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Theoretical Result of Deflection of Light Under General Relativity Could Be Wrong

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ABSTRACT: In previous works, it was established the correct equation that governs gravitational effect of a massive body, considered it with a fixed and constant mass M, on a planet of mass $m = M_0 \Big/ \left(1 - v^2/c^2\right)^{3/2}$, moving at velocity v and momentum p, became: $\dfrac{d^2r}{dt^2} - r.\omega^2 + \dfrac{dm}{m.dt} = -\mathcal{G}$, where the value of the gravitational field $\mathcal{G}$ exerted by the massive body on the planet's variable mass was found to be: $\mathcal{G} = \dfrac{\dfrac{2.G.M}{r^2}.\dfrac{v}{V_0} - v.\dfrac{dv}{dr}.\left(\dfrac{p_0}{p} - \dfrac{p}{p_0}\right)}{\dfrac{p_0}{p} + \dfrac{p}{p_0}}$, denoting by p_0 the constant value of the linear momentum for the planet attracted by the massive body, at its nearest point (r_0). Through these equations and that of Energy, $\Delta K = K - K_0 = m.(2.v^2 - c^2) - m_0.(2.V_0^2 - c^2)$, was obtained also in previous work a very approximate value of Mercury's precession. In present work we address a photon's motion around a massive body by doing the same treatment done for any mass. The clue to do this is due to realizing that photon has a real mass (depending on its linear momentum p, kinetic energy E or/and frequency ν) given $m = \dfrac{p}{c} = \dfrac{E}{c^2} = \dfrac{h.\nu}{c^2}$, and thus, it is susceptible to be attracted by another mass. According to this work, this is a consistent, general and natural way to attack the problem of photon's motion.

KEYWORDS: Universal Gravitation, Kepler Laws, Vectorial Lorentz Transformations, Orbital Precession, Bending of Light, STR, GTR.

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I. INTRODUCTION

This work is a continuation of previous one on vectorial relativity referred to: Gravitation and Precession of planets [1]. In present work it is derived an exact equation of a photon motion around a very massive body (a star or Sun) and a very approximate expression of angle of deflection as an effect of the variation of the photon's mass in its curved motion. In Section II, in order to put clear the used procedure for achieving such derivation we repeat that appeared in [2]. In following sub-sections of this part II is discussed how the consideration of variable mass of photon or a planet inside Vectorial Relativity Theory it allows calculating the effect of light deflection and precession observed in planets motion. Also, the consistency of our calculations and those given by the General Theory of Relativity is analyzed. At the end of this work are presented some conclusions.

II. *RELATIVISTIC (VECTORIAL) ANALYSIS FOR PHOTON'S MOTION.*

In this part it is shown that analysis of the mass of a photon, treated as a variable mass (or relativistic), attracted by a massive body with a gravitational force given by the Newton's Universal Law of Gravitation, leads to an erroneous and fallacious result. This could indicate that the treatment of gravitation when variable masses are taken into account either is not so simple as it was for constant masses, or indicates that something is overlooked in Newton's Universal Law of Gravitation. Let's see.

It is known that although photon does not have rest mass when it is traveling at speed c, it has a non-zero mass given by the correct relationship, $m = \frac{E}{c^2} = \frac{p}{c}$ [2]. Given this feature of photons, they can be attracted by the gravitational field of a massive body and take a curvilinear path as any other body does it. Let's recall that when a photon changes its mass only changes its frequency, so, its velocity magnitude, c, remains as it is: an universal constant.

By Letting a photon be attracted by a massive body, approaching onto it at a minimum distance denoted by a radio-vector R_0, measured from center of mass of the massive body to a point P_0, located at this minimum distance, such that radio-vector R_0, forms an angle of 90 degrees with the photon's vector velocity $\mathbf{c}$, at P_0. With this in mind, let's try to find the deflection of the photon, produced onto its variable mass by the gravitational field of the massive body.

By considering an unknown gravitational force F as central, to be determined [2], and putting photon mass m as the quotient between the linear Momentum p, and the speed of light c, $m = \frac{p}{c}$, we can write in general that:

$$\frac{d\mathbf{p}}{dt} = -F.\mathbf{U}_r \tag{1}$$

where $\mathbf{U}_r$ is the unit vector on the direction of radius. Minus sign indicates the contrary sense of the centrifugal force, $\frac{d\mathbf{p}}{dt}$. Thus, by making: $\mathbf{p} = \left(\frac{p}{c}\right).\mathbf{c}$:

$$\frac{d\mathbf{p}}{dt} = \frac{d\left(\frac{p}{c}.\mathbf{c}\right)}{dt} = \left(\frac{p}{c}.\frac{d(\mathbf{c})}{dt} + \frac{\mathbf{c}}{c}.\frac{dp}{dt}\right) = -F.\mathbf{U_r} \tag{2}$$

Expressing in polar form for plane curvilinear motion the vector velocity of light , $\mathbf{c}$, and its acceleration vector, $\frac{d\mathbf{c}}{dt}$, as function of the unit vectors $\mathbf{U}_r$ ($\mathbf{U}_r = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$) and $\mathbf{U}_\theta$ ($\mathbf{U}_\theta = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$). The angle θ, swept by radio-vector $\mathbf{r}$ of the photon's mass in its movement with origin in the center of mass of the massive body, beginning at P_0 for $\theta = t = 0$, until a generic point of the trajectory, P. Operating on $\mathbf{r} = r.\mathbf{U}_r$, we obtain:

$$\frac{d\mathbf{r}}{dt} = \mathbf{c} = \frac{dr}{dt}.\mathbf{U}_r + r.\frac{d\theta}{dt}.\mathbf{U}_\theta = \frac{dr}{dt}.\mathbf{U}_r + r.\omega.\mathbf{U}_\theta$$

$$\mathbf{c} = \frac{dr}{dt}.\mathbf{U}_r + r.\omega.\mathbf{U}_\theta \tag{3}$$

$$\frac{d\mathbf{c}}{dt} = \left[\frac{d^2r}{dt^2} - r.\left(\frac{d\theta}{dt}\right)^2\right].\mathbf{U}_r + \left[r.\frac{d^2\theta}{dt^2} + 2.\frac{dr}{dt}.\frac{d\theta}{dt}\right].\mathbf{U}_\theta \tag{4}$$

Substituting these results in (2), dividing by $\frac{p}{c}$, and simplifying, it follows that:

$$\left[\frac{d^2r}{dt^2} - r.\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{p}.\frac{dp}{dt}.\frac{dr}{dt}\right].\mathbf{U}_r + \left[r.\frac{d^2\theta}{dt^2} + 2.\frac{dr}{dt}.\frac{d\theta}{dt} + \frac{r}{p}.\frac{dp}{dt}.\frac{d\theta}{dt}\right].\mathbf{U}_\theta = -\mathcal{G}.\mathbf{U}_r \tag{5}$$

Given that gravitation is associated to central force this vector equation originates the following two conditions:

- **a)** What is multiplied by $\mathbf{U}_\theta$, must be zero because gravitational force only have component on the radial direction, $\mathbf{U}_r$, and
- **b)** What is multiplied by $\mathbf{U}_r$ equals $\mathcal{G} = \frac{F.c}{p}$.

By preserving the definition of the unknown gravitational field, of being the quotient between the gravitational force and mass, the following relationships are valid:

$$\frac{d\mathbf{p}}{dt} = -F.\mathbf{U_r} \quad \Rightarrow \quad \frac{d\mathbf{p}.d\mathbf{r}}{dt} = -F.d\mathbf{r}.\mathbf{U_r} \quad \Rightarrow \quad dp.c = -F.dr$$
$$\frac{dp}{p} = -\frac{F}{p.c}.dr = -\frac{\mathcal{G}}{c^2}.dr \quad \text{for, } F = \frac{\mathcal{G}.p}{c}; \quad \text{or, } \mathcal{G} = \frac{F.c}{p} \tag{6}$$

Where $\mathcal{G}$, denotes the unknown gravitational field. From here, we can have the following two presentations of the equation of motion:

1) $$\frac{d^2r}{dt^2} - r.\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{p}.\frac{dp}{dt}.\frac{dr}{dt} = -\mathcal{G} \tag{7}$$

$$\frac{d^2r}{dt^2} - r.\omega^2 - \frac{F}{p.c}.\frac{dr}{dt}.\frac{dr}{dt} = \frac{d^2r}{dt^2} - r.\omega^2 - \frac{\mathcal{G}}{c^2}.\left(c^2 - \omega^2.r^2\right) = -\mathcal{G}$$

2) $$\frac{d^2r}{dt^2} - r.\omega^2 + \frac{\mathcal{G}}{c^2}\omega^2.r^2 = 0 \tag{8}$$

From condition **a)** in equation (5) it is obtained the conservation of a constant angular momentum: $\frac{p}{c}.r^2.\omega = \frac{p_0}{c}.r_0^2.\omega_0 = L$. This also can be put as: $p.r^2.\omega = p_0.r_0^2.\omega_0 = L.c = K$, where K is another constant quantity. On the other side, we had obtained in [2] the following **exact** expression for photon's motion:

$$\left(\frac{dr}{dt}\right)^2 = q^2 = \omega^2.r^4.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) + p_0^2.c^2.\left(\frac{1}{p_0^2} - \frac{1}{p^2}\right) \tag{9}$$

In this same reference we also had assumed a general expression for the squared radial speed q^2, that contains an additional factor of linear momentums quotient multiplying its second term [2]:

$$q^2 = \frac{\omega_0^2.r_0^4.p_0^2}{p^2}.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - 2.G.M.\frac{p_0}{p}.\left(\frac{1}{r_0} - \frac{1}{r}\right) \tag{10}$$

Where, G is the known gravitational constant, and M the massive body's mass. The previous expression meets conservation of angular momentum law and consistently q equals speed of light at infinite radius. Also, reduces to the classical Newtonian expression for circular motion. *It takes implicit* ***assuming*** *the equality given next*:

$$p_0^2.c^2.\left(\frac{1}{p_0^2} - \frac{1}{p^2}\right) = -2.G.M.\frac{p_0}{p}.\left(\frac{1}{r_0} - \frac{1}{r}\right). \tag{11}$$

This relationship allows deriving the following mechanical relationships [2]:

$$\mathcal{G}=\frac{\dfrac{2.G.M}{r^2}}{\left(\dfrac{p}{p_0}+\dfrac{p_0}{p}\right)}=\frac{\dfrac{G.M}{r^2}}{\sqrt{\left(\dfrac{G.M}{c^2}.\left(\dfrac{1}{r_0}-\dfrac{1}{r}\right)\right)^2+1}} \tag{12}$$

$$p=p_0.\left[\sqrt{\left(\frac{G.M}{c^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right)^2+1}-\frac{G.M}{c^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right] \tag{13}$$

$$p_0=p.\left[\sqrt{\left(\frac{G.M}{c^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right)^2+1}+\frac{G.M}{c^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right] \tag{14}$$

$$r=\frac{r_0}{1-\dfrac{r_0.c^2}{2.G.M}\left(\dfrac{p_0}{p}-\dfrac{p}{p_0}\right)} \tag{15}$$

A) CONSISTENCY OF GRAVITATIONAL FIELD EXPRESSION FOR PHOTON.

In fact, additionally to controls done in [2] for angular momentum conservation, value of $q=c$ at infinite and classical value in circular motion, let's verify that expression for p in (14), meets the general relationship: $\dfrac{dp}{p.dr}=-\dfrac{\mathcal{G}}{c^2}$.

Let $a=\sqrt{\left(\dfrac{G.M}{c^2}.\left(\dfrac{1}{r_0}-\dfrac{1}{r}\right)\right)^2+1}$. Thus from (14), $p=p_0.\left(\sqrt{a^2+1}-a\right)$. Taking derivatives

$$\frac{dp}{p.dr}=\frac{\dfrac{p_0.d\left(\sqrt{a^2+1}-a\right)}{dr}}{p_0.\left(\sqrt{a^2+1}-a\right)}=\frac{\dfrac{d\left(\sqrt{a^2+1}-a\right)}{dr}}{\left(\sqrt{a^2+1}-a\right)}=\frac{\dfrac{a.\frac{da}{dr}}{\sqrt{a^2+1}}-\dfrac{da}{dr}}{\left(\sqrt{a^2+1}-a\right)}=\frac{\dfrac{a}{\sqrt{a^2+1}}-1}{\left(\sqrt{a^2+1}-a\right)}.\frac{da}{dr}=-\frac{1}{\sqrt{a^2+1}}.\frac{da}{dr}$$

$$\frac{da}{dr}=\frac{d}{dr}\left[\frac{G.M}{c^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right]=\frac{G.M}{c^2}.\frac{1}{r^2}$$

Substituting $\dfrac{1}{2}.\left(\dfrac{p_0}{p}+\dfrac{p}{p_0}\right)=\sqrt{\left(\dfrac{G.M}{c^2}.\left(\dfrac{1}{r_0}-\dfrac{1}{r}\right)\right)^2+1}$, and $\dfrac{da}{dr}=\dfrac{G.M}{c^2}.\dfrac{1}{r^2}$ into previous expression:

$$\frac{dp}{p.dr}=-\frac{1}{\sqrt{a^2+1}}.\frac{da}{dr}=-\frac{\frac{G.M}{c^2.r^2}}{\sqrt{\left(\frac{G.M}{c^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right)^2+1}}.=-\frac{\frac{\frac{G.M}{r^2}}{\frac{1}{2}.\left(\frac{p_0}{p}+\frac{p}{p_0}\right)}}{c^2}=$$

It is obtained effectively and consistently that: $\frac{dp}{p.dr}=-\frac{\mathcal{G}}{c^2}$ (16)

Another shorter way to demonstrate this, comes from the fact that for photon the expression of energy (also) in Vectorial Relativity is $E=m.c^2=\frac{p}{c}.c^2=p.c$. In fact, given that in general, $F=m.\mathcal{G}=\frac{p}{c}.\mathcal{G}$:

$$\frac{d\mathbf{p}.}{dt}.d\mathbf{r}=dE=-F.dr\Rightarrow\frac{dE}{dr}=c\frac{dp}{dr}=-F=-\frac{p}{c}.\mathcal{G}\Rightarrow c\frac{dp}{dr}=-\frac{p}{c}.\mathcal{G}\Rightarrow\frac{dp}{p.dr}=-\frac{\mathcal{G}}{c^2}$$

B) OTHER POSSIBLE CRITERIA FOR DETERMINING THE CORRECT FIELD EXPRESSION, FOR PHOTON AND ALSO FOR PLANETS.

The assumed expression for the second term in equation (9) passed some controls and criteria, but for the sake of generality let's start again, but with a generic family of possible solutions for photon's motion given by the expression:

$$q^2=\omega^2.r^4.\left(\frac{1}{r_0^2}-\frac{1}{r^2}\right)-2.GM.\frac{p_0^s}{p^s}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}}-\frac{1}{r^{2n+1}}\right)\quad \text{for } s,\ n=\text{generic exponents} \tag{17}$$

Where, its peer for masses is: $q^2=\omega^2.r^4.\left(\frac{1}{r_0^2}-\frac{1}{r^2}\right)-2.GM.\frac{m_0^s}{m^s}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}}-\frac{1}{r^{2n+1}}\right)$ (18)

Equation (18) is only given to compare later with that used in GR for calculation of precession of Mercury.

1) Let's check for Angular Momentum $L=\omega.r^2.\frac{p}{c}$ and $K=c.L$. Given that $c^2=q^2+\omega^2.r^2$:

$$c^2 - \omega^2.r^2 = \omega^2.r^4.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - 2.GM.\frac{p_0^s}{p^s}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r^{2n+1}}\right)$$

$$c^2 = \frac{\omega^2.r^4}{r_0^2}\frac{p^2}{p^2} - 2.GM.\frac{p_0^s}{p^s}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r^{2n+1}}\right) \Rightarrow c^2 = \frac{K^2}{r_0^2.p^2} - 2.GM.\frac{p_0^s}{p^s}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r^{2n+1}}\right)$$

$$K^2 = c^2.r_0^2.p^2.\frac{p_0^2}{p_0^2} + 2.GM.\frac{p_0^s}{p^{s-2}}.r_0^{2n+2}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r^{2n+1}}\right) \Rightarrow K^2 = \frac{2.GM.\frac{p_0^s}{p^{s-2}}.r_0^{2n+2}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r^{2n+1}}\right)}{1 - \frac{p^2}{p_0^2}}$$

On the other side, in elliptic motion, at aphelion variation of radius is zero. Thus, from (17)

$$q_a^2 = 0 = \omega_a^2.r_a^2.\left(\frac{1}{r_0^2} - \frac{1}{r_a^2}\right).\frac{p_a^2}{p_a^2} - 2.GM.\frac{p_0^s}{p_a^s}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r_a^{2n+1}}\right) \quad K_a^2 = \frac{2.GM.\frac{p_0^s}{p_a^{s-2}}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r_a^{2n+1}}\right)}{\left(\frac{1}{r_0^2} - \frac{1}{r_a^2}\right)}$$

Equaling K 's at aphelion:

$$K_a = \frac{2.GM.\frac{p_0^s}{p_a^{s-2}}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r_a^{2n+1}}\right)}{\left(\frac{1}{r_0^2} - \frac{1}{r_a^2}\right)} = \frac{2.GM.\frac{p_0^s}{p_a^{s-2}}.r_0^{2n+2}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r_a^{2n+1}}\right)}{1 - \frac{p_a^2}{p_0^2}}$$

$$\Rightarrow \quad 1 - \frac{p_a^2}{p_0^2} = 1 - \frac{r_0^2}{r_a^2} \quad \Rightarrow \quad p_a^2.r_a^2 = p_0^2.r_0^2 = L^2 = \frac{K^2}{c^2}$$

This means that any member of this family meets conservation of Angular Momentum.

2) Now, let's check that for infinite radius, $q^2 = c^2$:

$$q^2 = \omega^2.r^4.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - 2.GM.\frac{p_0^s}{p^s}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r^{2n+1}}\right) = \omega^2.r^2.\left(\frac{r^2}{r_0^2} - 1\right) - 2.GM.\frac{p_0^s}{p^s}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r^{2n+1}}\right)$$

$$q^2 = \left(c^2 - q^2\right)\left(\frac{r^2}{r_0^2} - 1\right) - 2.GM.\frac{p_0^s}{p^s}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r^{2n+1}}\right) \Rightarrow q^2 = \frac{c^2.\left(\frac{r^2}{r_0^2} - 1\right) + 2.GM.\frac{p_0^s}{p^s}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}} - \frac{1}{r^{2n+1}}\right)}{1 + \frac{r^2}{r_0^2} - 1}$$

$$\lim_{r\to\infty} q^2 = \lim_{r\to\infty} c^2.\left(\frac{r^2}{r_0^2} - 1\right).\frac{r_0^2}{r^2} + 2.GM.\frac{p_0^s}{p^s}.\frac{r_0^2}{r^2}.\left(\frac{1}{r_0} - \frac{r_0^{2n}}{r^{2n+1}}\right) = \lim_{r\to\infty} c^2.\left(1 - \frac{r_0^2}{r^2}\right) + 2.GM.\frac{p_0^s}{p^s}.\frac{r_0^2}{r^2}.\left(\frac{1}{r_0} - \frac{r_0^{2n}}{r^{2n+1}}\right) = c^2$$

As we see, any member of this family of possible values of this second term meets the control at infinite of $q^2 = c^2$ for finite values of $n \geq -1$ and $s \geq 1$. So, it is necessary to build some other criteria for discovering which one of this family could be the correct chose.

3) Let's derive a generic expression of the gravitational field relative to photons for this family:

$$-2.GM.\frac{p_0^s}{p^s}.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}}-\frac{1}{r^{2n+1}}\right)=p_0^2.c^2.\left(\frac{1}{p_0^2}-\frac{1}{p^2}\right)$$

$$-2.GM.r_0^{2n}.\left(\frac{1}{r_0^{2n+1}}-\frac{1}{r^{2n+1}}\right)=\frac{c^2}{p_0^{s-2}}.\left(\frac{p^s}{p_0^2}-p^{s-2}\right)$$

Taking derivatives relative to radius:

$$-2.GM.r_0^{2n}.\left(\frac{2n+1}{r^{2n+2}}\right)=\frac{c^2}{p_0^{s-2}}.\left(\frac{s.p^{s-1}}{p_0^2}-(s-2)p^{s-3}\right).\frac{dp}{dr}=\frac{c^2}{p_0^{s-2}}.\left(\frac{s.p^s}{p_0^2}-(s-2)p^{s-2}\right).\frac{dp}{p.dr}$$

$$\frac{dp}{p.dr}=\frac{-2.GM.r_0^{2n}.\left(\frac{2n+1}{r^{2n+2}}\right)}{\frac{c^2}{p_0^{s-2}}.\left(\frac{s.p^s}{p_0^2}-(s-2)p^{m-2}\right)}=\frac{\dfrac{-2.GM.r_0^{2n}.\left(\frac{2n+1}{r^{2n+2}}\right).p_0^{m-2}}{\left(\frac{s.p^s}{p_0^2}-(s-2)p^{s-2}\right)}}{c^2}=-\frac{\mathcal{G}}{c^2}\Rightarrow \mathcal{G}=\frac{2.GM.r_0^{2n}.\left(\frac{2n+1}{r^{2n+2}}\right)}{\left(\frac{s.p^s}{p_0^s}-(s-2)\frac{p^{s-2}}{p_0^{s-2}}\right)} \quad (19)$$

Let's build the possible candidates to be chosen for the Gravitational Field, $\mathcal{G}$, in the next table:

$\mathcal{G}$	$n=-1$	$n=0$	$n=1$	$n=2$
$s=1$	$\dfrac{-2.\frac{GM}{r_0^2}}{\left(\frac{p}{p_0}+\frac{p_0}{p}\right)}$	$\dfrac{2.\frac{GM}{r^2}}{\left(\frac{p}{p_0}+\frac{p_0}{p}\right)}$	$\dfrac{6.\frac{GM}{r^4}.r_0^2}{\left(\frac{p}{p_0}+\frac{p_0}{p}\right)}$	$\dfrac{10.\frac{GM}{r^6}.r_0^4}{\left(\frac{p}{p_0}+\frac{p_0}{p}\right)}$
$s=2$	$-\dfrac{GM}{r_0^2}.\dfrac{p_0^2}{p^2}$	$\dfrac{GM}{r^2}.\dfrac{p_0^2}{p^2}$	$3.GM.\dfrac{r_0^2}{r^4}.\dfrac{p_0^2}{p^2}$	$5.GM.\dfrac{r_0^4}{r6}.\dfrac{p_0^2}{p^2}$
$s=3$	$\dfrac{-2.\frac{GM}{r_0^2}}{\left(\frac{3.p^3}{p_0^3}-\frac{p}{p_0}\right)}$	$\dfrac{2.\frac{GM}{r^2}}{\left(\frac{3.p^3}{p_0^3}-\frac{p}{p_0}\right)}$	$\dfrac{6.\frac{GM}{r^4}.r_0^2}{\left(\frac{3.p^3}{p_0^3}-\frac{p}{p_0}\right)}$	$\dfrac{10.\frac{GM}{r^6}.r_0^4}{\left(\frac{3.p^3}{p_0^3}-\frac{p}{p_0}\right)}$

Table 1. Expressions of the Gravitational Field for photons. Our choosing in yellow.

Following a similar procedure was the expression of the gravitational field for a similar and more general family in connection with planets, whose applications can be seen in Table 2:

$$\mathcal{G}=\frac{2.GM.r_0^{2n}.\left(\frac{2n+1}{r^{2n+2}}\right).\frac{v}{V_0}-\left((s-2).\frac{m^s}{m_0^s}\frac{v}{V_0}-(s-2)\frac{m^{s-2}}{m_0^{s-2}}.\frac{V_0}{v}\right).\frac{v.dv}{dr}}{\left(s.\frac{m^s}{m_0^s}\frac{v}{V_0}-(s-2)\frac{m^{s-2}}{m_0^{s-2}}.\frac{V_0}{v}\right)} \tag{20}$$

$\mathcal{G}$	$n=-1$	$n=0$	$n=1$	$n=2$
$s=1$	$\frac{-2.\frac{GM}{r_0^2}\frac{v}{V_0}-\left(\frac{p_0}{p}-\frac{p}{p_0}\right).\frac{v.dv}{dr}}{\left(\frac{p}{p_0}+\frac{p_0}{p}\right)}$	$\frac{2.\frac{GM}{r^2}\frac{v}{V_0}-\left(\frac{p_0}{p}-\frac{p}{p_0}\right).\frac{v.dv}{dr}}{\left(\frac{p}{p_0}+\frac{p_0}{p}\right)}$	$\frac{6.\frac{GM}{r^4}.r_0^2.\frac{v}{V_0}-\left(\frac{p_0}{p}-\frac{p}{p_0}\right).\frac{v.dv}{dr}}{\left(\frac{p}{p_0}+\frac{p_0}{p}\right)}$	$\frac{10\frac{GM}{r^6}.r_0^4.\frac{v}{V_0}-\left(\frac{p_0}{p}-\frac{p}{p_0}\right).\frac{v.dv}{dr}}{\left(\frac{p}{p_0}+\frac{p_0}{p}\right)}$
$s=2$	$\mathcal{G}=-\frac{GM}{r_0^2}\frac{m_0^2}{m^2}$	$\frac{GM}{r^2}\frac{m_0^2}{m^2}$	$3.\frac{GM}{r^4}.r_0^2.\frac{m_0^2}{m^2}$	$5.\frac{GM}{r^6}.r_0^4\frac{m_0^2}{m^2}$
$s=3$	$\frac{-2.\frac{GM}{r_0^2}\frac{v}{V_0}-\left(\frac{m^2}{m_0^2}\frac{p}{p_0}-\frac{p_0}{p}\right).\frac{v.dv}{dr}}{\left(3.\frac{m^2}{m_0^2}\frac{p}{p_0}-\frac{p_0}{p}\right)}$	$\frac{2.\frac{GM}{r^2}\frac{v}{V_0}-\left(\frac{m^2}{m_0^2}\frac{p}{p_0}-\frac{p_0}{p}\right).\frac{v.dv}{dr}}{\left(3.\frac{m^2}{m_0^2}\frac{p}{p_0}-\frac{p_0}{p}\right)}$	$\frac{\frac{6.GM}{r^4}.r_0^2.\frac{v}{V_0}-\left(\frac{m^2}{m_0^2}\frac{p}{p_0}-\frac{p_0}{p}\right).\frac{v.dv}{dr}}{\left(3.\frac{m^2}{m_0^2}\frac{p}{p_0}-\frac{p_0}{p}\right)}$	$\frac{10\frac{GM}{r^6}.r_0^4-\left(\frac{m^2}{m_0^2}\frac{p}{p_0}-\frac{p_0}{p}\right).\frac{v.dv}{dr}}{\left(3.\frac{m^2}{m_0^2}\frac{p}{p_0}-\frac{p_0}{p}\right)}$

Table 2. Expressions of the Gravitational Field for planets. Our choosing in yellow.

One first additional criterion is that $\mathcal{G}$ should behave inversely proportional to the squared distance to the center of the massive body (or approximately at least, according to the inverse-square law of fields). This control should lead to discard those values of $\mathcal{G}$ for $n=-1$, 1, and 2, and leave only $\mathcal{G}$ for $n=0$ for any value of s. Another reason to be eliminated $\mathcal{G}$ for $n=-1$, comes to consider that $\mathcal{G}$ should be positive (attractive), and as it can be seen form tables, such values are negative.

4) Another criterion to use in choosing the adequate value of the gravitational field to be assumed could be one such that in case of circular motion of photon it should reduce to that of Newton. For circular motion $r=r_0$, $p=p_0$ and we observe from the table that those expressions of $\mathcal{G}$ in which:

a) $n=-1$, for any value of s it is obtained that $\mathcal{G}$ reduces always to the **non-Newtonian** value $\mathcal{G}=-\frac{GM}{r_0^2}$.

b) $n=0$, for any value of s it is obtained that $\mathcal{G}$ reduces always to the **Newtonian** value $\mathcal{G}=\frac{GM}{r_0^2}$.

c) $n=1$, for any value of s it is obtained that $\mathcal{G}$ reduces always to a **non-Newtonian** value $\mathcal{G}=\frac{3.GM}{r_0^2}$.

d) $n=2$, for any value of s it is obtained that $\mathcal{G}$ reduces always to a **non-Newtonian** value $\mathcal{G}=5.\frac{GM}{r_0^2}$.

According to this criterion those $\mathcal{G}$ where $n=-1,1,\ 2$ should be discarded as a valid selection.

5) Another criterion is that the expression of gravitational field should be valid either for a planet of mass m_{planet}, or for a particle, such a photon, of mass $m_{photon}=\frac{p}{c}$. Say, the expression of the gravitational field for a planet, with variable velocity v should reduce to that for photon with constant velocity c.

As it can be checked from Tables 1 and 2, the whole family meets this criterion. Nevertheless, what this means is that if in any theory we find solutions where the expression for masses does not reduce to that of photon for $v=V_0=c$, we can say that there is an inconsistency of such theory.

6) Last criterion: The result of calculating the angular momentum of a planet moving in an elliptical path must either be the same as taking the origin of the movement perihelion or aphelion. In fact, deriving such expressions for $n=0$:

$$v^2-\omega^2.r^2=\omega^2.r^4.\left(\frac{1}{r_0^2}-\frac{1}{r^2}\right)-2.GM.\frac{m_0^s}{m^s}.\left(\frac{1}{r_0}-\frac{1}{r}\right)$$

$$v^2=\frac{\omega^2.r^4}{r_0^2}\frac{m^2}{m^2}-2.GM.\frac{m_0^s}{m^s}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\ \Rightarrow\ v^2=\frac{L^2}{r_0^2.m^2}-2.GM.\frac{m_0^s}{m^s}.\left(\frac{1}{r_0}-\frac{1}{r}\right)$$

$$\Rightarrow\quad L^2=v^2.r_0^2.m^2+2.GM.\frac{m_0^s}{m^{s-2}}.r_0^2.\left(\frac{1}{r_0}-\frac{1}{r}\right)$$

$$L^2=v^2.r_0^2.m^2.\frac{m_0^2}{m_0^2}\frac{V_0^2}{V_0^2}+2.GM.\frac{m_0^s}{m^{s-2}}.r_0^2.\left(\frac{1}{r_0}-\frac{1}{r}\right)\ \Rightarrow\ L^2=\frac{2.GM.\frac{m_0^s}{m^{s-2}}.r_0^2.\left(\frac{1}{r_0}-\frac{1}{r}\right)}{1-\frac{p^2}{p_0^2}} \qquad (21)$$

The value at Aphelion, coming from perihelion:

$$L_A^2 = v_A^2.r_0^2.m_A^2 + 2.GM.\frac{m_0^s}{m_A^{s-2}}.r_0^2.\left(\frac{1}{r_0} - \frac{1}{r_A}\right) = v_A^2.r_0^2.m_A^2.\frac{r_A^2}{r_A^2} + 2.GM.\frac{m_0^s}{m_A^{s-2}}.r_0^2.\left(\frac{1}{r_0} - \frac{1}{r_A}\right)$$

$$L_A^2 = L_A^2.\frac{r_0^2}{r_A^2} + 2.GM.\frac{m_0^s}{m_A^{s-2}}.r_0^2.\left(\frac{1}{r_0} - \frac{1}{r_A}\right) \Rightarrow L_A^2 = \frac{2.GM.\frac{m_0^s}{m_A^{s-2}}.r_0^2.\left(\frac{1}{r_0} - \frac{1}{r_A}\right)}{1 - \frac{r_0^2}{r_A^2}} = \frac{2.GM.\frac{m_0^s}{m_A^{s-2}}.\left(\frac{1}{r_0} - \frac{1}{r_A}\right)}{\frac{1}{r_0^2} - \frac{1}{r_A^2}}$$

$$L_A^2 = \frac{2.GM.\frac{m_0^s}{m_A^{s-2}}}{.\left(\frac{1}{r_0} + \frac{1}{r_A}\right)} \tag{22}$$

Angular momentum calculated at perihelion coming from aphelion will be:

$$L_0^2 = \frac{2.GM.\frac{m_A^s}{m_0^{s-2}}}{.\left(\frac{1}{r_A} + \frac{1}{r_0}\right)} \tag{23}$$

For being $L_0^2 = L_A^2$ it is necessary that $\frac{m_A^s}{m_0^{s-2}} = \frac{m_0^s}{m_A^{s-2}}$. This is only possible for $s = 1$, in order to have the same result in any case, namely, $m_A.m_0 = m_0.m_A$:

$$L_A^2 = \frac{2.GM.m_0.m_A}{.\left(\frac{1}{r_0} + \frac{1}{r_A}\right)} = L_0^2 = \frac{2.GM.m_A.m_0}{.\left(\frac{1}{r_A} + \frac{1}{r_0}\right)}$$

Thus, the best expression (and unique, in our opinion) for the gravitational field, for planets or for photons, meeting all previous criteria is:

$$\mathcal{G}_{planet} = \frac{2.\frac{GM}{r^2}\frac{v}{V_0} - \left(\frac{p_0}{p} - \frac{p}{p_0}\right).\frac{v.dv}{dr}}{\left(\frac{p}{p_0} + \frac{p_0}{p}\right)}, \text{ which reduces for photon to: } \mathcal{G}_{photon} = \frac{2.\frac{GM}{r^2}}{\left(\frac{p}{p_0} + \frac{p_0}{p}\right)} \tag{24}$$

C) EXACT EQUATION FOR PHOTON’S MOTION AROUND A MASSIVE BODY.

The following conversions can be used adequately for obtain a suitable presentation of the motion equation [5], [6], [7]:

$$\frac{d}{dt}=\frac{d\theta}{dt}\frac{d}{d\theta}=\frac{K}{p.r^2}\frac{d}{d\theta};\qquad u=\frac{1}{r};\qquad \frac{d\left(\frac{1}{r}\right)}{d\theta}=\frac{du}{d\theta}=-\frac{1}{r^2}\frac{dr}{d\theta}; \tag{25}$$

In fact in the equation, $\frac{d^2r}{dt^2}-r.\left(\frac{d\theta}{dt}\right)^2+\frac{1}{p}.\frac{dp}{dt}.\frac{dr}{dt}=-\mathcal{G}$, by operating on the first term:

$$\frac{d^2r}{dt^2}=\frac{d}{dt}\left[\frac{dr}{dt}\right]=\frac{d}{dt}\left[\frac{K}{p.r^2}\frac{dr}{d\theta}\right]=-.K.\frac{K}{p.r^2}\frac{d}{d\theta}\left[\frac{1}{p}\frac{du}{d\theta}\right]=-\frac{K^2}{p.r^2}\left[\left(\frac{d}{d\theta}\left(\frac{1}{p}\right)\right)\frac{du}{d\theta}+\frac{1}{p}\frac{d^2u}{d\theta^2}\right]$$

$$\frac{d^2r}{dt^2}=-\frac{K^2}{p}.u^2\left[\frac{du}{d\theta}\frac{(-1)}{p^2}\frac{dp}{d\theta}+\frac{1}{p}\frac{d^2u}{d\theta^2}\right]=\frac{K^2}{p^3}.u^2.\frac{du}{d\theta}\frac{dp}{d\theta}-\frac{K^2}{p^2}.u^2.\frac{d^2u}{d\theta^2}$$

By using these conversions and others into the equation of motion for photon:

$$\frac{d^2r}{dt^2}-r.\left(\frac{d\theta}{dt}\right)^2+\frac{1}{p}.\frac{dp}{dt}.\frac{dr}{dt}=\frac{K^2}{p^3}.u^2.\frac{du}{d\theta}\frac{dp}{d\theta}-\frac{K^2}{p^2}.u^2.\frac{d^2u}{d\theta^2}-\frac{K^2}{p^2}u^3+\frac{1}{p}.\frac{dp}{dt}.\frac{dr}{dt}=-\mathcal{G}$$

Using previous conversions on last term, $\frac{1}{p}.\frac{dp}{dt}.\frac{dr}{dt}$, we get:

$$\frac{1}{p}.\frac{dp}{dt}.\frac{dr}{dt}=\frac{1}{p}.\left(\frac{K}{p}.u^2\frac{dp}{d\theta}\right).\left(\frac{K}{p}.u^2\frac{dr}{d\theta}\right)=\frac{1}{p}.\left(\frac{K}{p}.u^2\frac{dp}{d\theta}\right).\left(\frac{K}{p}.(-)\frac{du}{d\theta}\right)=-\frac{K^2}{p^3}.u^2.\frac{dp}{d\theta}.\frac{du}{d\theta}$$

Substituting and simplifying we arrive at ***an exact equation of motion for photon*** (for $K=c.r_0.p_0=c.L$):

$$\frac{d^2r}{dt^2}-r.\left(\frac{d\theta}{dt}\right)^2+\frac{1}{p}.\frac{dp}{dt}.\frac{dr}{dt}=-\frac{K^2}{p^2}.u^2.\frac{d^2u}{d\theta^2}-\frac{K^2}{p^2}u^3=-\mathcal{G}\quad\Rightarrow\quad u^2.\left(\frac{d^2u}{d\theta^2}+u\right)=\frac{p^2}{c^2.L^2}\mathcal{G} \tag{26}$$

D) AN APPROXIMATE SOLUTION FOR PHOTON'S MOTION.

For $a=\frac{G.M}{c^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)$, $p=p_0.\left(\sqrt{a^2+1}-a\right)$, and $\frac{p}{p_0}+\frac{p_0}{p}=2.\sqrt{a^2+1}$

$$u^2.\left(\frac{d^2u}{d\theta^2}+u\right)=\frac{p^2}{c^2.L^2}\frac{\frac{2.G.M}{r^2}}{\left(\frac{p}{p_0}+\frac{p_0}{p}\right)}=\frac{p_0^2.\left(\sqrt{a^2+1}-a\right)^2}{c^2.L^2}\frac{\frac{G.M}{r^2}}{\sqrt{a^2+1}};$$

$$a = \frac{G.M}{c^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right) = 1474.14411.\left(\frac{1}{r_0} - \frac{1}{r}\right)$$

For $r_0 \approx GM/c^2$, and in its vicinities $r \approx r_0$ the value of a is $a = \frac{G.M}{c^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right) << 1$. Also, when $r_0 >> GM/c^2$ always $a = \frac{G.M}{c^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right) << 1$. Thus, for analyzing these two cases (black holes and bending of light) we can make the following approximations:

$$u^2.\left(\frac{d^2u}{d\theta^2} + u\right) = \frac{p_0^2}{c^2.L^2}\frac{G.M}{r^2}\left(\frac{1+2.a^2}{\sqrt{a^2+1}} - 2.a\right) \cong \frac{p_0^2}{c^2.L^2}\frac{G.M}{r^2}\left[\left(1+2.a^2\right)\left(1-\frac{1}{2}a^2\right) - 2.a\right]$$

$$u^2.\left(\frac{d^2u}{d\theta^2} + u\right) \cong \frac{p_0^2}{c^2.L^2}\frac{G.M}{r^2}\left(1+\frac{3}{2}a^2 - a^4 - 2.a\right) \cong \frac{p_0^2}{c^2.L^2}\frac{G.M}{r^2}\left(1-2.a\right)$$

In last expression terms containing squared a or with a greater exponent were discarded. Substituting a by its expression, and simplifying:

$$u^2.\left(\frac{d^2u}{d\theta^2} + u\right) \cong \frac{p_0^2}{c^2.L^2}\frac{G.M}{r^2}\left(1 - 2.\frac{G.M}{c^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right) = \frac{\frac{p_0^2}{c^2.L^2}.\left(G.M - 2.\frac{G^2M^2}{r_0.c^2}\right)}{r^2} + 2.\frac{\frac{p_0^2}{c^2.L^2}\frac{G^2M^2}{c^2}}{r^3}$$

For $\alpha_1 = \frac{p_0^2}{c^2.L^2}.\left(G.M - 2.\frac{G^2M^2}{r_0.c^2}\right)$ and $\beta_1 = \frac{p_0^2}{c^2.L^2}\frac{G^2M^2}{c^2} = \frac{1}{c^2.r_0^2}\frac{G^2M^2}{c^2}$

$$\Rightarrow u^2.\left(\frac{d^2u}{d\theta^2} + u\right) = \alpha_1.u^2 + 2.\beta_1.u^3 \Rightarrow \frac{d^2u}{d\theta^2} + \left(1 - 2.\beta_1\right).u = \alpha_1 \qquad (27)$$

This second order equation has the following exact solution:

$$u = A + B.\cos(\Delta\theta) \quad \text{or} \quad u = A + B.\sin(\Delta\theta) \qquad (28)$$

Where, the solution can be done by taking twice the derivatives of u. Thus:

$$u = A + B.\cos(\Delta\theta) \qquad \frac{du}{d\theta} = -B.\Delta.\sin(\Delta\theta) \qquad \frac{d^2u}{d\theta^2} = -B.\Delta^2.\cos(\Delta\theta)$$

$$u = A + B.\sin(\Delta\theta) \qquad \frac{du}{d\theta} = B.\Delta.\cos(\Delta\theta) \qquad \frac{d^2u}{d\theta^2} = -B.\Delta^2.\sin(\Delta\theta)$$

Substituting in the second order equation, we have:

$$\frac{d^2u}{d\theta^2} + \left(1 - 2.\beta_1\right).u = \alpha_1 \quad \Rightarrow \quad -B.\Delta^2.\cos(\Delta\theta) + \left(1 - 2.\beta_1\right).\left[A + B.\cos(\Delta\theta)\right] = \alpha_1$$

$$\frac{d^2u}{d\theta^2}+(1-2.\beta_1).u=\alpha_1 \quad\Rightarrow\quad -B.\Delta^2.\sin(\Delta\theta)+(1-2.\beta_1).[A+B.\sin(\Delta\theta)]=\alpha_1$$

Grouping,

$\left[-\Delta^2+(1-2.\beta_1)\right]B\cos(\Delta\theta)+(1-2.\beta_1).A=\alpha_1$, or $\left[-\Delta^2+(1-2.\beta_1)\right]B.\sin(\Delta\theta)+(1-2.\beta_1).A=\alpha_1$

Due to trigonometric functions $\cos(\Delta\theta)$ and $\sin(\Delta\theta)$ are independent orthogonal functions, we can obtain values of constants A, B and Δ, by equaling coefficients:

$$-\Delta^2+(1-2.\beta_1)=0 \quad\Rightarrow\quad \Delta=\sqrt{1-2.\beta_1} \tag{29}$$

$$(1-2.\beta_1).A=\alpha_1 \quad\Rightarrow\quad A=\frac{\alpha_1}{(1-2.\beta_1)} \tag{30}$$

Initial conditions: for $\theta=0$, $u=\dfrac{\alpha_1}{1-2.\beta_1}+B.\cos[(1-2.\beta_1).0]=u_0 \quad\Rightarrow\quad B=u_0-\dfrac{\alpha_1}{1-2.\beta_1}$ (31)

And for $\theta=\dfrac{\frac{\pi}{2}}{1-2.\beta_1}$, $u=\dfrac{\alpha_1}{1-2.\beta_1}+B.\sin\dfrac{\pi}{2}=u_0 \quad\Rightarrow\quad B=u_0-\dfrac{\alpha_1}{1-2.\beta_1}$

As it can be observed, photon's precession is implicitly contained in the factor Δ. The complete equation for photon's trajectory becomes in both cases given by:

$$u=\frac{\alpha_1}{1-2.\beta_1}+\left(u_0-\frac{\alpha_1}{1-2.\beta_1}\right).\cos\left(\sqrt{1-2.\beta_1}.\theta\right) \tag{32}$$

$$u=\frac{\alpha_1}{1-2.\beta_1}+\left(u_0-\frac{\alpha_1}{1-2.\beta_1}\right).\sin\left(\sqrt{1-2.\beta_1}.\theta\right) \tag{33}$$

$$\alpha_1=\frac{p_0^2}{c^2.L^2}.\left(G.M-2.\frac{G^2M^2}{r_0.c^2}\right) \text{ and } \beta_1=\frac{p_0^2}{c^2.L^2}\frac{G^2M^2}{c^2} \quad \frac{p_0^2}{c^2.L^2}=\frac{p_0^2}{c^2.c^2.\frac{p_0^2}{c^2}.r_0^2}=\frac{1}{c^2.r_0^2} \tag{34}$$

It is worth noticing that first equation is referred to an increasing radius, as for example when photon goes from r_0 (perihelion) to infinite, or to a radius greater than that of perihelion, for example to aphelion, describing an elliptic path. Second equation is applicable for a decreasing radius, for r coming from infinite, or coming from aphelion.

E) DEFLECTION OF STARLIGHT OR BENDING OF LIGHT

Defining $h'=\dfrac{\alpha_1}{1-2.\beta_1}$ we can obtain a suitable expression for angle: $\theta=\dfrac{1}{\Delta}.arccos\left(\dfrac{\frac{1}{r}-h'}{\frac{1}{r_0}-h'}\right)$ (35)

And another one for radius: $r = \dfrac{1/h'}{1+e.\cos(\Delta\theta)}$ for $e = \dfrac{1}{r_0.h'} - 1 \Rightarrow h' = \dfrac{u_0}{r_0.(1+e)}$ (36)

Angle θ is the angle that radius sweeps relative to the center of the massive body, when photon moves around the mass M. Deflection angle, φ, of a photon is that formed by its vector velocity at infinity in its parabolic path relative to the rectilinear trajectory as if massive body has null mass, $M = 0$. Measuring is done from r_0, $\theta = 0$, the closest point to mass M, to $r = \infty$. In this way, φ is the excess to $\dfrac{\pi}{2}$ radians and it will be given by:

$$\varphi = \theta_\infty - \frac{\pi}{2} = \frac{1}{\sqrt{1-2.\beta_1}}.\arccos\left(\frac{-h'}{\frac{1}{r_0} - h'}\right) - \frac{\pi}{2} \cong (1+\beta_1).\arccos\left(\frac{-h'}{\frac{1}{r_0} - h'}\right) - \frac{\pi}{2}$$

$$\varphi = \arcsin\left(\frac{-h'}{\frac{1}{r_0} - h'}\right) + \beta_1.\arccos\left(\frac{-h'}{\frac{1}{r_0} - h'}\right)$$

By considering symmetric paths at both sides of r_0, $\theta = 0$, the total deflection, $\phi = 2.\varphi$, of photon will be:

$$\phi = 2.\varphi = 2\arcsin\left(\frac{-h'}{\frac{1}{r_0} - h'}\right) + 2\beta_1.\arccos\left(\frac{-h'}{\frac{1}{r_0} - h'}\right) \qquad (37)$$

Calculated total deflection is: 0.874. This value is half of observed measurements. Given that this calculation takes into account relativistic considerations, in the sense of considering a variable energy and momentum of photon ($E = pc = h.\nu$, where h is the Planck constant and ν the frequency of the light photon) and that our equations meet general laws in physics, one explanation for this result, according to us, is the incandescence and density of Sun's atmosphere, greater at the near surface and smaller with increasing altitude, which produces an additional refraction that doubles in excess the deflection of the starlight. Very known is the refraction that the atmosphere produces on radio electric waves, when it changes its density, because of clouds or rains. The following section is devoted to a similar analysis to this of photon, but in agreement with observed measurements.

F) COMPARISON WITH GENERAL RELATIVITY (GR) RESULTS FOR PHOTONS.

In the General Theory of Relativity path of photon originating from infinity in the direction $\phi = 0$ and going off to infinity in the direction $\phi = \pi$ for large values of radius r is governed by the equation (4.6.4) that appears in reference [5], repeated here as:

$$\left(\frac{du}{d\phi}\right)^2 + u^2 = u_0^2.\left(1 - u_0.\frac{2.GM}{c^2}\right) + \frac{2.GM}{c^2}.u^3 \tag{38}$$

Under this equation the total deflected angle obtained is $\alpha = \frac{4.GM}{r_0.c^2}$, which gives the famous value of 1.75'', that coincides good enough with the experimental measured value.

Let's make some evaluation of this equation. By doing $\frac{du}{d\phi} = \frac{\frac{du}{dt}}{\frac{d\phi}{dt}}$ and $\frac{du}{dt} = -\frac{dr}{r^2.dt}$, and putting $\frac{d\phi}{dt} = \omega$, we arrive at the following equivalent equation:

$$\left(\frac{-\frac{dr}{r^2.dt}}{\omega}\right)^2 + u^2 = u_0^2.\left(1 - u_0.\frac{2.GM}{c^2}\right) + \frac{2.GM}{c^2}.u^3 = u_0^2 - u_0^3.\frac{2.GM}{c^2} + \frac{2.GM}{c^2}.u^3$$

$$\frac{\left(\frac{dr}{dt}\right)^2}{\omega^2.r^4} = \left(u_0^2 - u^2\right) - \frac{2.GM}{c^2}.\left(u_0^3 - u^3\right)$$

$$\left(\frac{dr}{dt}\right)^2 = q^2 = \omega^2 r^4.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - \frac{2.GM.r^4\omega^2.}{c^2}\left(\frac{1}{r_0^3} - \frac{1}{r^3}\right) = \omega^2 r^4.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - \frac{2.GM.r^4\omega^2.p^2}{\omega_0^2.r_0^2.p^2}\left(\frac{1}{r_0^3} - \frac{1}{r^3}\right)$$

$$\left(\frac{dr}{dt}\right)^2 = q^2 = \omega^2 r^4.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - \frac{2.GM.K^2}{\omega_0^2.r_0^2.p^2}\left(\frac{1}{r_0^3} - \frac{1}{r^3}\right) = \omega^2 r^4.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - \frac{2.GM.K^2 p_0^2.r_0^2}{p_0^2.\omega_0^2.r_0^4.p^2}\left(\frac{1}{r_0^3} - \frac{1}{r^3}\right)$$

We finally obtain the expression:

$$\left(\frac{dr}{dt}\right)^2 = q^2 = \omega^2 r^4.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - \frac{2.GM.r_0^2.p_0^2}{p^2}\left(\frac{1}{r_0^3} - \frac{1}{r^3}\right) \tag{39}$$

By comparing it with the generic expression in (17), it can be realized that equation (39) is a particular case of that family applicable to photons, for $n = 1$ and $s = 2$.

The corresponding Einstein's expression of the Gravitational Field for a photon, deduced from (39) through equation (19), for $n = 1$ and $s = 2$, is:

$$\mathcal{G} = \frac{2.GM.r_0^{2n}.\left(\dfrac{2n+1}{r^{2n+2}}\right)}{\left(\dfrac{s.p^s}{p_0^s} - (s-2)\dfrac{p^{s-2}}{p_0^{s-2}}\right)} \quad \Rightarrow \quad \mathcal{G}_{photon} = 3.GM.\frac{r_0^2}{r^4}.\frac{p_0^2}{p^2} \tag{40}$$

It can be realized that expression (40) does not fulfill the criterion of being inversely proportional to the squared distance between centers of photon and massive body, as it is suggested by Field Theory, but to the fourth power of such distance.

For circular motion they do not reduce to Newtonian value $\frac{GM}{r_0^2}$, but to three times this value: $\frac{3.GM}{r_0^2}$.

According to these controls, definition of Gravitational Field for photon expressed in expression (40), coming from Einstein's GR, is wrong.

Finally, previous controls on developed criteria showed that the value given by GR for bending of light, although almost coincident with the experimental measured value could be theoretically erroneous. Thus, measured value, approximately double of calculated in a correct way in this work, could be justifiable mainly due to additional effect of refraction done by sun's atmosphere onto the photon path, previously commented in the last paragraph of part E.

It is noteworthy to mention that in our calculation of Mercury's precession [1], using left expression in (24) for planets, was obtained a numerical result very close to the experimental measured value and very similar to that given by GR. We think that the criterion of using expressions with consistent structure like that of "what is true for planets should also be valid for photons" is a firm and reliable one, because the only different aspect in case of photon is that its speed is constant. Equation (24) fulfills such criterion.

G) COMPARISON WITH GENERAL RELATIVITY (GR) RESULTS FOR PLANETS.

By recalling the general-relativistic equation (4.4.6) from reference [5] and using Schwarzschild solution as a model for the solar system, it is obtained:

$$\left(\frac{du}{d\phi}\right)^2 + u^2 = E + \frac{2.GM}{h^2}.u + \frac{2.GM}{c^2}.u^3 \tag{41}$$

Where E is a constant related to the energy of the orbit, which for circular motion, $u = u_0$, $\frac{du}{d\phi} = 0$:

$$E = u_0^2 - \frac{2.GM}{h^2}.u_0 - \frac{2.GM}{c^2}.u_o^3 \tag{42}$$

Substituting and simplifying:

$$\left(\frac{du}{d\phi}\right)^2 = u_0^2 - u^2 - \frac{2.GM}{c^2}.\left(u_o^3 - u^3\right) - \frac{2.GM}{h^2}.\left(u_0 - u\right) \qquad (43)$$

By doing, as before, $\frac{du}{d\phi} = \frac{\frac{du}{dt}}{\frac{d\phi}{dt}}$ and $\frac{du}{dt} = -\frac{dr}{r^2.dt}$, and putting $\frac{d\phi}{dt} = \omega$, we arrive at the following equivalent GR equation for planets:

$$\left(\frac{du}{dt}\right)^2 = q^2 = \omega^2.r^4.\left(u_0^2 - u^2\right) - \frac{2.GM.\omega^2.r^4}{c^2}.\left(u_o^3 - u^3\right) - \frac{2.GM.\omega^2.r^4}{h^2}.\left(u_0 - u\right) \qquad (44)$$

Substituting $u = 1/r$, $h = \omega.r^2$, in previous equation and simplifying:

$$q^2 = \omega^2.r^4.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - \frac{2.GM.\omega^2.r^4.m^2}{c^2.m^2}.\left(\frac{1}{r_0^3} - \frac{1}{r^3}\right) - \frac{2.GM.\omega^2.r^4}{\omega^2.r^4}.\left(\frac{1}{r_0} - \frac{1}{r}\right)$$

$$q^2 = \omega^2.r^4.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - \frac{2.GM.r_0^2.m_0^2}{m^2}.\left(\frac{1}{r_0^3} - \frac{1}{r^3}\right) - 2.GM.\left(\frac{1}{r_0} - \frac{1}{r}\right) \qquad (45)$$

Under equation (43) or equivalently from this equation, deduced from (43), perihelion advance (or precession) of Mercury was calculated [5]. Although value given by GR with this development, 43 arc seconds per century, is in excellent agreement with observed measurements, equation (43) seems not to be as correct as we would have expected from this result. Let's evaluate this equation in a way similar to that we have followed for photon. We will start by making some controls for an "elliptic" motion of a planet by applying the sixth criterion in part (B), which says "The result of calculating the angular momentum of a planet moving in an elliptical path must either be the same as taking the origin of the movement perihelion or aphelion". Calculation for aphelion at $r = r_a$, where $q_a = 0$, coming from perihelion, gives the expression of the angular momentum at aphelion.

$$\left(\frac{dr}{dt}\right)^2 = q^2 = \omega^2 r^4.\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - \frac{2.GM.r_0^2.m_0^2}{m^2}\left(\frac{1}{r_0^3} - \frac{1}{r^3}\right) - 2.GM.\left(\frac{1}{r_0} - \frac{1}{r}\right)$$

$$\Rightarrow \omega_a^2 r_a^4.\left(\frac{1}{r_0} + \frac{1}{r_a}\right) = \frac{2.GM.r_0^2.m_0^2}{m_a^2}\left(\frac{1}{r_0^2} + \frac{1}{r_a.r_0} + \frac{1}{r_a^2}\right) + 2.GM$$

$$\omega_a^2 r_a^4.m_a^2 = \frac{2.GM.r_0^2.m_0^2.\left(\frac{1}{r_0^2} + \frac{1}{r_a.r_0} + \frac{1}{r_a^2}\right) + 2.GM}{\left(\frac{1}{r_0} + \frac{1}{r_a}\right)} \qquad (46)$$

Calculation for perihelion, $r = r_0$, coming from aphelion, gives the expression of the angular momentum at perihelion, where $q_0 = 0$.

$$\left(\frac{dr}{dt}\right)^2\Bigg|_{r=r_0} = 0 = \omega_0^2 r_0^2.\left(\frac{1}{r_a^2}-\frac{1}{r_0^2}\right) - \frac{2.GM.r_a^2.m_a^2}{m_0^2}\left(\frac{1}{r_a^3}-\frac{1}{r_0^3}\right) - 2.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)$$

$$\Rightarrow \quad \omega_0^2 r_0^2.\left(\frac{1}{r_a^2}-\frac{1}{r_0^2}\right) = \frac{2.GM.r_a^2.m_a^2}{m_0^2}\left(\frac{1}{r_a^3}-\frac{1}{r_0^3}\right) + 2.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)$$

$$\omega_0^2 r_0^4.m_0^2 = \frac{2.GM.r_a^2.m_a^2.\left(\frac{1}{r_0^2}+\frac{1}{r_a.r_0}+\frac{1}{r_a^2}\right)+2.GM}{\left(\frac{1}{r_0}+\frac{1}{r_a}\right)} \tag{47}$$

By equaling angular momentums $\omega_0^2 r_0^4.m_0^2 = \omega_a^2 r_a^4.m_a^2 \;\Leftrightarrow\; r_0^2.m_0^2.V_0^2 = r_a^2.m_a^2.V_a^2$ leads to the equality of second members, or simplifying, $r_0^2.m_0^2 = r_a^2.m_a^2$, which forces $V_0^2 = V_a^2$. Given that this is not a true result in an elliptic path of a planet, it proves that equivalent equations (43), (44) or (45) are not consistent expressions for determining the motion of a planet. However, this inconsistency was an expected one given that the expressions for any value of n and $s = 2$ in the family defined in equation (18) were previously discarded under used criteria. We repeat it here for $n = 1$, $s = 2$, to compare with expression (45), just as an illustrative parenthesis:

$$q^2 = \omega^2.r^4.\left(\frac{1}{r_0^2}-\frac{1}{r^2}\right) - 2.GM.\frac{m_0^2}{m^2}.r_0^2.\left(\frac{1}{r_0^3}-\frac{1}{r^3}\right)$$

$$q^2 = \omega^2.r^4.\left(\frac{1}{r_0^2}-\frac{1}{r^2}\right) - \frac{2.GM.r_0^2.m_0^2}{m^2}.\left(\frac{1}{r_0^3}-\frac{1}{r^3}\right) - 2.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)$$

This expression is very similar to that in (45), except only that it does not contain the last term present in that equation and in this we found this type of inconsistencies.

But, let's go back again to our analysis of equation (45), in order to calculate its corresponding gravitational field in order to compare it with the classical Newtonian expression. As before, we use an exact expression to establish similarities. Thus, such equation must be equal to that appearing in reference [2], repeated here:

$$q^2 = \omega^2.r^4.\left(\frac{1}{r_0^2}-\frac{1}{r^2}\right) + \frac{\omega_0^2.r_0^4.m_0^2}{r_0^2}.\left(\frac{1}{m_0^2}-\frac{1}{m_0^2}\right) - \left(V_0^2 - v^2\right) \tag{48}$$

This implies that following equality should hold:

$$-\frac{2.GM.r_0^2.m_0^2}{m^2}.\left(\frac{1}{r_0^3}-\frac{1}{r^3}\right)-2.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)=\frac{\omega_0^2.r_0^4.m_0^2}{r_0^2}.\left(\frac{1}{m_0^2}-\frac{1}{m^2}\right)-\left(V_0^2-v^2\right)$$

Simplifying,

$$-2.GM.r_0^2.\left(\frac{1}{r_0^3}-\frac{1}{r^3}\right)-2.GM.\frac{m^2}{m_0^2}\left(\frac{1}{r_0}-\frac{1}{r}\right)=\frac{m^2}{m_0^2}.v^2-V_0^2$$

Taking derivatives relative to radius:

$$-6.GM.r_0^2.\frac{1}{r^4}-4.GM.\frac{m}{m_0^2}\left(\frac{1}{r_0}-\frac{1}{r}\right)\frac{dm}{dr}-2.GM.\frac{m^2}{m_0^2}\frac{1}{r^2}=\frac{m^2}{m_0^2}.2.v.\frac{dv}{dr}+2.m.\frac{dm}{dr}\frac{v^2}{m_0^2}$$

Simplifying and substituting for $\frac{dm}{m}=\frac{dp}{p}+\frac{dv}{v}$:

$$-3.GM.\frac{r_0^2}{r^4}-GM.\frac{m^2}{m_0^2}\frac{1}{r^2}=\frac{m^2}{m_0^2}.v.\frac{dv}{dr}+v^2.\frac{dm}{m.dr}\frac{m^2}{m_0^2}+2.GM.\frac{m^2}{m_0^2}\left(\frac{1}{r_0}-\frac{1}{r}\right)\frac{dm}{m.dr}$$

$$-3.GM.r_0^2.\frac{dr}{r^4}-GM.\frac{m^2}{m_0^2}\frac{dr}{r^2}=\frac{m^2}{m_0^2}.v.dv+\left[v^2+2.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right].\frac{m^2}{m_0^2}.\frac{dm}{m}$$

$$-3.GM.\frac{m_0^2}{m^2}.r_0^2.\frac{dr}{r^4}-GM.\frac{dr}{r^2}=v^2.\frac{dv}{v}+\left[v^2+2.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right]\left(\frac{dp}{p}+\frac{dv}{v}\right)$$

$$-3.GM.\frac{r_0^2}{r^4}.\frac{m_0^2}{m^2}-\frac{GM}{r^2}=\left[2.v^2+2.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right]\left(\frac{dv}{v.dr}\right)+\left[v^2+2.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right]\left(\frac{dp}{p.dr}\right)$$

Substituting in the last relationship the general relationship for the gravitational field: $\frac{dp}{p.dr}=-\frac{\mathcal{G}}{v^2}$, we finally have expression of the gravitational field corresponding to equation (45):

$$\mathcal{G}=\frac{3.GM.\frac{r_0^2}{r^4}.\frac{m_0^2}{m^2}+\frac{GM}{r^2}+\left[2.v^2+2.GM.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right]\left(\frac{dv}{v.dr}\right)}{\left[1+\frac{2.GM}{v^2}.\left(\frac{1}{r_0}-\frac{1}{r}\right)\right]} \tag{49}$$

Expression (49) for circular motion of planets does not reduce to the known Newtonian expression $\frac{GM}{r_0^2}$, but to $\mathcal{G} = 4.\frac{GM}{r_0^2}$. Such expression applied to photon (which differs from a planet only in that it is a constant-velocity particle), gives a field for photon: $\mathcal{G} = \frac{3.GM.\frac{r_0^2}{r^4}.\frac{p_0^2}{p^2} + \frac{GM}{r^2}}{\left[1 + \frac{2.GM}{c^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right]}$. Say, it does not reduce to the Field for photon obtained from GR in equation (40): $\mathcal{G}_{photon} = 3.GM.\frac{r_0^2}{r^4}.\frac{p_0^2}{p^2}$. In sum, according to the criteria developed in this work, the two equations given by GR for analyzing the gravitational behavior for photons and that for planets given in (38) and (41) respectively, have enough inconsistencies as to conclude that they are not correct. Say, they don't reproduce correctly the real trajectory followed by photons or planets in their motion around a massive body. In such those cases, we are proposing in this work and in previous one [1] the optic of Vectorial Relativity, which as we have demonstrated meets all discussed criteria and is easier to handle than GR.

III. CONCLUSIONS

It was demonstrated that expressions used in GR for calculating bending of light and precession of Mercury are inconsistent from a theoretical point of view.

Nonetheless, It is important to clarify that we are only criticizing the theoretical way used in GR for calculating bending of light result, we are not saying that GR is wrong. It is noteworthy the fact that Einstein, applying also GR calculated the correct half value in 1911, but later in 1916 he changed his result to that commented and coincident with the known value of 1.75 arc-seconds "as the correct" one.

It is noteworthy to mention that in our calculation of Mercury's precession [1], using left expression in (24) for planets, was obtained a numerical result very close to the experimental measured value and very similar to that given by GR. We think that the criterion of using expressions with consistent structure like that of "what is true for planets should also be valid for photons" is a firm and reliable one, because the only different aspect in case of photon is that its speed is constant. Our equation (24) fulfills such criterion.

According to our opinion, results obtained here and in previous works confirm the validity of Vectorial Relativity in aspects as: Criticisms to Lorentz Transformations [4], the new definitions of Relativistic Mass [4], Relativistic Energy [11], Gravitational Field [2], and that the consideration of Gravitational Force as a Central Force in a relativistic analysis [2] also leads to correct results. Thus, its applications to Quantum Mechanics in order to improve its results [9] [10] should be taken into account.

Along this long period of searching started in 1996, it has been shown that it is possible to have only one and consistent relativistic theory for explaining the physics of our universe, inside our known space of three dimensions, using the same and known fundamental concepts of physics plus considering speed of light as an universal constant and considering relativistic variation of mass with velocity and speed of light as it was achieved under Vectorial Relativity.

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