the general science journal

Is Special Relativity really tested empirically?

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Abstract

All elements of special relativity, like the Lorentz transform, time dilation, space contraction, moving mass and $E = mc^2$ *are said to be verified empirically beyond any doubt. While checking a discrete model of space I was surprised to notice that these experiments cannot distinguish between the Lorentz transform and the coordinate transform in my discrete model. There were also an unclarity in what frame of reference the formula for moving mass is differentiated to give* $E = mc^2$. *In the discrete model* $E = mc^2$ *follows easily from basic concepts. My conclusion is that the special relativity theory is not tested well enough and the principle that there is no preferred frame of reference should be revised.*

Keywords: Special Relativity, Lorentz invariance, discrete model

Newtonian physics is based on the absolute time and the coordinate transform from rest frame of reference R to a moving frame of reference R' is the Galileo transform

$$x' = x - vt \tag{1}$$

$$t' = t$$

$$y' = y$$

$$z' = z$$

for R' moving with the speed v with respect to R. If a signal sent from R' has the velocity u' in the x-direction the signal has the velocity

$$u = v + u' \tag{2}$$

in the R frame of reference. The Galileo transform has the inverse transform

$$x = x' + vt' \tag{3}$$

$$t = t'$$

$$y = y'$$

$$z = z'$$

i.e., the inverse is obtained by changing the sign of the velocity. This is the principle of relativity: there is no preferred frame of reference, all frames of reference moving with a constant velocity are equivalent. We can consider R' as the moving frame and R as the rest frame, or think about R' as the rest frame and R as a moving frame of reference, moving to the opposite direction with the same speed v.

The Lorentz transform for these two frames of reference is

$$x' = \gamma(x - vt) \tag{4}$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$y' = y$$

$$z' = z$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{5}$$

The Lorentz transform has the inverse transform

$$x = \gamma(x' + vt') \tag{6}$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$y = y'$$

$$z = z'$$

Also this transform has no preferred frame of reference. The value of γ is determined from the requirement that there is no preferred frame of reference: inverting (4) gives

$$x = \frac{1}{1 - \frac{v^2}{c^2}}\gamma^{-1}(x' + vt')$$

and setting

$$x = \gamma(x' + vt')$$

gives γ the value in (5). If a signal with the velocity u' in the x-direction is sent from R' the velocity observed in R is

$$u = \frac{v + u'}{1 + (vu'/c^2)} \tag{7}$$

It was this velocity addition formula that made me look for a discrete model.

A discrete space consists of finite size space volume elements which in the x direction have the length Δx_i and a time which is divided into discrete size time elements Δt_i. Initially we can assume that all space elements have the same length Δx and all time elements have the length Δt. Movement can proceed only to the neighboring element. A finite model has a natural maximum speed that we can assign the value c

$$c = \frac{\Delta x}{\Delta t} \tag{8}$$

I do not assume that the space volume elements are in a lattice and cannot move relative to each other. The model can be best compared to liquid: space volume elements have a fixed size and nearest neighbord but they can move in the space. The space is three-dimensional, thus the time is not considered a coordinate, though we can treat it as a coordinate in a coordinate transform.

It may feel that a discrete mode is unnatural and a continuous space is more natural for the reality. The motivation for a discrete model is that all interactions with the exception of gravitation have been modelled as quantum gauge field theories. Gauge fields introduce new dimensions for the symmetries. If the space is continuous, we have to add more dimensions to the space: in each point in the space there is needed a number of compactified dimensions, like small circles where the symmetries are realized. This model does not seem very natural: we have no evidence that there are more dimensions in the reality. The only way to dispense with these dimensions is to realize them as constructions in the dimensions that we have, but constructions require volume. Thus, the space has to have finite size volume elements. Our space has a maximum speed c. If the space consists of finite size space elements and there is a maximum speed, then the time must consist of finite size time units so that moving one space unit in one time unit gives the maximum speed c. This reasoning leads to the discrete model, but in this very simple paper I will not go to the model at all.

Let us assume that the space volume elements keep a state and a test particle in such a discrete space is some kind of a construction of space elements in certain states. A test particle can move to any direction with a constant speed v. How can such a movement be realized in a discrete model? The test particle can only move in discrete steps. We could assume that the test particle keeps counters and counts how many steps it has taken to each direction in a number of time steps, but this seems unnatural. A better way it to assume that the test particle keeps a probability for a step in a given direction in a time unit. In each time step it makes a probabilistic choice whether to move to a given direction or not to move. If the space and time elements are small, this results into a fairly straight movement to a chosen direction with a given velocity v and the model does not need a more complicated state than a test mass has in a continuous space.

The velocity addition formula (7) seems very unnatural for a discrete model. This is so because in a discrete model there is a preferred frame of reference: the space elements have a rest frame. All movement is in reality happening in the rest frame and the test particle would have to implement (7) in some way. The formula (7) requires calculations that a test particle cannot be expected to do.

A natural velocity addition formula that a test particle can be expected to do is the following. In each time step the test particle has a rule to move to the nearest neighbor in a given direction or not to move, depending on the outcome of the probabilistic decision. If the test particle is sent from a moving frame of reference with the velocity u' in the x-direction it follows the same rule: it makes the same number of decisions but if the frame of reference already has a move to a given direction in a given time unit, the test particle omits this time unit. Thus, it makes a decision only for those time units when the frame of reference has no move in the given direction. I give the rule here only for a movement in the x-direction. If the frame of reference moves with the speed v in the x-direction, the proportion of time units when the frame does not move is

$$P_1 = 1 - \frac{v}{c}$$

For these time units the test particle makes the decision to move to the x-direction or not to move. The proportion of time units when it will not move is

$$P_2 = 1 - \frac{u'}{c}$$

When observed from the rest frame R the test particle does not move with the probability P_1P_2 in a given time unit and therefore it moves with the velocity

$$u = c(1 - P_1P_2) = v + u' - \frac{vu'}{c} \tag{9}$$

in the frame of reference R. This natural velocity addition formula gives the maximum velocity c to all test particles. It is not more complicated than keeping the velocity in a continuous space.

Let (x', y', z', t') be the coordinates in R' for the test particle sent from R' with the velocity u'. In R the coordinates are (x, y, z, t). We fix the origins for these coordinates so that the test particle is sent from the origin in each coordinates. In R' it moves with the speed u', thus $x' = u't'$, while in R it moves with the speed u, thus $x = ut$. The frame R' moves with the velocity v with respect to R, therefore

$$x' = x - vt \tag{10}$$

The time transform is obtained from

$$t' = \frac{x'}{u'} = \frac{1}{u'}(x - vt) = \frac{1}{u'}(ut - vt) = \frac{1}{u'}(u - v)t$$

$$= \frac{1}{u'}(v + u' - \frac{vu'}{c} - v)t = \left(1 - \frac{v}{c}\right) t$$

Coordinates transverse to the movement do not change: $y' = y$, $z' = z$. The unit of measure in R' can be different from the unit of measure in R. If the unit of measurement is different, we should introduce a multiplier and write (10) as

$$x' = \gamma_1(x - vt)$$

for some γ. In the Lorentz transfer this change of the unit of measurement affects only the direction of the movement. We can make a similar assumption. The transform gets the form

$$x' = \gamma_1(x - vt) \tag{11}$$

$$t' = \gamma_1 \left(1 - \frac{v}{c}\right) t$$

$$y' = y$$

$$z' = z$$

where γ_1 is not yet determined. This transform has the inverse transform if $v \neq c$

$$x = \gamma_1^{-1} \left(x' + \frac{vc}{c-v} t'\right) \tag{12}$$

$$t = \gamma_1^{-1} \frac{c}{c-v} t'$$

$$y = y'$$

$$z = z'$$

The discrete model has a rest frame of reference and the inverse transform is not obtained by inverting the sign of v. It is not possible to set γ_1 to any value so that there is no preferred frame of reference. If we require that

$$x = \gamma_1 (x' + vt')$$

and equate this with the form in (12) we get

$$\gamma_1 = \sqrt{1 + \frac{v^2}{u'(c-v)}}$$

while if we require that

$$t = \gamma_1 \left(1 + \frac{v}{c}\right) t'$$

we get $\gamma_1 = \gamma$ in (5).

The transform (11) is clearly not the Lorentz transform (4) and we would expect that surely experiments have ruled out (11). But this is not the case. If $u' = c$ in (11), the transform (11) is exactly (4) if γ_1 is set to γ. Also if $v = c$ the transform (11) is (4). Classical experiments by Michelson-Morley and Kennedy-Thorndyle used light which implies that $v = u' = c$. In the Mössbauer experiment the signal is gamma rays and again $v = u' = c$. These experiments cannot distinguish between (11) and (4). The Ives-Stilwell experiment has $v < c$ but $u' = c$ and it also cannot tell the difference. The same seems to be true also to all later experiments: they always use a signal that travels with the speed of light.

The Ives-Stilwell experiment is of special interest as it verifies the value γ in the Lorentz transform. Or does it? The problem is that the Ives-Stilwell experiment, like modern experiments of the similary type, measured the time dilation from which we get the proper time. In special relativity the proper time is

$$\Delta t' = \sqrt{1 - \frac{v^2}{c^2}}\Delta t \tag{13}$$

but let us calculate the time dilation directly from the Lorentz transform (4). We have a clock in the R' frame of reference and measure signals (x_1, t_1) and (x_2, t_2) sent by the clock. In the Ives-Stilwell experiment the clock is ions speeded to a fraction of c, in the classical experiment $v = 0.005c$ but in modern experiments v is much closer to c. These ions emit light and the experiment has a way to separate the frequency shift caused by the time dilation from the frequency shift caused by the Doppler effect. Thus, the signal is light and travels with the speed $u' = c$. Two signals come from a clock that is in rest in R' if $x'_2 - x'_1 = u'(t'_2 - t'_1)$. That is, $x'_2 = x'_1$ is not the condition that the two signals come from a clock which is at rest in R'. The positions x'_2 and x'_1 can be the same for an object at rest in R' only if $v = u'$. The condition $x'_2 - x'_1 = u'(t'_2 - t'_1)$ is always filled as $x' = u't'$, thus we always measure signals that are in rest in R'. Assigning $u' = c$ the inverse transform (6) gives

$$x_2 - x_1 = \gamma(x'_2 - x'_1 + v(t'_2 - t'_1)) = \gamma(c + v)(t'_2 - t'_1)$$

As $x = ut$ and $u = c$ in the experiment we get

$$c\Delta t = c(t_1 - t_2) = x_2 - x_1 = \gamma(c + v)(t'_2 - t'_1)$$

Thus the time dilation is

$$\begin{aligned}\Delta t' &= t'_1 - t'_2 = \gamma^{-1}(c + v)^{-1}\Delta t \\ &= \sqrt{(1 - \frac{v}{c})(1 + \frac{v}{c})}\frac{c}{c + v}\Delta t \\ &= \sqrt{(1 - \frac{v}{c})(1 + \frac{v}{c})}\frac{1}{1 + \frac{v}{c}}\Delta t\end{aligned}$$

$$= \frac{\sqrt{1-\frac{v}{c}}}{\sqrt{1+\frac{v}{c}}}\frac{\sqrt{1-\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}}\Delta t$$

$$= \gamma\left(1-\frac{v}{c}\right)\Delta t \tag{14}$$

Clearly, (14) is not (13), which was measured in the Ives-Stilwell experiment.

This is because (14) is the time delay of an oscillator oscillating in the x-direction, but in the we measure an oscillator that oscillates in the transverse direction. We can consider oscillation to be in the y-direction. The transform (4) and the inverse (6) do not treat this case as in those formulae x' is in the x-direction, but let us assume that an oscillator moves between two positions y'_1 and y'_2 with the speed $u' = c$. This move is half an oscillation and in R' it takes the time $\Delta t' = (y'_2 - y'_1)/u'$. The frame R' moves with the velocity v, thus in Δt it has moved the distance $v\Delta t$ along the x-axis in R. The y coordinate is unchanged in (4), thus $y_2 - y_1 = y'_2 - y'_1$ and this distance is along the y-axis in R. The total distance the signal has travelled in R is from the Pythagoras theorem

$$L = \sqrt{(y_2-y_1)^2 + (v\Delta t)^2}$$

In R the signal travels with the speed u, thus

$$\Delta t = \frac{L}{u}$$

Inserting L and $y_2 - y_1 = y'_2 - y'_1 = c\Delta t'$ and solving gives

$$(\Delta t')^2 = \frac{u^2}{u'^2}\left(1-\frac{v^2}{u^2}\right)(\Delta t)^2 \tag{15}$$

In the Ives-Stilwell experiment, as in all later experiments, $u' = u = c$, thus we get the equation (13)

$$(\Delta t')^2 = \left(1-\frac{v^2}{c^2}\right)(\Delta t)^2$$

The parameter γ in (4) does not appear in this calculation. The calculation uses the Pythagoras theorem and assumes that in the y direction there is no length change.

For the transformation (11) time dilation in the x-direction is calculated in the same way as for the Lorentz transform. Inserting the always valid expression

$x_2' - x_1' = u'(t_2' - t_1')$ to the inverse formula (12) yields the relation of $\Delta t'$ and Δt and it corresponds to the time ditation of a clock that is in rest in R' and oscillates in the x-direction. Thus

$$x_2 - x_1 = \gamma_1^{-1}(x_2' - x_1' + \frac{vc}{c-v}(t_2' - t_1'))$$

$$= \gamma_1^{-1}\left(u' + \frac{cv}{c-v}\right)(t_2' - t_1')$$

Inserting $x = ut = (v + u' - \frac{vu'}{c})t$ yields

$$\gamma_1\left(v + u' - \frac{vu'}{c}\right)(t_2 - t_1) = \frac{c(v+u')}{c-v}(t_2' - t_1')$$

Thus

$$\frac{t_2' - t_1'}{t_2 - t_1} = \gamma_1 \frac{c-v}{c(v+u') - vu'} \frac{c(v+u') - vu'}{c} = \gamma_1\left(1 - \frac{v}{c}\right)$$

We get a result similar to (14)

$$\Delta t' = \gamma_1\left(1 - \frac{v}{c}\right)\Delta t \tag{16}$$

The derivation of (15) only needs the Pythagoras theorem and that there is no length change in the transverse direction. It follows that (14) is valid also for the transform (11). We cannot insert the u' from velocity summation formula (9) to (15) because in (15) the velocities v and u' are orthogonal and (9) only gives the summation for parallel velocities. Let us define summation of orthogonal velocities in the transform (11) by the formula

$$u^2 = v^2 + u'^2 - \frac{v^2 u'^2}{c^2} \tag{17}$$

This is similar to (9) and also limits the maximal velocity to c. Solving

$$\frac{1}{u'^2} = \frac{1}{u^2 - v^2}\left(1 - \frac{v^2}{c^2}\right)$$

and inserting to (15) gives

$$\Delta t'^2 = \frac{u^2}{u^2 - v^2}\left(1 - \frac{v^2}{c^2}\right)\frac{u^2 - v^2}{u^2}\Delta t^2 = \left(1 - \frac{v^2}{c^2}\right)\Delta t^2$$

Thus, it gives (13) for every u, not only for $u = c$. It is very good that the result does not depend on u since if the time dilation is a real phenomenon it cannot depend on u, the speed of the signal sent from R'. The velocity summation formula (17) is the only formula giving (13) for all values of u. The summation formula (17) is not the same as (7) in the Lorentz transform.

The time delay has been experimentally measured many times and can be accepted as a real phenomenon. Time is a scalar, therefore the time dilation in the x-direction must be the same as the time delay in the y-direction. Thus, (14) must give the same time dilation as (13). We get the equation

$$\sqrt{1-\frac{v^2}{c^2}} = \gamma\left(1-\frac{v}{c}\right)$$

Thus

$$\gamma = \sqrt{1-\frac{v^2}{c^2}}\left(1-\frac{v}{c}\right)^{-1} = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}} \qquad (18)$$

This value of γ does not give the form (6) for the inverse transform and therefore it rules out the principle that there is no preferred frame of reference.

This observation solves the twin paradox: a twin, who travels with a speed close to c ages slower than the one who stays at home, but if R can be considered moving and R' at rest, the result is the opposite. The solution is that for speeds v close to c we cannot consider R' to be at rest and R as moving.

We can set γ_1 to the value in (18)

$$\gamma_1 = \gamma = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(1+\frac{v}{c}\right)$$

Inserting this value of γ_1 to (11) and (12) shows that the inverse transform is almost obtained by changing v to $-v$

$$x' = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}}(x-vt) \qquad (19)$$

$$t' = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}}\left(1-\frac{v}{c}\right)t$$

$$x = \frac{\sqrt{1-\frac{v}{c}}}{\sqrt{1+\frac{v}{c}}}\left(x' + vt'\left(1-\frac{v}{c}\right)^{-1}\right) \approx \frac{\sqrt{1-\frac{v}{c}}}{\sqrt{1+\frac{v}{c}}}(x'+vt')$$

$$t = \frac{\sqrt{1-\frac{v}{c}}}{\sqrt{1+\frac{v}{c}}}\left(1-\frac{v}{c}\right)^{-1}t' \approx \frac{\sqrt{1-\frac{v}{c}}}{\sqrt{1+\frac{v}{c}}}\left(1+\frac{v}{c}\right)t'$$

The length contraction in (11) is the same as in the Lorentz transform assuming γ has the same value. For two points (x_2', t') and (x_1', t') at the same time the length in R' and in R relate as

$$\Delta x' = x_2' - x_1' = \gamma(x_1 - x_2) \tag{20}$$

For the transform (11) holds

$$t'^2 - c^{-2}x'^2 = t^2 - c^{-2}x^2 + \frac{2vt}{c^2}(x - ct) \tag{21}$$

Thus, for $u' = c$ holds

$$ds^2 = dt'^2 - c^{-2}dx'^2 - c^{-2}dy'^2 - c^{-2}dz'^2 = t^2 - c^{-2}dx^2 - c^{-2}dy^2 - c^{-2}dz^2 = ds^2$$

and the transform leaves the space element invariant, but this is not true for $u' < c$. The invariant property for (11) is

$$x' - ct' = x - ct$$

The transform (11) does not describe the geometry of a flat Minkowski space.

As $ds'^2 \neq ds^2$ and the inverse transform is not obtained by changing the sign of the velocity, there is no argument that in some way proves that the Lorentz transform (4) is the correct one.

It has been known for a long time that the mass depends on the velocity and becomes infinite when the velocity approaches c. These experiments were started by Thomson (1893) and Searle (1897) and later continued by Kaufman and others. The formula for the moving mass

$$\frac{m_t}{m} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{22}$$

was derived by Lorentz from his theory where an electron gets deformed in the direction of movement. The index t in m_t refers to the transversal mass as the growing mass seemed to be transversal to the movement. Now it is called the moving mass and Einstein's totally different derivation of this formula is considered as the correct one, but perhaps it should be reconsidered.

In Kaufman's experiments the mass growth was observed by measuring the trajectory of an electron in different speeds. The charge of an electron does not seem to depend on the velocity. The trajectory shows that the mass grows as in (22) and it must be in the frame of reference of the moving electron as it is measured from the equations of motion of the electron. That is, the equations of motion are given in the local coordinates of the moving particle. The mass grows in R', not necessarily in R. Indeed, if mass is conserved in R, mass cannot grow in R.

There are several ways of deriving (22), the first derivation being given by Lorentz. I suggest the following simple explanation, which I will not work out to a proof in this paper. In the discrete model I consider space as liquid consisting of space volume elements which can flow and gravitation not as geometry where point masses move along geodesic lines but as a flow where space volume elements flow to point masses and disappear to a hole in the point mass. This model means that there must be some universe where the space volume elements go and that there must be antigravitation because otherwise we run out of space volume elements. Antigravitation would not appear as massive objects. It would appear as new space volume being created. Such sources of space elements would be as common as mass points in the universe, but stars in the universe are far apart and there is no reason to assume that there would be a source of space volume elements anywhere close to us. Some phenomena, possibly supernovas, would be explainable with antigravitational sources if this model were correct.

Starting from this simple model, a point mass eats space volume elements proportionally to its transverse size. If it has a higher speed it meets and eats more space volume elements. The higher speed can also be understood as a shorter time unit: because of the time dilation the time unit in R' is smaller in the same proportion as in (22). The smaller time unit makes it possible for the mass point to eat space elements faster. This is seen in the equations of motion as a larger mass. Because the mass gain is caused by the time dilation in the transverse direction it has the same dependency as (13).

This explanation I leave only on the idea level, but let us look at the usual derivation how from (22) we get to $E = mc^2$. If the mass m and the velocity v are both variable, the differential of the force can be expressed as

$$F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

The differential of the kinetic energy is then

$$dW_K = Fds = m\frac{dv}{dt}ds + v\frac{dm}{dt}ds = m\frac{ds}{dt}dv + v\frac{ds}{dt}dm = mvdv + v^2dm$$

Assuming that we show

$$c^2dm = mvdv + v^2dm \tag{23}$$

it only remains to integrate from the rest mass m_0 to the moving mass m

$$W_K = \int_0^{W_K} dW_k = \int_{m_0}^{m} c^2dm = (m - m_0)c^2$$

Assigning

$$W_0 = m_0c^2$$

we get

$$E = W_k + W_0 = mc^2$$

Naturally it was known from atomic masses long before Einstein that there is mass loss which can be connected to binding energy and atoms may contain energy. The formula $E = mc^2$ was discovered before Einstein, thus he had a reason to expect that $W_0 = m_0c^2$ holds. It remains to derive (23). It is derived from (22)

$$m_0 = m\sqrt{1 - \frac{v^2}{c^2}}$$

Squaring and multiplying by c^2

$$m_0^2c^2 = m^2c^2 - m^2v^2$$

Then this expression is differentiated as

$$0 = 2mc^2dm - 2mv^2dm - m^2 2vdv \tag{24}$$

that is

$$c^2dm = mvdv + v^2dm$$

That seems initially like a correct way to get (23), but in what frame of reference is the differentiation (24) done? The frame R' moves with the speed v. If v is changing there are some forces seen in R', the accelerating frame experiences these forces as gravitation. Nothing in this calculation indicates that it is done in R'. Indeed, it cannot be done in R' because $v = 0$ in R' all the time even if v is not constant. It must be that the differentiation is done in the rest frame R. The problem is that the mass grows as (22) in R', not in R. Experiments that show how mass grows are experiments where the growth of mass is seen from the trajectory of the particle. The trajectory is determined by the forces in the local coordinates of the moving particle, in R'.

It is very possible that the mass of the particle does not change in R. Mass is conserved in R and the reasons why the mass grows in R' can be that it is caused by the time unit becoming smaller, or the length grows in R'. In any case, (22) does not describe what happens to the mass in R. Therefore the differentiation cannot be made in R. If not in R and not in R', then the step (24) is not justified.

There is a simple way to prove $E = mc^2$ in the discrete model. The discrete space with only local interactions necessarily has a maximum speed:

$$c = \frac{s_u}{t_u}$$

where s_u is the length of a space unit and t_u is the time unit. The maximum acceleration a_u is speeding a mass from zero speed to the speed c in a single time unit t_u:

$$a_u = \frac{c}{t_u} = \frac{s_u}{t_u^2}$$

The parameters s_u, t_u, c and a_u are universal constants in this model. Space can be considered as incompressible liquid.

Force fields are created by another set of parameters: F_u, p_u, m_u and E_u where the index u refers to *unit* and F is force, p is pressure, m is mass and E is energy. These parameters are not universal constants: they get their value from the whole mass of the universe. It is possible to think of space volume elements flowing into holes in point masses. As the gravitation force points towards a point pass and a force grows to the direction where the pressure decreases it is better to think of the pressure as decreasing when approaching a point mass. Any of these parameters F_u, p_u, m_u, E_u can be taken as the cause of the others. I take the pressure as the cause. Thus

$$F_u = p_u s_u^3$$

$$m_u = \frac{F_u}{a_u} = \frac{p_u s_u^2}{a_u}$$

$$\frac{p_u}{m_u} = \frac{a_u}{s_u^2}$$

$$\frac{E_u}{m_u} = \frac{p_u s_u^3}{m_u} = a_u s_u = c^2$$

So $E = mc^2$. This calculation should not be interpreted in the way that the mass of the space element turns into energy. It should be understood as describing that if one space element is crushed in some way, the pressure created by of all outside space elements releases energy. This energy causes the fallout from the crushed space element to accelerate from the zero initial speed to the speed of c and to escape as a photon or some other particle.

We can compare energy released in nuclear reactions to energy released in a collapse of demolished building. When one floor is crushed e.g. by colliding the atom nucleus with a neutron (or a skyscraper with an airplane) the outside space elements (or the upper floors) fill the space left after the collapse. The energy released is the potential energy of the outside space elements and not some binding energy that used to keep the collapsed space element together. In a similar way, the energy that is released when a building collapses is not some binding energy that the concrete and steel of a destroyed floor had stored in their materia. Some energy is needed to destroy the structures of a floor, just like it is necessary to bombard atoms with neutrons, but the released energy is from the potential energy the upper floors had before they fell, i.e., it is the pressure times the changed volume.

Let us draw some conclusions from this simple article. The Lorentz transform is not the correct one and it has not been empirically verified. The alternative transform (11) has not been ruled out and it has the summation formula (17) which is the only formula that gives the correct proper time (13) so that it does not depend on the velocity u' of the signal sent from the moving frame. The principle that there is no preferred frame of reference is incorrect. The derivation of the mass growth and the $E = mc^2$ in the special relativity may be questioned and they are not the only ways to derive those properties. Geometrization is not the correct paradigm for gravitation.

References:

Basics of the special relativity theory can be assumed known.