac{6GM.V_0^2}{c^2}+18.GM.\frac{V_0^4}{c^4}-12.\frac{G^2M^2}{r_0.c^2}-18.\frac{V_0^2}{c^4}.\frac{G^2M^2}{r_0}}{r^2}+\frac{\left[-12.\frac{G^2M^2}{c^2}-18.\frac{V_0^2.G^2M^2}{c^4}\right]}{r^3}$$

Dividing by 2 and simplifying,

$$v.\frac{dv}{dr}=\frac{\left(-GM+3.GM.\frac{V_0^2}{c^2}+9.GM.\frac{V_0^4}{c^4}-6.\frac{G^2M^2}{c^2.r_0}-9.\frac{V_0^2}{c^2}.\frac{G^2M^2}{c^2.r_0}\right)}{r^2}-\frac{\left(6.\frac{G^2M^2}{c^2}+9.\frac{V_0^2}{c^2}.\frac{G^2M^2}{c^2}\right)}{r^3}$$

Discarding those terms being at least eight orders of magnitude smaller than others,

$$v.\frac{dv}{dr}\approx-\frac{GM+6.\frac{G^2M^2}{c^2.r_0}}{r^2}-6.\frac{G^2M^2}{c^2r^3} \tag{26}$$

Introducing last result into $u^2.\left(\frac{d^2u}{d\theta^2}-u\right)=\frac{m^2}{L^2}\mathcal{G}$, for $\mathcal{G}=-\frac{c^2+2.v^2}{c^2-v^2}.v.\frac{dv}{dr}$

$$\mathcal{G}=-\frac{c^2+2.v^2}{c^2-v^2}.\left(-\frac{GM+6.\frac{G^2M^2}{c^2.r_0}}{r^2}-6.\frac{G^2M^2}{c^2r^3}\right)=\frac{c^2+2.v^2}{c^2-v^2}.\left(\frac{GM+6.\frac{G^2M^2}{c^2.r_0}}{r^2}+6.\frac{G^2M^2}{c^2.r^3}\right)$$

For $f(r)=\left(\frac{GM+6.\frac{G^2M^2}{c^2.r_0}}{r^2}+6.\frac{G^2M^2}{c^2r^3}\right)$

Introducing, $m=\frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$, for M_0, the mass at rest, $\Rightarrow m^2=\frac{M_0^2}{\left(1-\frac{v^2}{c^2}\right)^3}$ and simplifying:

$$u^2.\left(\frac{d^2u}{d\theta^2}+u\right)\cong\frac{m^2}{L^2}.\frac{(c^2+2.v^2)}{(c^2-v^2)}.f(r)=\frac{1}{L^2}.\frac{M_0^2}{\left(1-\frac{v^2}{c^2}\right)^3}\frac{\left(1+2.\frac{v^2}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\right)}.f(r)=\frac{M_0^2}{L^2}.\frac{\left(1+2.\frac{v^2}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\right)^4}.f(r)$$

$$u^2.\left(\frac{d^2u}{d\theta^2}+u\right)\cong\frac{M_0^2}{L^2}.\left(1+2.\frac{v^2}{c^2}\right).\left(1+4\frac{v^2}{c^2}\right).f(r)\cong\left(1+6.\frac{v^2}{c^2}+8.\frac{v^4}{c^4}\right).\frac{M_0^2}{L^2}.f(r)\approx\frac{M_0^2}{L^2}.f(r)$$

Substituting

$$u^2.\left(\frac{d^2u}{d\theta^2}+u\right)\cong\frac{M_0^2}{L^2}.\left(\frac{GM+6.\frac{G^2M^2}{c^2.r_0}}{r^2}+6.\frac{G^2M^2}{c^2r^3}\right)=\frac{M_0^2}{L^2}.\left(\left(GM+6.\frac{G^2M^2}{c^2.r_0}\right).u^2+6.\frac{G^2M^2}{c^2}.u^3\right)\ \Rightarrow$$

$$\frac{d^2u}{d\theta^2}+u\cong\frac{M_0^2}{L^2}.\left(GM+6.\frac{G^2M^2}{c^2.r_0}+6.\frac{G^2M^2}{c^2}.u\right)$$

$$\Rightarrow\quad\frac{d^2u}{d\theta^2}+\left(1-6.\frac{M_0^2.G^2M^2}{L^2c^2}\right).u=\frac{M_0^2}{L^2}.\left(GM+6.\frac{G^2M^2}{c^2.r_0}\right)$$

For $\alpha=\frac{M_0^2}{L^2}.\left(GM+6.\frac{G^2M^2}{c^2.r_0}\right)$ and $\beta=\frac{M_0^2.G^2M^2}{L^2c^2}\quad\Rightarrow\quad\frac{d^2u}{d\theta^2}+(1-6.\beta).u=\alpha$ (27)

This second order differential equation has a known solution given by:

$u=A+B.\cos(\Delta\theta)$

Where, by taking twice the derivatives of u :

$$u=A+B.\cos(\Delta\theta)\qquad\frac{du}{d\theta}=-B.\Delta.\sin(\Delta\theta)\qquad\frac{d^2u}{d\theta^2}=-B.\Delta^2.\cos(\Delta\theta)$$

And substituting in the second order equation, we have:

$$\frac{d^2u}{d\theta^2}+(1-6.\beta).u=\alpha\quad\Rightarrow\quad-B.\Delta^2.\cos(\Delta\theta)+(1-6.\beta).[A+B.\cos(\Delta\theta)]=\alpha$$

$$\left[-\Delta^2+(1-6.\beta)\right]B\cos(\Delta\theta)+(1-6.\beta).A=\alpha$$

Given that the trigonometric function $\cos(\Delta\theta)$ is an independent orthogonal function, we can obtain the values of constants A, B and Δ, by equaling coefficients:

$$-\Delta^2+(1-6.\beta)=0\quad\Rightarrow\quad\Delta=\sqrt{1-6.\beta}\tag{28}$$

$$(1-6.\beta).A=\alpha\quad\Rightarrow\quad A=\frac{\alpha}{(1-6.\beta)}\tag{29}$$

Initial conditions imply:

$$\theta=0,\quad u=\frac{\alpha}{1-6.\beta}+B.\cos[(1-6.\beta).0]=u_0\quad\Rightarrow\quad B=u_0-\frac{\alpha}{1-6.\beta}\tag{30}$$

The complete equation of planet trajectory becomes:

$$u=\frac{\alpha}{1-6.\beta}+\left(u_0-\frac{\alpha}{1-6.\beta}\right).\cos\left(\sqrt{1-6.\beta}.\theta\right)\tag{31}$$

For $\alpha = \frac{M_0^2}{L^2}.\left(GM + 6.\frac{G^2 M^2}{c^2.r_0} \right)$ and $\beta = \left(\frac{M_0.GM}{L.c} \right)^2 \Rightarrow \frac{d^2u}{d\theta^2} + (1-6.\beta).u = \alpha$ (32)

Or defining $h' = \frac{\alpha}{1-6.\beta}$ we can obtain a suitable expression for angle: $\theta = \frac{1}{\Delta}.\arccos\left(\frac{\frac{1}{r} - h'}{\frac{1}{r_0} - h'} \right)$, (33)

or for radius: $r = \frac{1/h'}{1 + \left(\frac{1}{r_0.h'} - 1 \right).\cos(\Delta\theta)}$ (34)

D) PLANET'S PRECESSION.

For one cycle of the function u the angle is 2π, then angle θ_C, greater than 2π, swept by radius meets $\Delta\theta_C = 2\pi$. So, the positive precession Ξ per revolution is:

$$\Xi = \theta_C - 2\pi = \frac{2\pi}{\Delta} - 2\pi = 2\pi\left(\frac{1}{\sqrt{1-6.\beta}} - 1 \right) \cong 2\pi(1 + 3.\beta - 1) = 6.\pi.\beta = 6.\pi.\left(\frac{M_0.GM}{L.c} \right)^2 \quad (35)$$

Observe that this approximate value of orbital precession is very close to those given by observations and General Theory of Relativity. Precession for planet Mercury is the value:

$$\Xi \cong 6.\pi.\left(\frac{M_0.GM}{L.c} \right)^2 = 42.9195$$

IV CONCLUSION

According to this work, obtained result for Mercury's precession, confirm the validity of: our criticisms to Lorentz Transformations [4], our new definitions of Relativistic Mass [4], Relativistic Energy [1], Gravitational Field [2] and consideration of Gravitational Force as a Central Force. In this sense it is important to observe its possible application to Quantum Mechanics [8] [9]. Along all this work started in 1996, it has been shown that it is possible to have only one and consistent theory for explaining the physics of our universe, inside the known three dimensions (by now) with the same fundamental concepts of physics plus considering variation of mass with velocity and speed of light as an universal constant (Vectorial Relativity). This work also leads to establish as a first conclusive fact that Special Theory of Relativity (SRT) is not correct.

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