

# Time is not a Vector: Corrections to the Article “Vectorial Relativity versus Special or General Relativity?”

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**ABSTRACT:** In this work the main conceptual errors that sequentially were introduced in the development of the Special Theory of Relativity (SR) are explained. A simple presentation of the Lorentz Transformations (LT) is given, within where it is emphasized that, by using an incomplete configuration, repeated by more than one hundred years, it leads to a first error of a chain of them, when it is assumed that the cross sectional components, in the movable inertial reference system, are invariant or not affected by its relative movement respect a fixed inertial system, on the basis of a presumed “postulate of isotropy” that equals the referred moving components to those of the fixed inertial system. Making use of the general configuration of an inertial system moving on an inclined line relative to the X- axis, and by considering parallel axes between them, correct results for the transformations of distance, time and speed are obtained. These new transformations were called Vectorial Transformations of Lorentz (VLT). In this work is given a discussion of the different possibilities of interpreting the vector structure of time inside the VLT. This concludes with the correction of the mistake of considering time as a vector that we have used in previous papers. Based on the development of the VLT arose the definitions of the Local Transformations of Lorentz (LLT) which allowed as well to as much demarcate more complete the concepts of local time as those of local physical magnitudes. As a consequence of the referred assumptions previously indicated within SR, it was obtained an erroneous definition of the concept of relativistic mass, groundwork of all the definitions of the dynamic physics magnitudes (Energy, Moments, Fields, etc.). A correct analysis for the characterization of relativistic mass was obtained and consequently, the route to correct defining other dynamic magnitudes, as that of the relativistic energy, which allows a correct development of Relativistic Quantum Mechanics. Finally, the expression of the relativistic field of gravitation arises and this concept applied to the calculation of orbits and the precession of planets perihelion of the Solar System. Very precise results in agreement with experimental measurements were obtained. The general validity of Vectorial Relativity (VR) in the whole physics solves the problem of the “theoretical separation” for their applicability existing between Special Relativity (SR) and General Relativity (GR).

**KEYWORDS:** Lorentz Transformations; Vectorial Lorentz Transformations; Relativistic Time, Mass, Energy, Quantum Mechanics, Fields, Gravitation. Calculation of the Planet Orbits, of the Precession of Mercury Perihelion.

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**I. INTRODUCTION**

**1) Lorentz Transformations versus Galilean Transformations.** Galilean Transformations are obtained by considering that time is the same in any inertial system and that it is independent of the space. The basic configuration used to obtain them and to compare with those of the Lorentz transformations is constructed by considering an inertial system restricted to move rectilinear and at constant speed  $v$ , on the coordinate  $X$  of another fixed system. Both coordinate systems with their axes parallel (see Fig. 1). When both systems’ origins coincide a pulse of light is sent to the space. Galilean Transformations (GT) obtained from tis configuration are:

$$\begin{aligned}
 x' &= x - v.t, & y' &= y, & z' &= z, & t' &= t \\
 x^2 + y^2 + z^2 &= c^2.t^2, & x'^2 + y'^2 + z'^2 &= c'^2.t'^2
 \end{aligned}
 \tag{1}$$

These results, plus the following speculation: “since axes  $Y$  and  $Z$  are parallel to  $Y'$  and  $Z'$  ones and the movement of the movable system is on  $X$ -axis, leads to assume the equality of the components”:

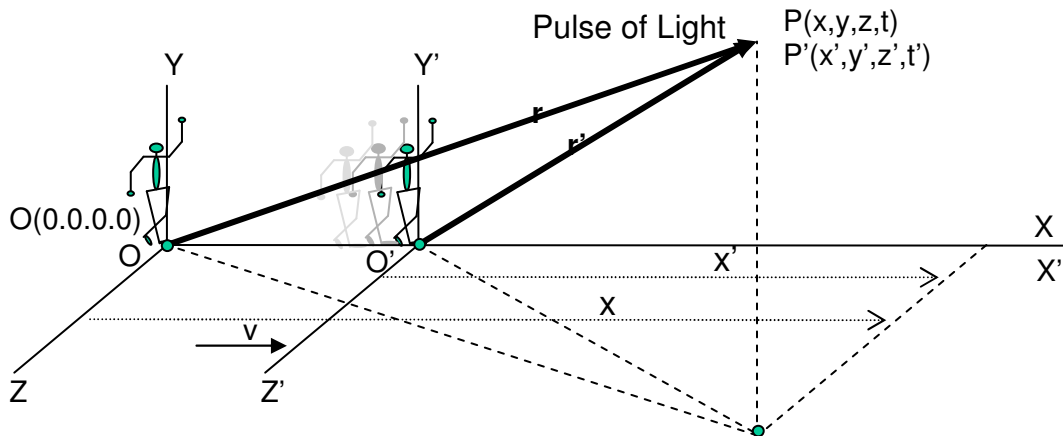


Fig. 1 “Three-dimensional motion” of Lorentz Transformations

As it is known, although Newton equations (laws) are invariant to these transformations, the GT modify the structure of Maxwell equations, against the principle of relativity enunciated by Galileo himself. And not only that, GT considered the speed of light as a non constant value,  $c' \neq c$ , an aspect that that the referred Maxwell equations show as constant and independent of the velocity of the device that emits it. The scientists at that time rejected GT because this failure.

Lorentz Transformations on the basis of recognizing the speed of the light as a universal constant and that the laws of physics are the same in any inertial system, are achieved by the introduction of a factor  $k$ , to be determined, which allows mathematically preserving the constancy of the speed of light,  $c' = c$ . This is done only with the X-component:

$$x' = k(x - v.t)
 \tag{2}$$

Then, the following assumption is done: “Given that motion of inertial system O’ is on the X-axis of the fixed one O, and that Y and Y’ are parallel and also Z and Z’, thus by isotropy,  $y' = y$ ,  $z' = z$ ”. Namely,

$$x^2 + y^2 + z^2 = c^2 \cdot t^2, \quad x'^2 + y'^2 + z'^2 = c^2 \cdot t'^2 \tag{3}$$

If we imagine, inside the conditions of Lorentz transformations, the case of an one-dimensional motion of the moving system O’ at speed  $v$  and a pulse of light is sent along the only axis of the fixed system O, then the following is met:  $c = x/t = x'/t'$ , and so, the expression of the structure of time for this case can be obtained:

$$x' = k(x - v \cdot t) \Rightarrow c \cdot t' = k \left( c \cdot t - v \frac{x}{c} \right) \Rightarrow t' = k \left( t - \frac{v}{c^2} x \right) \tag{4}$$

Of course, this expression is valid only for this particular case of the one-dimensional case. Nevertheless, given that in the variables concerned in the assumption  $y' = y$ ,  $z' = z$ , is not observed any dependency with time, let’s check if the time structure is also valid for the three-dimensional case of Fig. 1. Substituting (2) and (4) in the second equation of (3), we obtain:

$$k^2(x^2 + 2vxt + v^2t^2) + y^2 + z^2 = c^2k^2(t^2 - 2xtv/c^2 + x^2v^2/c^4),$$

Grouping and arranging suitably for comparing coefficients with the first equation in (3):

$$(k^2 - k^2v^2/c^2)x^2 + y^2 + z^2 = c^2t^2(k^2 - k^2v^2/c^2) \Leftrightarrow x^2 + y^2 + z^2 = c^2t^2 \Rightarrow k^2 - k^2v^2/c^2 = 1$$

And we observe that given that the assumption obliges to the equality of terms  $y'^2 = y^2$ ,  $z'^2 = z^2$ , validity of expression (4) is automatically guaranteed for this restricted three-dimensional configuration and the same value  $k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ , of the one-dimensional case. As we had already

observed, this configuration restricts the movement of the movable system to only move on X-axis of the fixed system. Nevertheless, in the literature this limitation by means of rotations of the axis of movement of the movable system is solved, or assigning an arbitrary axis to the movement of the movable system, to which they call axis of movement, and perpendicular axes to the other two axes. Thus, the assumption stays intact to consider invariants the perpendicular components to the direction of movement of the movable inertial system.

**2) Vectorial Lorentz Transformations versus Lorentz Transformations.** In order to obtain general and correct transformations between components of both systems in the three coordinate axes (by making no type of restrictions, assumptions nor speculations), the movable system was released to move on an inclined line that formed an angle  $\beta$  with plane XY of the fixed system and in which the projection of this line, on the same plane XY, formed an angle  $\alpha$  with respect to X-axis (see fig. 2). As before, the Galilean transformations (GT) for this general configuration, obtained between the spatial components according to the three coordinate axes, are the following ones:

$$\begin{aligned} x' &= x - v \cdot t \cdot \cos \alpha \cdot \cos \beta, & y' &= y - v \cdot t \cdot \sin \alpha \cdot \cos \beta, & z' &= z - v \cdot t \cdot \sin \beta, & t' &= t \\ x^2 + y^2 + z^2 &= c^2 \cdot t^2, & x'^2 + y'^2 + z'^2 &= c'^2 \cdot t'^2 \end{aligned} \tag{5}$$

The Vectorial Lorentz Transformation, in which equally by recognizing the speed of the light as a universal constant and that the laws of physics are the same in any inertial system, are achieved by the introduction of a factor  $k$  in the spatial components of the Galilean Transformations, to be determined, which allows mathematically preserving the constancy of the speed of light, become:

$$x' = k.(x - v.t.\cos\alpha.\cos\beta), \quad y' = k.(y - v.t.\sin\alpha.\cos\beta), \quad z' = k.(z - v.t.\sin\beta), \quad (6a)$$

In where the following is met:

$$x^2 + y^2 + z^2 = c^2.t^2, \quad x'^2 + y'^2 + z'^2 = c^2.t'^2 \quad (6b)$$

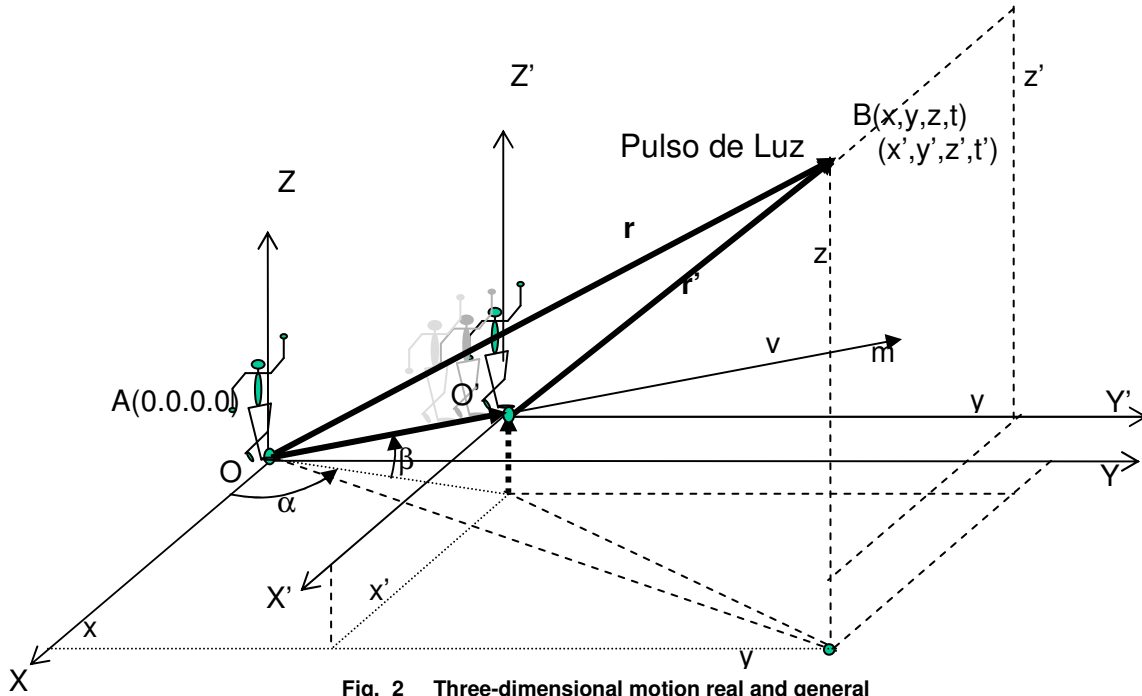


Fig. 2 Three-dimensional motion real and general

Based on the previous relationships and doing suitable changes, we are going to obtain the transformation of time:

$$c^2.t'^2 = x'^2 + y'^2 + z'^2 = k^2.[(x - v.t.\cos\alpha.\cos\beta)^2 + (y - v.t.\sin\alpha.\cos\beta)^2 + (z - v.t.\sin\beta)^2]$$

$$c^2.t'^2 = k^2 \left[ x^2 + y^2 + z^2 + v^2.t^2.(cos\alpha.\cos\beta)^2 + v^2.t^2.(sin\alpha.\cos\beta)^2 + v^2.t^2.sin^2\beta \right]$$

$$c^2.t'^2 = k^2 \left[ c^2.t^2 + v^2.t^2 - 2.v.x.(t.\cos\alpha.\cos\beta) - 2.v.y.(t.\sin\alpha.\cos\beta) - 2.v.z.(t.\sin\beta) \right]$$

Given that  $\cos^2\alpha.\cos^2\beta + \sin^2\alpha.\cos^2\beta + \sin^2\beta \equiv 1 \Rightarrow c^2.t^2 \equiv c^2.t^2.(cos^2\alpha.\cos^2\beta + sin^2\alpha.\cos^2\beta + sin^2\beta)$ ,

and that  $t^2 \equiv \frac{x^2 + y^2 + z^2}{c^2} \Rightarrow v^2.t^2 \equiv v^2.\frac{x^2 + y^2 + z^2}{c^2}$ . Substituting and grouping again:

$$c^2.t'^2 = k^2 \left\{ \left[ c^2.(t.\cos\alpha.\cos\beta)^2 - 2.v.x.(t.\cos\alpha.\cos\beta) + v^2.\frac{x^2}{c^2} \right] + \left[ c^2.(t.\sin\alpha.\cos\beta)^2 - 2.v.y.(t.\sin\alpha.\cos\beta) + v^2.\frac{y^2}{c^2} \right] \right. \\ \left. + \left[ c^2.(t.\sin\beta)^2 - 2.v.z.(t.\sin\beta) + v^2.\frac{z^2}{c^2} \right] \right\}$$

$$c^2.t'^2 = c^2.k^2.\left[\left(t.\cos\alpha.\cos\beta - \frac{v}{c^2}.x\right)^2 + \left(t.\sin\alpha.\cos\beta - \frac{v}{c^2}.y\right)^2 + \left(t.\sin\beta - \frac{v}{c^2}.z\right)^2\right] \quad (7)$$

From the last expresión we arrive at the transformation of times:

$$t'^2 = k^2.\left[\left(t.\cos\alpha.\cos\beta - \frac{v}{c^2}.x\right)^2 + \left(t.\sin\alpha.\cos\beta - \frac{v}{c^2}.y\right)^2 + \left(t.\sin\beta - \frac{v}{c^2}.z\right)^2\right]. \quad (8)$$

By carefully observing this last expression (8) it suggests a **vector interpretation of time** measured in the moving system as the following one:

$$\mathbf{t}' = k.\left(\mathbf{t} - \frac{v}{c^2}\mathbf{r}\right) \quad (9)$$

Although this concept is going to be discussed later on, in previous works, we had used this interpretation of time as a vector, but this interpretation is not apparently correct , as it will be shown, and whose correction or amendment is the most important aspect of this work.

In fact, we will develop all the possible interpretations in the following paragraphs.

a) **Time as vector.** Obviously The expressions of the spatial transformations (6) can be written as only one vector expression, which together with the equation (9) would form an independent set of equations, indicated as follows:

$$\begin{aligned} \mathbf{r}' &= k.(\mathbf{r} - \mathbf{v}.t) & \mathbf{r}' &= k.(\mathbf{r} - \mathbf{v}.t) \\ \mathbf{t}' &= k.\left(\mathbf{t} - \frac{v}{c^2}\mathbf{r}\right) & \Rightarrow & \mathbf{t}' &= k.\left(\mathbf{t} - \frac{v}{c^2}\mathbf{r}\right) \end{aligned} \quad (10)$$

If we assign the vector interpretation to time (9), a first problem arises: **such vector time in fixed system should have the direction of the speed of the movable system**, as it comes out from equations (6a) and (6b) to become the set of equations presented in the second part of equations (10). Thus, it will imply that we have to consider now the speed of the movable system as a scalar. Let's force the acceptance that in this case time behaves as a vector and speed as a scalar!. Despite this there is one more problem: **what is the direction of the vector time measured in the movable system?** With these doubts let's see the following interpretation.

b) **Time as scalar and speed as a vector. First part.** In order to investigate what is the meaning of the vector structure of the time transformation measured from the movable system, let's try on interpreting equation (7). This equation,

$$c^2.t'^2 = k^2.\left[\left(c.t.\cos\alpha.\cos\beta - \frac{v}{c}.x\right)^2 + \left(c.t.\sin\alpha.\cos\beta - \frac{v}{c}.y\right)^2 + \left(c.t.\sin\beta - \frac{v}{c}.z\right)^2\right]$$

after working on a little, originates the following vector expression:

$$\text{For } \mathbf{u}_v = (\cos\alpha.\cos\beta).\mathbf{i} + (\sin\alpha.\cos\beta).\mathbf{j} + (\cos\beta).\mathbf{k} \text{ and } \mathbf{c}'t' = \mathbf{r}' \Rightarrow \mathbf{c}'t' = k.t.\left(\mathbf{c}.\mathbf{u}_v - \frac{v}{c}\mathbf{c}\right) \quad (11)$$

But, first of all, the development preceding equation (8), showed that equation (11) is not independent of the equation of velocities,  $\mathbf{r}' = k.(\mathbf{r} - \mathbf{v}.t) \Rightarrow \mathbf{c}'t' = k.(c.t - \mathbf{v}.t)$ . This only characteristic of dependency of equation (11) makes the interpretation (b) to be discarded, because it should generate an independent vector equation. Secondly, in this situation it would imply that the answer to the question

done at the end of the previous part (a) is that the direction of the vector time, measured in the movable system would be the one of the vector speed of the light measured in the movable system; and the direction of the vector time, measured in the fixed system, would be the one of the vector speed of the movable system measured in the fixed system, as it was said in (a). All these characteristics are inconsistent, which would also discard the interpretation (a) like valid.

c) **Time as scalar and speed as a vector. Second part.** Another way to interpret the vector structure of the expression (8), comes to multiply both sides of the equation by  $v^2$ , to obtain:

$$v^2 \cdot t'^2 = k^2 \cdot [(v \cdot t \cdot \cos \alpha \cdot \cos \beta - \frac{v^2}{c^2} \cdot x)^2 + (v \cdot t \cdot \sin \alpha \cdot \cos \beta - \frac{v^2}{c^2} \cdot y)^2 + (v \cdot t \cdot \sin \beta - \frac{v^2}{c^2} \cdot z)^2] \tag{12}$$

This gives the following independent equation:

$$\mathbf{v}' \cdot t' = k \cdot \left( \mathbf{v} \cdot t - \frac{v^2}{c^2} \cdot \mathbf{c} \cdot t \right) \Leftrightarrow v \cdot t' \cdot \mathbf{u}'_v = k \cdot \left( v \cdot t \cdot \mathbf{u}_v - \frac{v^2}{c^2} \cdot c \cdot t \cdot \mathbf{u}_c \right) \Leftrightarrow \mathbf{u}'_v = k \cdot \frac{t}{t'} \left( \mathbf{u}_v - \frac{v}{c} \cdot \mathbf{u}_c \right) \tag{13}$$

It can be observed, similar to which happens with the speed of the light ( $\mathbf{c}' \neq \mathbf{c}$ ), that its direction is not measured equal in both reference systems, but the measurements of its magnitude is the same,  $|\mathbf{c}'| = |\mathbf{c}| = c$ , the direction of the vector speed of the movable system,  $\mathbf{v}'$ , with respect to the fixed system, measured in the movable system, is different from that measured in the fixed system,  $\mathbf{v}$ , but they have the same magnitude,  $v$ . The equation  $\mathbf{r}' = k \cdot (\mathbf{r} - \mathbf{v} \cdot t)$ , expressed based on  $\mathbf{u}_v$  and  $\mathbf{u}_c$ , joined with the previous obtained equation, constitute the following pair of independent equations:

$$\begin{aligned} \mathbf{u}'_c &= k \cdot \frac{t}{t'} \left( \mathbf{u}_c - \frac{v}{c} \cdot \mathbf{u}_v \right) \\ \mathbf{u}'_v &= k \cdot \frac{t}{t'} \left( \mathbf{u}_v - \frac{v}{c} \cdot \mathbf{u}_c \right) \end{aligned} \Rightarrow \begin{bmatrix} \mathbf{u}'_c \\ \mathbf{u}'_v \end{bmatrix} = k \cdot \frac{t}{t'} \begin{bmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{u}_c \\ \mathbf{u}_v \end{bmatrix} \tag{14}$$

d) **Time as scalar and speed as a vector. Third part.** Another interpretation with the same previous basement is the following one: defining the product of the vector speed of the movable system, measured with respect to the fixed system,  $\mathbf{v}$ , times the time  $t$ , like the vector of position of the movable system  $\mathbf{r}_0 = \mathbf{v} \cdot t$ , and to  $\mathbf{r}_0'$  like  $\mathbf{r}_0' = \mathbf{v}' \cdot t'$ , we obtain the following vector representation of the equations (15), being obtained a system of vector equations that mathematically models what is indicated in Fig. 2, and that can be represented as a matrix, just as the equations (14):

$$\begin{aligned} \mathbf{r}' &= k \cdot (\mathbf{r} - \mathbf{r}_0) & \mathbf{r}' &= k \cdot (\mathbf{r} - \mathbf{r}_0) \\ \mathbf{r}'_0 &= k \cdot \left( \mathbf{r}_0 - \frac{v^2}{c^2} \cdot \mathbf{r} \right) & \mathbf{r}'_0 &= k \cdot \left( -\frac{v^2}{c^2} \cdot \mathbf{r} + \mathbf{r}_0 \right) \end{aligned} \Rightarrow \begin{bmatrix} \mathbf{r}' \\ \mathbf{r}'_0 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -\frac{v^2}{c^2} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{r}_0 \end{bmatrix} \Rightarrow \mathbf{y} = \mathbf{A} \mathbf{x} \tag{15}$$

It can be observed that the second equation also can be visualized spatially more easily because it would be the sum of a negative vector,  $-\frac{v^2}{c^2} \cdot \mathbf{r}$ , plus the vector of position of the movable system,  $\mathbf{r}_0$ .

That is to say that this form of representation is most advisable to take into account the vector structure of the equation (8).

If the roles of the inertial systems change, in the sense that we consider now to the system O as the moving one and to the O' system as that fixed, we obtain the following relations:

$$\begin{aligned} \mathbf{r} &= k \cdot (\mathbf{r}' + \mathbf{r}_0') & \mathbf{r} &= k \cdot \left( \mathbf{r}' + \mathbf{r}_0' \right) \\ \mathbf{r}_0 &= k \cdot \left( \mathbf{r}_0' + \frac{v^2}{c^2} \cdot \mathbf{r}' \right) & \mathbf{r}_0 &= k \cdot \left( \frac{v^2}{c^2} \cdot \mathbf{r}' + \mathbf{r}_0' \right) \end{aligned} \Rightarrow \begin{bmatrix} \mathbf{r} \\ \mathbf{r}_0 \end{bmatrix} = k \begin{bmatrix} 1 & 1 \\ \frac{v^2}{c^2} & k \end{bmatrix} \begin{bmatrix} \mathbf{r}' \\ \mathbf{r}_0' \end{bmatrix} \Rightarrow \mathbf{x} = B\mathbf{y} \quad (16)$$

It can be observed that if we replaced in the previous vector matrix equation (16), the matrix by its value in (15), we will obtain that the product of the scalar matrices  $BA$  should be equal to the unity matrix, which gives the condition to obtain the value of the factor  $k$  :

$$\mathbf{x} = B\mathbf{y} = BA\mathbf{x} \Rightarrow BA = I \Rightarrow \begin{bmatrix} k & k \\ \frac{k \cdot v^2}{c^2} & k \end{bmatrix} \begin{bmatrix} k & -k \\ -\frac{k \cdot v^2}{c^2} & k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Thus, from this discussion we determined that the vector interpretation (c) is the valid one. We can observe that the interpretation (d) is really another version of the same (c), although visually much more maneuverable. This conclusion would correct our previous works where we chose to consider, wrongly, like valid the interpretation (a). Nevertheless, as this error does not modify the definition of the Local Transformations of Lorentz (LLT), considered later on, it does not have later repercussions, that is to say, it does not influence in the later calculations of this work, which have been achieved based on the application of the referred LLT.

We will let the treatment so far vectorial to continue with the before interrupted scalar calculation in the equation (8), for this general case. We will accomplish this way similar to that done previously to obtain the value of the factor  $k$ , comparing the obtained coefficients coming from replacing the equations (6a) and the expression of the transformation of time (8) in the second equation of (6b) with those of the first equation of (6b).

$$\begin{aligned} c^2 \cdot k^2 \cdot \left[ \left( t \cdot \cos \alpha \cdot \cos \beta - \frac{v}{c^2} \cdot x \right)^2 + \left( t \cdot \sin \alpha \cdot \cos \beta - \frac{v}{c^2} \cdot y \right)^2 + \left( t \cdot \sin \beta - \frac{v}{c^2} \cdot z \right)^2 \right] = \\ = k^2 \cdot \left[ (x - v \cdot t \cdot \cos \alpha \cdot \cos \beta)^2 + (y - v \cdot t \cdot \sin \alpha \cdot \cos \beta)^2 + (z - v \cdot t \cdot \sin \beta)^2 \right] \\ \left\{ \begin{aligned} (k^2 - k^2 v^2 / c^2) x^2 + (k^2 - k^2 v^2 / c^2) y^2 + (k^2 - k^2 v^2 / c^2) z^2 &= (k^2 - k^2 v^2 / c^2) c^2 t^2 \\ x^2 + y^2 + z^2 &= c^2 \cdot t^2 \end{aligned} \right\} \Rightarrow k = 1 / \sqrt{1 - \frac{v^2}{c^2}} \end{aligned}$$

As we see, all the coefficients must be equal to unity and the value of the factor  $k$  for this case is ratified like a same result of the comparison of the coefficients, agreed to the expected result.

On the other hand, the transformations (VLT) preserve invariant the interval of the space-time of the Special Theory of Relativity for a space of any number of dimensions, which means that this new presentation of the Lorentz transformations, to which we have denominated Vectorial Transformations of Lorentz is valid not only for the Light but for any projectile that moves to speeds smaller than those

of the light. That is to say, the following equality is preserved:  $c^2 \cdot t'^2 - r'^2 = \frac{c^2}{v^2} \cdot v^2 \cdot t'^2 - r'^2 = \frac{c^2}{v^2} \cdot r_0'^2 - r'^2$ .

In fact:

$$\begin{aligned} c^2 \cdot t'^2 - r'^2 &= \sum_{j=1}^N \frac{c^2}{v^2} \cdot k^2 \cdot \left( x_{0j} - \frac{v^2}{c^2} \cdot x_j \right)^2 - k^2 \cdot \sum_{j=1}^N (x_j - v \cdot t_j)^2 = k^2 \cdot \sum_{j=1}^N \left[ \frac{c^2}{v^2} \left( x_{0j} - \frac{v^2}{c^2} \cdot x_j \right)^2 - (x_j - v \cdot t_j)^2 \right] = \\ &= k^2 \cdot \sum_{j=1}^N \left[ \frac{c^2}{v^2} \left( x_{0j}^2 - 2 \frac{v^2}{c^2} x_{0j} \cdot x_j + \frac{v^4}{c^4} \cdot x_j^2 \right) - (x_j^2 - 2 \cdot x_{0j} \cdot x_j + x_{0j}^2) \right] = k^2 \cdot \sum_{j=1}^N \left[ \frac{c^2}{v^2} x_{0j}^2 + \frac{v^2}{c^2} \cdot x_j^2 - x_j^2 - x_{0j}^2 \right] \\ &= \frac{1}{1 - \frac{v^2}{c^2}} \cdot \sum_{j=1}^N \left[ \frac{c^2}{v^2} x_{0j}^2 \left( 1 - \frac{v^2}{c^2} \right) - x_j^2 \left( 1 - \frac{v^2}{c^2} \right) \right] = \sum_{j=1}^N \left[ \frac{c^2}{v^2} x_{0j}^2 - x_j^2 \right] = \frac{c^2}{v^2} \sum_{j=1}^N [x_{0j}^2] - \sum_{j=1}^N [x_j^2] = \frac{c^2}{v^2} r_0'^2 - r^2 = c^2 \cdot t'^2 - r^2 \\ &\Rightarrow c^2 \cdot t'^2 - r'^2 = c^2 \cdot t'^2 - r^2 \end{aligned}$$

**Particular case.** We will apply the Vectorial Transformations of Lorentz (VLT) to the original configuration of the well-known Transformations of Lorentz (LT) in which the line inclined on which the movable system moves coincides with X-axis, that is to say, it forms angles,  $\alpha = \beta = 0$ , see Fig. 1, and in this way to obtain the correct transformations, without assumptions. The general equations of the VLT with the coincident origins at the beginning of the measurements (3) and (4), repeated here, are:

$$\begin{aligned} x' &= k \cdot (x - v \cdot t \cdot \cos \alpha \cdot \cos \beta), \quad y' = k \cdot (y - v \cdot t \cdot \sin \alpha \cdot \cos \beta), \quad z' = k \cdot (z - v \cdot t \cdot \sin \beta), \quad \text{con } k = \left( 1 - v^2 / c^2 \right)^{\frac{1}{2}} \\ t'^2 &= k^2 \cdot \left[ \left( t \cdot \cos \alpha \cdot \cos \beta - \frac{v}{c^2} \cdot x \right)^2 + \left( t \cdot \sin \alpha \cdot \cos \beta - \frac{v}{c^2} \cdot y \right)^2 + \left( t \cdot \sin \beta - \frac{v}{c^2} \cdot z \right)^2 \right]. \end{aligned}$$

Applying them for  $\alpha = \beta = 0$ , in order to repeat the rigid configuration of the original LT, as we will see it give transformations for the LT, different from the known ones (by more than one hundred years):

$$x' = \frac{x - v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad y' = \frac{y}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad z' = \frac{z}{\sqrt{1 - \frac{v^2}{c^2}}} \quad t' = \sqrt{\frac{\left( t - \frac{v}{c^2} \cdot x \right)^2 + \left( -\frac{v}{c^2} \cdot y \right)^2 + \left( -\frac{v}{c^2} \cdot z \right)^2}{1 - \frac{v^2}{c^2}}} \tag{17}$$

As it is demonstrated in (17), the referred assumptions of the LT,  $y' = y$ ,  $z' = z$  are not true. As a control of which the new transformations obtained in (17) are correct even for projectiles whose speed is smaller than that one of the light, the demonstration of the invariance of the spacetime interval,  $s$ , cause of which these are valid transformations, occurs next:

$$s^2 = r'^2 - c^2 \cdot t'^2 = \frac{(x - v \cdot t)^2 + y^2 + z^2}{1 - \frac{v^2}{c^2}} - c^2 \cdot \frac{\left( t - \frac{v}{c^2} \cdot x \right)^2 + \left( \frac{v}{c^2} \cdot y \right)^2 + \left( \frac{v}{c^2} \cdot z \right)^2}{1 - \frac{v^2}{c^2}} =$$



$$s^2 = r'^2 - c^2 \cdot t'^2 = \frac{x^2 + y^2 + z^2 - 2 \cdot v \cdot t \cdot x + v^2 \cdot t^2 - c^2 \cdot t^2 + 2 \cdot v \cdot t \cdot x - \frac{v^2}{c^2} \cdot (x^2 + y^2 + z^2)}{1 - \frac{v^2}{c^2}} =$$

$$s^2 = r'^2 - c^2 \cdot t'^2 = \frac{(x^2 + y^2 + z^2) \left[ 1 - \frac{v^2}{c^2} \right] - \left[ 1 - \frac{v^2}{c^2} \right] c^2 \cdot t^2}{1 - \frac{v^2}{c^2}} = r^2 - c^2 \cdot t^2$$

Operating on the VLT of the speeds distinguished to the configuration of the original LT, we obtain:

$$u'_x = \frac{u_x - v}{\sqrt{\left(1 - \frac{v \cdot u_x}{c^2}\right)^2 + \left(\frac{v \cdot u_y}{c^2}\right)^2 + \left(\frac{v \cdot u_z}{c^2}\right)^2}} \quad u'_y = \frac{u_y}{\sqrt{\left(1 - \frac{v \cdot u_x}{c^2}\right)^2 + \left(\frac{v \cdot u_y}{c^2}\right)^2 + \left(\frac{v \cdot u_z}{c^2}\right)^2}}$$

$$u'_z = \frac{u_z}{\sqrt{\left(1 - \frac{v \cdot u_x}{c^2}\right)^2 + \left(\frac{v \cdot u_y}{c^2}\right)^2 + \left(\frac{v \cdot u_z}{c^2}\right)^2}}$$

It is also ratified, as expected, that if the projectile is a light photon also both observers will have to measure the same speed of the light,  $u'^2_x + u'^2_y + u'^2_z = u^2_x + u^2_y + u^2_z = c^2$ . In fact:

$$u'^2_x + u'^2_y + u'^2_z = \frac{(u_x - v)^2 + u^2_y + u^2_z}{\left(1 - \frac{v \cdot u_x}{c^2}\right)^2 + \left(\frac{v \cdot u_y}{c^2}\right)^2 + \left(\frac{v \cdot u_z}{c^2}\right)^2} = \frac{u^2_x - 2 \cdot v \cdot u_x + v^2 + u^2_y + u^2_z}{1 - \frac{2 \cdot v \cdot u_x}{c^2} + \frac{v^2 \cdot u^2_x}{c^4} + \frac{v^2 \cdot u^2_y}{c^4} + \frac{v^2 \cdot u^2_z}{c^4}} =$$

$$= \frac{(u^2_x + u^2_y + u^2_z) - 2 \cdot v \cdot u_x + v^2}{1 - \frac{2 \cdot v \cdot u_x}{c^2} + \frac{v^2}{c^2} \cdot (u^2_x + u^2_y + u^2_z)} = \frac{c^2 - 2 \cdot v \cdot u_x + v^2}{1 - \frac{2 \cdot v \cdot u_x}{c^2} + \frac{v^2}{c^2}} = \frac{c^2 \cdot \left(1 - \frac{2 \cdot v \cdot u_x}{c^2} + \frac{v^2}{c^2}\right)}{1 - \frac{2 \cdot v \cdot u_x}{c^2} + \frac{v^2}{c^2}} = c^2$$

**3) Generalization of the Vectorial Lorentz Transformations.** Both configurations studied in **1)** and **2)** are restricted to that the origins of the systems fixed and movable must coincide at the time of initiating the measurements. That is to say, the exposition invokes that an inertial system movable O' moves to constant speed  $v$  with respect to the fixed system O. For observers in both systems a projectile is shot at the moment  $t_0 = t'_0 = 0$ , when both origins agree in  $x_0 = x'_0 = 0$ ,  $y_0 = y'_0 = 0$  and  $z_0 = z'_0 = 0$  and the projectile reaches a point  $P(x, y, z)$  after a time interval  $t$ .

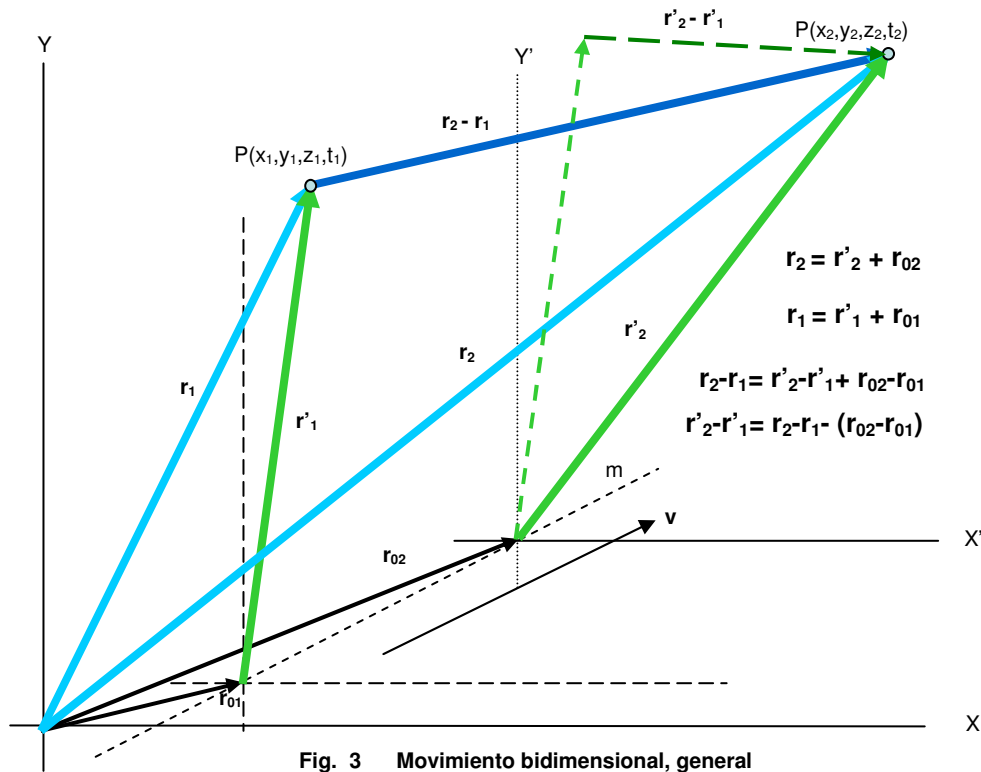


Fig. 3 Movimiento bidimensional, general

Thus, the idea is to release the restriction to that a projectile can be shot from any point  $P_1(x_1, y_1, z_1, t_1)$  until it reaches the point  $P_2(x_2, y_2, z_2, t_2)$  and that the movable system moves to speed  $v$  on any straight line,  $m$ , not coincident with any axis. We will make use of the two-dimensional vectorial diagram of Fig. 3, which does not diminish the generality of the vectorial representation and simplifies the visualization.

From Fig. 3 we observed that the vectorial Galilean Transformation between the distances comes given by:  $\mathbf{r}_2' - \mathbf{r}_1' = \mathbf{r}_2 - \mathbf{r}_1 - (\mathbf{r}_{02} - \mathbf{r}_{01}) \Rightarrow \Delta \mathbf{r}' = \Delta \mathbf{r} - \Delta \mathbf{r}_0$ . From here, according to the procedure that previously we have followed, we obtain the Vectorial Lorentz Transformations referred to the vector displacements. That is to say:

$$\mathbf{r}_2' - \mathbf{r}_1' = k[\mathbf{r}_2 - \mathbf{r}_1 - (\mathbf{r}_{02} - \mathbf{r}_{01})] \Rightarrow \Delta \mathbf{r}' = k(\Delta \mathbf{r} - \Delta \mathbf{r}_0) \tag{18}$$

Where, they equally should meet,

$$\begin{aligned} \mathbf{r}_2' - \mathbf{r}_1' = \mathbf{c} \cdot (t_2' - t_1') &\Rightarrow \Delta \mathbf{r}' = \mathbf{c} \cdot \Delta t' & \Delta x'^2 + \Delta y'^2 + \Delta z'^2 &= c^2 \cdot \Delta t'^2 \\ \mathbf{r}_2 - \mathbf{r}_1 = \mathbf{c} \cdot (t_2 - t_1) &\Rightarrow \Delta \mathbf{r} = \mathbf{c} \cdot \Delta t & \Delta x^2 + \Delta y^2 + \Delta z^2 &= c^2 \cdot \Delta t^2 \end{aligned}$$

From the scalar expressions and following the same procedure, the time transformation is obtained:

$$\Delta t'^2 = k^2 \cdot [(\Delta t \cdot \cos \alpha \cdot \cos \beta - \frac{v}{c^2} \cdot \Delta x)^2 + (\Delta t \cdot \sin \alpha \cdot \cos \beta - \frac{v}{c^2} \cdot y)^2 + (\Delta t \cdot \sin \beta - \frac{v}{c^2} \cdot \Delta z)^2]$$

Multiplying by  $v^2$ , the corresponding vector expression is obtained, to complete the set of equations:

$$\begin{aligned}
 v^2 \Delta t'^2 &= k^2 \cdot [(\Delta t \cdot v \cdot \cos \alpha \cdot \cos \beta - \frac{v^2}{c^2} \cdot \Delta x)^2 + (\Delta t \cdot v \cdot \sin \alpha \cdot \cos \beta - \frac{v^2}{c^2} \cdot y)^2 + (\Delta t \cdot v \cdot \sin \beta - \frac{v^2}{c^2} \cdot \Delta z)^2] \\
 v^2 \Delta t'^2 &= k^2 \cdot [(\Delta t \cdot v_x - \frac{v^2}{c^2} \cdot \Delta x)^2 + (\Delta t \cdot v_y - \frac{v^2}{c^2} \cdot y)^2 + (\Delta t \cdot v_z - \frac{v^2}{c^2} \cdot \Delta z)^2] \\
 \Delta r_0'^2 &= k^2 \cdot [(\Delta x_0 - \frac{v^2}{c^2} \cdot \Delta x)^2 + (\Delta y_0 - \frac{v^2}{c^2} \cdot y)^2 + (\Delta z_0 - \frac{v^2}{c^2} \cdot \Delta z)^2] \\
 \Delta \mathbf{r}_0' &= k \cdot \left( \Delta \mathbf{r}_0 - \frac{v^2}{c^2} \cdot \Delta \mathbf{r} \right) \tag{19}
 \end{aligned}$$

By eliminating the restriction of the coincidence of the origins at the beginning of the measurements leads to the natural and intuitive form to compare relative movements, with independent measurements. On the other hand, the incremental presentation of the Vectorial Transformations of Lorentz allows to the analysis of displacements differentials. Indeed, the previous expressions applied to the displacement differential, which is excellent for its use in curvilinear trajectories, are expressed in the following way:

$$d\mathbf{r}' = k \cdot (d\mathbf{r} - d\mathbf{r}_0) \quad d\mathbf{r}_0' = k \cdot \left( d\mathbf{r}_0 - \frac{v^2}{c^2} \cdot d\mathbf{r} \right) \quad d\mathbf{r}' = \mathbf{c}' \cdot dt' \quad d\mathbf{r} = \mathbf{c} \cdot dt \tag{20}$$

And in scalar form:

$$\begin{aligned}
 dx' &= k \cdot (dx - dx_0) & dy' &= k \cdot (dy - dy_0) & dz' &= k \cdot (dz - dz_0) \\
 dx_0' &= k \cdot \left( dx_0 - \frac{v^2}{c^2} \cdot dx \right) & dy_0' &= k \cdot \left( dy_0 - \frac{v^2}{c^2} \cdot dy \right) & dz_0' &= k \cdot \left( dz_0 - \frac{v^2}{c^2} \cdot dz \right) \\
 dx_0 &= dt \cdot v \cdot \cos \alpha \cdot \cos \beta, & dy_0 &= dt \cdot v \cdot \sin \alpha \cdot \cos \beta, & dz_0 &= dt \cdot v \cdot \sin \beta, & dr_0^2 &= v^2 dt^2 = v_x^2 \cdot dt^2 + v_y^2 \cdot dt^2 + v_z^2 \cdot dt^2
 \end{aligned}$$

$$dt' = k \cdot dt \cdot \sqrt{\left( \cos \beta \cdot \cos \alpha - \frac{v}{c^2} \cdot \frac{dx}{dt} \right)^2 + \left( \cos \beta \cdot \sin \alpha - \frac{v}{c^2} \cdot \frac{dy}{dt} \right)^2 + \left( \sin \beta - \frac{v}{c^2} \cdot \frac{dz}{dt} \right)^2}$$

The transformation for the velocity of the projectile is given as:

$$\frac{d\mathbf{r}'}{dt'} = \mathbf{u} = \frac{(u_x - v \cdot \cos \alpha \cdot \cos \beta) \mathbf{i} + (u_y - v \cdot \sin \alpha \cdot \cos \beta) \mathbf{j} + (u_z - v \cdot \sin \beta) \mathbf{k}}{\sqrt{\left( \cos \beta \cdot \cos \alpha - \frac{v}{c^2} \cdot u_x \right)^2 + \left( \cos \beta \cdot \sin \alpha - \frac{v}{c^2} \cdot u_y \right)^2 + \left( \sin \beta - \frac{v}{c^2} \cdot u_z \right)^2}}$$

If the projectile is a photon, or a pulse of light then,

$$\left\{ \begin{aligned} dx'^2 + dy'^2 + dz'^2 &= c^2 \cdot dt'^2 \\ dx^2 + dy^2 + dz^2 &= c^2 \cdot dt^2 \end{aligned} \right\} \Rightarrow \left\{ \begin{aligned} c^2 \cdot dt'^2 - (dx'^2 + dy'^2 + dz'^2) &= 0 \\ c^2 \cdot dt^2 - (dx^2 + dy^2 + dz^2) &= 0 \end{aligned} \right\}$$

## II. REAL ANALYSIS OF THE VECTORIAL LORENTZ TRANSFORMATIONS

When the previous obtained expressions are applied to projectiles different from a ray of light with speeds smaller than  $c$ , then the last indicated expressions,  $r'^2 - c^2.t'^2$  y  $r^2 - c^2.t^2$ , are different from zero, but they take a same value without mattering who measures them ( $s' = s$ ), which it is known like the invariant interval of the space-time in the jerk of the Special Theory of Relativity (SRT):

$$\left\{ \begin{array}{l} c^2.dt'^2 - (dx'^2 + dy'^2 + dz'^2) = ds^2 \\ c^2.dt^2 - (dx^2 + dy^2 + dz^2) = ds^2 \end{array} \right\} \quad (21)$$

In addition, the speed of the projectile measured in both systems responds to the following relations:

$$\left\{ \begin{array}{l} u'^2.dt'^2 - (dx'^2 + dy'^2 + dz'^2) = 0 \\ u^2.dt^2 - (dx^2 + dy^2 + dz^2) = 0 \end{array} \right\}, \text{ Namely: } \left\{ \begin{array}{l} u'^2 = \frac{(dx'^2 + dy'^2 + dz'^2)}{dt'^2} \\ u^2 = \frac{(dx^2 + dy^2 + dz^2)}{dt^2} \end{array} \right\}, \text{ for } u'^2 \neq u^2 \quad (22)$$

And by all means, the same happens with the discreet displacements, as much for the ray of light like for any projectile that moves to a speed  $v$  smaller than  $c$ :

$$\left\{ \begin{array}{l} (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2 = c^2.\Delta t'^2 \\ (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 = c^2.\Delta t^2 \end{array} \right\} \quad (23)$$

$$\left\{ \begin{array}{l} c^2.\Delta t'^2 - [(x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2] = S_{12} \\ c^2.\Delta t^2 - [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2] = S_{12} \end{array} \right\} \quad (24)$$

The experience that we can take from this analysis of the comparison of inertial systems with relative movements is that:

- a) Indeed, an interdependence between the time and the space must exist so that the speed of light has a unique and constant value, no matter the speed of the reference system from which the speed of light is measured and no matter the speed of the device producing such light.
- b) That is to say, the observers who measure the speed of light do not need to put themselves in agreement to measure the physical characteristics of the light. They are totally independent in their measurements. Their reference can be a fixed point of a planet, or the spaceship in which it travels, in short, they can choose any fixed point with respect to their selves. They will always obtain the same value of the speed of light that others obtained.
- c) Nevertheless, if an observer measured the speed of the light with respect to a non-fixed point regarding his position, his measurement no longer would be the same constant value that it would obtain if he did it from and, as before, relative its fixed position. For example, it is the case of an observer who travels in a spaceship, and passes near the Sun, to which it supposes fixed in the space and he sets out to measure the speed of a sparkle of blue light that happens once in a while in a point of the surface of the star. When his spaceship is there, the sparkle is happening next to the point (perpendicular to the direction of the speed of the sparkle) and traveling at speed  $v$  parallel to the direction of

the ray of blue light the observer, during a time  $t$ , measures the whole range by the ray of blue light as if had emitted it, taking like reference his spaceship, and obtains the speed of light  $c$ . Nevertheless, if he measured the distance traveled by the ray from the point in the surface of the Sun (to which he left back), during  $t$  would be equal to  $(v+c)t$ , reason why the speed of the blue ray by this route would be greater than  $c$  and equal to  $v+c$ .

- d) In sum, the Vectorial Lorentz Transformations imply that the distances and times of an event anyone, measured by observers located in different inertial systems (with relative movement among them) must be done by each observer with respect to a fixed point of his reference system, in an independent way and without knowledge of the inertial movement of the other observer, or of the relative movement among them. If the measurements, of the speed of a light ray for example, do not take control rigorously of these references in the way before indicated, they will obtain measurements of speeds different from that of light.

### III. PROPER TIME, SPECIAL AND GENERAL RELATIVITY

The proper time is defined, within the Special Relativity Theory, like the time related at the interval  $ds$ . This interval, for an equal speed to the one of the light has a null value, but for speeds smaller than the one of the light it has variable magnitude. Nevertheless, for being equal for both observers it is called the invariant interval:

$$\left\{ \begin{array}{l} c^2 \cdot dt'^2 - (dx'^2 + dy'^2 + dz'^2) = ds'^2 \\ c^2 \cdot dt^2 - (dx^2 + dy^2 + dz^2) = ds^2 \end{array} \right\}; \quad ds' = ds \quad (25)$$

H. Minkowski [3] defines the invariant interval  $ds$ , within a four-dimensional space, like the product of the proper time multiplied by the speed of the light  $ds = c \cdot d\tau$ , namely,

$$\left\{ \begin{array}{l} c^2 \cdot dt'^2 - (dx'^2 + dy'^2 + dz'^2) = c^2 \cdot d\tau^2 \\ c^2 \cdot dt^2 - (dx^2 + dy^2 + dz^2) = c^2 \cdot d\tau^2 \end{array} \right\} \quad (26)$$

An introduction to the physical explanation of this definition is boarded by L.D. Landau and L.M. Lifshitz [4] with a particular example. The following exhibition summarizes the fundamental thing of such introduction: “a coordinate system rigidly joined to clocks is considered that move with respect to a fixed observer. The clocks constitute an inertial reference system. During an infinitesimal time interval  $dt$  (measured by a clock in the fixed reference system) the movable clocks move a distance  $\sqrt{dx^2 + dy^2 + dz^2}$ . While, in the coordinate system rigidly joined to the movable clocks, any displacement is not detected:  $dx' = dy' = dz' = 0$ . Due to the invariance of the interval, the following is fulfilled:

$$ds^2 = c^2 \cdot d\tau^2 = c^2 \cdot dt^2 - (dx^2 + dy^2 + dz^2) = c^2 dt'^2$$

Where, for being invariant the speed of light, so does the proper time  $d\tau$ :

$$d\tau = dt' = dt \sqrt{1 - \frac{dx^2 + dy^2 + dz^2}{c^2 dt^2}}$$

(27)

But,

$$\frac{dx^2 + dy^2 + dz^2}{dt^2} = v^2 \tag{28}$$

Where  $v$  is the speed of the movable clocks for this particular case; but  $d\tau$ , in his more complete definition, is referred to the time during which the four-dimensional interval is developed, which, we repeated, in this special case it is equal to the time measured by the movable clocks, and is given by the following expression:

$$d\tau = dt' = dt \sqrt{1 - \frac{v^2}{c^2}} \tag{29}$$

This relation is accepted, without better explanations, as much in the Special Theory of Relativity (SR) like in the General Relativity (GR) as the general expression in four dimensions of the proper time. This generalization, as it can be deduced rigorously, is not true.

The generalization of the expression-definition of the proper time, as well as it was inlaid within these theories is confused, even that it is an incorrect or incomplete definition, and it entails along with the referred assumption in the Lorentz transformations to incorrect transformations of the physical magnitudes, which leads to establish that the scope of application of SR is not general. And, obviously it is not so for both reasons. The proper time so defined is in general referred to a time that is not  $dt$  or  $dt'$ , namely, it can not be thought of any of the proper times of the reference systems. In this sense, in our opinion, it is a “vague” concept of time, or at least an incomplete or incorrect definition.

As well as we have managed to obtain the correct Lorentz transformations (Vectorial Transformations of Lorentz), we have observed that the vectorial concept of the time arose in a natural way as a vector depending on the familiar three-dimensional space components, giving consequently uniform transformations to all the components. In next section we will try to define the **local time** with a more logical and complete definition within a three-dimensional environment, “in contrast” to the four-dimensional one of the proper time, following a reasoning within the scope of that of the Vectorial Lorentz Transformations that we have been exposing and that lead, as we see, to the development of Vectorial Relativity (VR).

#### **IV. LOCAL LORENTZ TRANSFORMATIONS. LENGTH CONTRACTION AND TIME DILATION AND NEW DEFINITION OF RELATIVISTIC MASS**

Since we have already indicated, to obtain the Vectorial Lorentz Transformations between distances and times measured by observers located in different inertial systems with a relative movement between them, they will have to be realized by each observer with respect to, and from, a fixed point of their own reference system, independent of the other reference point of the other observer. It will imply that, besides the imperceptible relativistic differences, there will exist that introduced by the distance of separation between both reference points. Given the existence of a relative movement between them, the separation between the origins of both reference systems is an amount that is variable and equal to:  $r_0 = v \cdot \Delta t$ .

But, how results of measures of physical magnitudes in the reality must be compared? Generally, to compare measurements of physical characteristics it is tried to do them under the same conditions of measurement. For example, it is the case of an observer that travels in an airplane and a second observer that is fixed in a place of the Earth surface, and both observers wish to compare results of their measurements of the speed of the airplane. Obviously these measurements must be done with respect to a common Earth fixed reference point. The Earth observer does not have problems to measure the speed of the airplane, because he will measure the whole range by the airplane with respect to the origin O, where he is located in his reference system and the passed time in his clock. But the observer who is in the airplane will have to be ingenious to be able to measure the speed of the airplane with respect to a fixed point of the Earth surface, which can be the same datum point of the fixed system. Consider first the observer seated in his armchair in the airplane, traveling in uniform movement (constant rotation speed to a constant radius with respect to the center of mass of the Earth). For him any fixed point of the airplane, will be at rest. And a fixed point on Earth surface will really behave for him as a movable point. That is to say, for him what is moving is the fixed point taken as reference point for measurement in the terrestrial surface.

In order to solve how to measure the speed of the airplane, in magnitude and direction, and being able to compare it with the one of the fixed Earth observer, the observer who travels in the airplane decides to follow the following procedure to obtain his result: he will try to measure the speed with which the point O of the terrestrial surface moves away from the airplane, to which it will take as permanent reference point for his measurements.

For doing it he will determine the vector of position of the point O on Earth,  $\mathbf{R}'_0$ , with respect to its airplane. Obviously this vector of position  $\mathbf{R}'_0$  will have the same direction, but it will be of opposite sense to the vector of position of the airplane with respect to O, (observe that we did not talk about  $\mathbf{r}'_0$ , whose direction is different from that of  $\mathbf{r}_0$ , see fig 4), whose value will be defined as  $\mathbf{R}'_0 = k(-\mathbf{r}_0)$ . Then, he will measure the variation of the distance of the point O with respect to the airplane in the time, say the velocity  $\frac{d\mathbf{R}'_0}{dt'}$ . Obtained this measurement, he subtracts from the speed of the airplane with respect to him (which has a null value), and thus will obtain the magnitude and direction of the speed of its airplane with respect to the point of reference O (movable for him, but fixed for the other observer) on the terrestrial surface.

The previous procedure is the logical manner to compare speeds and displacements between both observers, which would allow to observe only relativistic differences, eliminating those that come from the relative movement (distance between the origins of the reference systems). The comparison of times between two observers of inertial systems with relative movement among them does not have apparent problems of accomplishment, because they are those that are read in the clocks located in each reference system. The same would happen with the comparison of measurements of magnitudes of masses.

To this method of comparison of physical magnitudes to find transformations having a single reference of measurement, we will denominate Local Transformations of Lorentz [1]. But, as we aimed before in the separate c of section II, the measurement of the speed of the light by both observers will not give a same result, nor will stay invariant either the tetra-dimensional interval, because since it has been seen they will be varying the original conditions of measurement of the VLT. Nevertheless, since a single datum point of measurement for both observers settles down, one first equality of conditions of measurement for lengths is being respected. By the results that we will

see in ahead it demonstrates that the local transformation thus defined continues being a valid transformation for the comparison of physical magnitudes.

**1)** In order systematize the analysis of the Local Transformations of Lorentz (LLT), from now on both observers will make their measurements, as already it has been said, taking as reference the same point. Let this point be the origin O of the fixed system. For example, if the fixed observer in Or measures a radio-vector  $\mathbf{R}$  (capital letters in LLT and small letters in VLT will be used), from O to a point P of a projectile sent to the space (the conditions for this measurement are equal to those of the VLT and  $\mathbf{R} = \mathbf{r}$ ), the observer located in the movable system will measure commented and defined, before equal similar a radio-vector  $\mathbf{R}'$ , between the same two points, whose magnitude would be, according to a:

$$\mathbf{R}' = \mathbf{r}' - \mathbf{R}'_0 = k(\mathbf{r} - \mathbf{r}_0) - k(-\mathbf{r}_0) = k.\mathbf{r} \Rightarrow \mathbf{R}' = k.\mathbf{r} \quad (30)$$

From here we obtain, consistently that  $\mathbf{r}'$  as much in LLT as in VLT it has the same value.

$$\mathbf{r}' = \mathbf{R}' + \mathbf{R}'_0 = k(\mathbf{r}) + k(-\mathbf{r}_0) = k(\mathbf{r}) - k(\mathbf{r}_0) = k(\mathbf{r} - \mathbf{r}_0)$$

Observe that  $\mathbf{r}' = k(\mathbf{r}) - k(\mathbf{r}_0)$ , for being a difference between two radio-vectors that leave from a common reference, it maintains equivalence with the originating vector of the VLT. Nevertheless,  $\mathbf{r}'_0$  is a vector that although it leaves from the origin O, it is not measured with respect to the fixed system O, but with respect to the movable system O', these differences can be seen in Fig. 4.

This result indicates to us that when two observers wit relative motion between them make measures from a common point, the distances or displacements are related as Local Transformations of Lorentz and take the expressions in referred in (30). If the distances measured in the movable system O', are considered like known, the quantification of the phenomenon of the contraction of lengths is obtained.

**2)** The following systematization or convention that finally defines the Local Transformations of Lorentz talks about to that any projectile will not go off. This way the displacement of the projectile will be null with respect to the movable system,  $\mathbf{r}' = 0 = k(\mathbf{r} - \mathbf{v}.t) = \mathbf{R}' + \mathbf{R}'_0 = k(\mathbf{r} - \mathbf{r}_0) \Rightarrow \mathbf{R}' = -\mathbf{R}'_0, \Rightarrow \mathbf{R}' = -\mathbf{R}'_0$  and length and direction will agree in with  $\mathbf{R} = \mathbf{r}$  and  $\mathbf{R}_0 = -\mathbf{v}.t$ . For this case of null displacement of the projectile, the previous equality is reduced to:

$$\mathbf{r} = \mathbf{v}.t \Rightarrow x = t.v.\cos\alpha.\cos\beta; y = t.v.\sin\alpha.\cos\beta; z = t.v.\sin\beta$$

Substituting these results in the original time transformation, equation (8):

$$t'^2 = k^2.[(t.\cos\alpha.\cos\beta - \frac{v}{c^2}.t.v.\cos\alpha.\cos\beta)^2 + (t.\sin\alpha.\cos\beta - \frac{v}{c^2}.t.v.\sin\alpha.\cos\beta)^2 + (t.\sin\beta - \frac{v}{c^2}.t.v.\sin\beta)^2]$$

We obtain the corresponding local transformation of time, similar to that indicated by L. D. Landau and L. M. Lifshitz [4].

$$t'^2 = k^2.t^2.\left(1 - \frac{v^2}{c^2}\right)[(\cos\alpha.\cos\beta)^2 + (t.\sin\alpha.\cos\beta)^2 + (t.\sin\beta)^2] = t^2.\sqrt{1 - \frac{v^2}{c^2}} \quad (31)$$



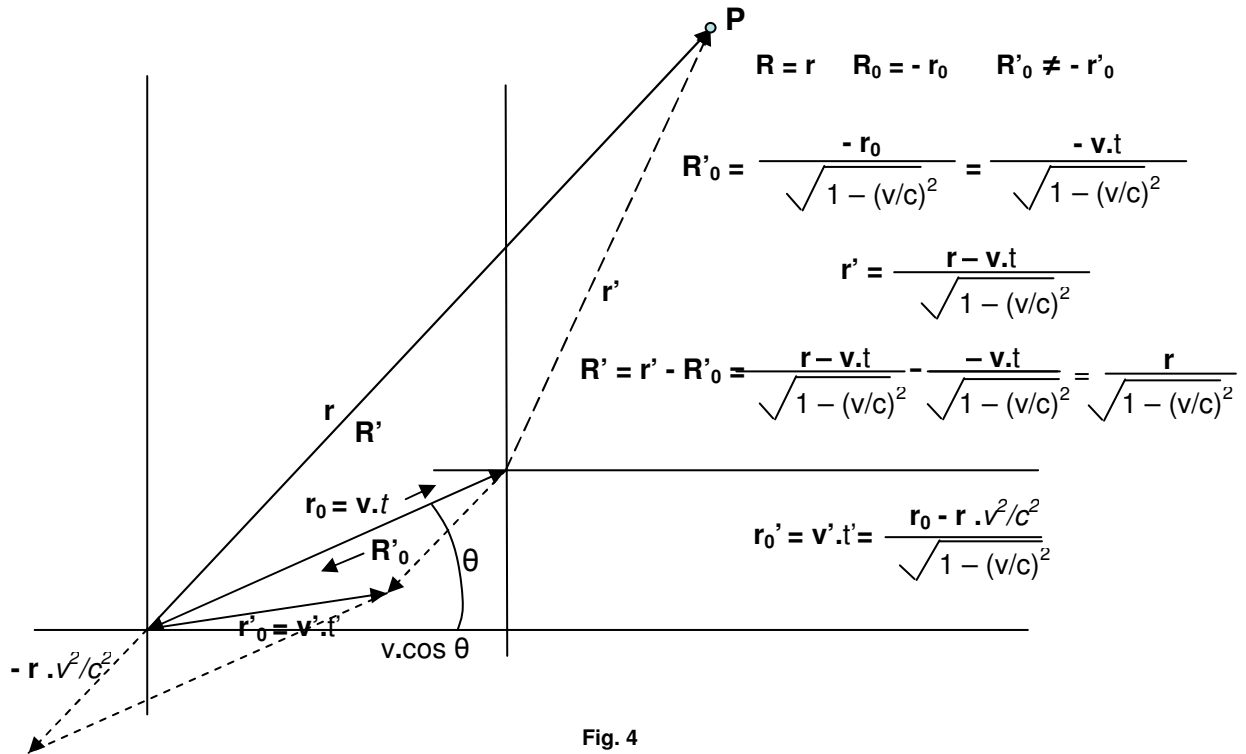


Fig. 4

3) The fact that in the LLT generally  $\mathbf{R}' = k.\mathbf{r}$  and  $\mathbf{R}'_0 = -k.\mathbf{r}_0$ , and that for this particular case,  $\mathbf{R}' = k.\mathbf{R} = -\mathbf{R}'_0 = -k.\mathbf{R}_0 = k.\mathbf{r}_0$  referred to the point origin  $O'$  of the movable system, which is a fixed point for an observer located in the movable system, we can establish that referring to any other **fixed** point  $P'(x', y')$  of the movable system  $O'$ , it is fulfilled that the vectors of position within the LLT, transform of equal way. That is to say, the vector of position of a point  $P'_1(x'_1, y'_1)$  located fixed in the movable system, measured from the point  $O$  by an observer of the fixed system, will be  $\mathbf{R}_1 = \mathbf{r}_1$ , and “the same” vector of position measured from the same point  $O$  by an observer of the movable system will be  $\mathbf{R}'_1 = k.\mathbf{r}_1$ . Also for another point  $P'_2(x'_2, y'_2)$ , it is met that  $\mathbf{R}_2 = \mathbf{r}_2$  and  $\mathbf{R}'_2 = k.\mathbf{r}_2$ , respectively. The distance, between those two point, becomes  $\Delta\mathbf{R}' = \mathbf{R}'_2 - \mathbf{R}'_1 = k(\mathbf{R}_2 - \mathbf{R}_1)$ . With the obtained results we can simplify the Local Transformations of Lorentz, for times and distances as follows:

$$d\mathbf{R}' = \frac{d\mathbf{R}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow dX' = \frac{dx}{\sqrt{1 - \frac{v^2}{c^2}}}; dY' = \frac{dy}{\sqrt{1 - \frac{v^2}{c^2}}}; dZ' = \frac{dz}{\sqrt{1 - \frac{v^2}{c^2}}}; dt' = dt.\sqrt{1 - \frac{v^2}{c^2}} \quad (32)$$

It can be seen that the Lorentz factors act uniformly in each component, expanding or contracting its shape.

If we consider the time measured in the movable system as  $t_0$ , the transformation of this time viewed by the observer located in the fixed system is:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (\text{Time dilation}) \quad (33)$$

With a similar reasoning, the Local Lorentz Transformation of a length  $L_0$  between any two fixed points located in the movable system,  $P'_1(x'_1, y'_1)$  and  $P'_2(x'_2, y'_2)$ , with any orientation, is the following one:

$$\mathbf{R}'_2 - \mathbf{R}'_1 = \frac{\mathbf{r}_2 - \mathbf{r}_1}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow L = L_0 \cdot \sqrt{1 - \frac{v^2}{c^2}} \quad (\text{Length contraction}) \quad (34)$$

Already in this stage of the presentation of the local transformation of Lorentz, it is relevant to emphasize kindness of its application, namely: we do not have to be worried about the coincidence of the location of events or its simultaneity to notice the dilation of the time or the contraction of lengths, anticipated by Fitzgerald and Lorentz. Nor we do not have either to worry about the version of magnitude, if it is cross-sectional or longitudinal because all the components of the magnitude are affected uniformly!

For example, the area  $A'$  of a fixed rectangle in the movable system of sides  $\ell'_1$  and  $\ell'_2$ , anyone their direction is, will have the following local transformation:

$$A' = A_0 = \ell'_1 \cdot \ell'_2 = \frac{\ell_1}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{\ell_2}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{A}{1 - \frac{v^2}{c^2}} \Rightarrow A_0 = \frac{A}{1 - \frac{v^2}{c^2}} \Rightarrow A = A_0 \cdot \left(1 - \frac{v^2}{c^2}\right) \quad (35)$$

And also a box of volume of  $V_0$ , of sides  $\ell'_1$ ,  $\ell'_2$  and  $\ell'_3$  fixed to the movable system:

$$V' = V_0 = \ell'_1 \cdot \ell'_2 \cdot \ell'_3 = \frac{\ell_1 \cdot \ell_2 \cdot \ell_3}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} = \frac{V}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow V_0 = \frac{V}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow V = V_0 \cdot \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}} \quad (36)$$

As it can be seen, the transformations that are obtained differ from those indicated in the SRT.

An excellent feature of the Local Transformations of Lorentz (LLT), is that the factor of Lorentz is characteristic itself for each physical magnitude and acts like a scaling factor without mattering if differential or integral is the magnitude.

For example, given that angle is a magnitude that comes from the quotient of two lengths, the arc and the radius, then we conclude that the is an invariant amount to the LLT:

$$\alpha' = \frac{s'}{R'} = \frac{\frac{s}{\sqrt{1 - \frac{v^2}{c^2}}}}{\frac{R}{\sqrt{1 - \frac{v^2}{c^2}}}} = \frac{s}{R} \Rightarrow \alpha' = \alpha; \quad d\alpha' = \frac{ds'}{R'} = \frac{\frac{ds}{\sqrt{1 - \frac{v^2}{c^2}}}}{\frac{R}{\sqrt{1 - \frac{v^2}{c^2}}}} = \frac{ds}{R} \Rightarrow d\alpha' = d\alpha \quad (37)$$

In this way, the angular velocity becomes of the following form:

$$\omega' = \frac{d\alpha'}{dt'} = \frac{d\alpha}{dt \cdot \sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{d\alpha}{dt}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \omega' = \frac{\omega}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (38)$$

**Local transformation of the Speed.** The same are obtained differentiating the displacement with respect to the time:

$$\mathbf{v}' = \frac{d\mathbf{R}'}{dt'} = \frac{\frac{d\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}}}{dt \cdot \sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{d\mathbf{r}}{dt}}{1 - \frac{v^2}{c^2}} \Rightarrow \mathbf{v}' = \frac{\mathbf{v}}{1 - \frac{v^2}{c^2}} \quad (39)$$

Observe that if the movable observer traveled at the speed of the light, and measured his speed with respect to a fixed point, in the terrestrial surface (not with respect to its origin  $O'$  because it is zero), he would obtain  $c' = \frac{c}{1 - \frac{v^2}{c^2}}$ , namely, the speed of light is not constant, the aspect previously indicated

in point c) of section II, consequence of the change of conditions already commented.

Similarly, the transformation for the acceleration is quickly obtained:

$$\mathbf{a}' = \frac{d\mathbf{v}'}{dt'} = \frac{\frac{d\mathbf{v}}{1 - \frac{v^2}{c^2}}}{dt \cdot \sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{d\mathbf{v}}{dt}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow \mathbf{a}' = \frac{\mathbf{a}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad (40)$$

**Local transformation of the Force and the Mass.** Since we do not know the transformation the mass, we must resort to a configuration in which we pruned to deduce some useful physical relations. We consider two masses  $m_1$  and  $m_2$  that rotate circularly, due to the gravitational attraction, around the center of mass of the  $CM$  system formed by both masses. They move such that the line joining the centers of mass of each one of the masses always pass through  $CM$ .

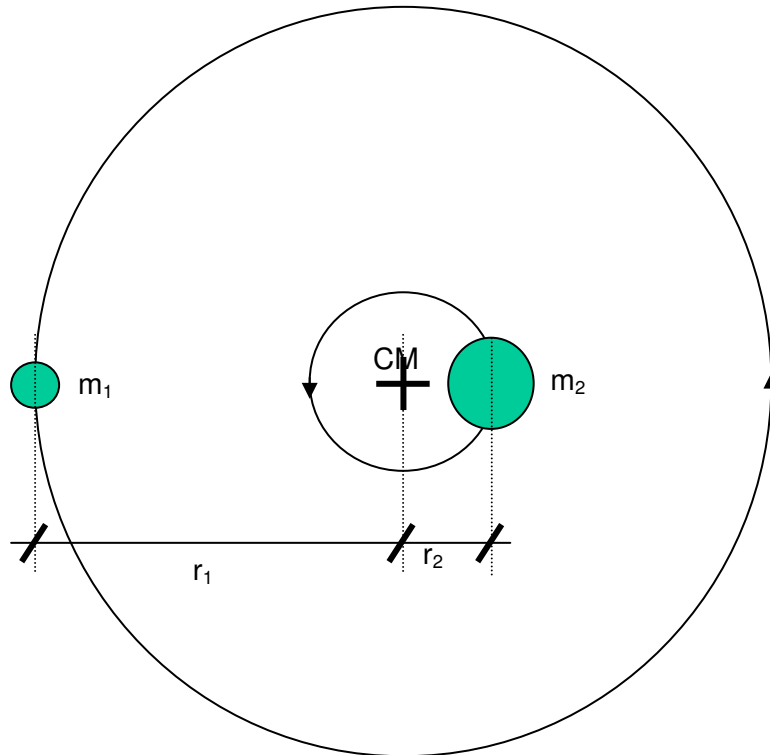
Thanks to the circular motion of the masses that we have imposed, it will allow us to only consider centrifugal forces in the analysis. Now we suppose a Hercules located in the  $CM$ , fixed, maintaining each mass through strong cords with each arm. With this configuration we will consider three observers, Hercules in the fixed and unique reference  $CM$ . First movable observer 1 on the mass  $m_1$  to distance  $r_1$ , and second movable observer 2, on the mass  $m_2$  to distance  $r_2$  of  $CM$ .

Considerations.

1. Because Hercules is in balance, he will measure equal and opposite tensions in each arm, produced by equal centrifugal forces. Thus, since  $T_1 = T_2 = T$ , then

$$m_1 \cdot \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 .$$

2. Because the centrifugal force, measured by observer 1 located in the mass  $m_1$ , equals the tension that exerts Hercules on the mass  $m_1$ , we will have as a result of the measurements done by observer 1 that  $m'_1 \cdot \omega'^2 \cdot r'_1 = T'_1$ .
3. Because the centrifugal force measured by observer 2 located in mass  $m_2$  is equal to the tension that exerts Hercules on the mass  $m_2$ , we will have as a result of the measurements done by the observer 2 that  $m''_2 \cdot \omega''^2 \cdot r''_2 = T''_2$ .



**Fig. 3.**

4. With Hercules in equilibrium,  $T'_1 = T''_2$ . Using the LLT of known distances (34) and that of the angular velocities (38), and the obtained thing in 2 and 3, we will obtain:

$$m'_1 \cdot \omega'^2 \cdot r'_1 = m'_1 \cdot \frac{\omega^2}{\left(1 - \frac{v_1^2}{c^2}\right)} \cdot \frac{r_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} \equiv m''_2 \cdot \omega''^2 \cdot r''_2 = m''_2 \cdot \frac{\omega^2}{\left(1 - \frac{v_2^2}{c^2}\right)} \cdot \frac{r_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} \quad (42)$$

The unique way so that this relation is always consistent for any value of  $v_1$  and  $v_2$  is that the masses have the following local transformation:

$$m'_1 = \left(1 - \frac{v_1^2}{c^2}\right)^{\frac{3}{2}} \cdot m_1 \quad \text{and} \quad m''_2 = \left(1 - \frac{v_2^2}{c^2}\right)^{\frac{3}{2}} \cdot m_2 \quad (43)$$

And so the factors of Lorentz are cancelled and it is arrived at valid consideration 1, from where we left. Thus, the LLT of the mass and the force (invariant) are obtained, for this particular case.

Since the masses  $m'_1$ ,  $m''_2$  are at rest with respect to each observer,  $m'_1 = m_{01}$  and  $m''_2 = m_{02}$ , it is found that the Lorentz local transformation of any mass, would come given by:

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \tag{44}$$

It is observed that this result is different from the used one for the relativistic mass in SR.

Moreover, making the analysis of the local transformation of the Force by a more general route, the same result is obtained. In fact, consider now an example similar to the previous one but without the restriction on the mass to move circularly at constant speed, but that it will have a generic ecliptic orbit, product of the gravitational attraction, with variable speed and acceleration. We will consider in addition, that there does not exist other gravitational forces acting, reason why the constancy of the Angular momentum will be preserved. We establish that the mass  $m_2$  in fig. 3, is the unique and “fixed” reference for measurements to be used to construct to the LLT and to which we will call  $M$ , Thus, the mass  $m_1$  will be the movable mass, to which we will denominate  $m$ . This way, the angular momentum of the movable mass  $m$  made by the fixed observer located at  $M$  is  $L = m.r^2.\omega$ , which must be a constant value because there are no more acting forces,  $m.r^2.\omega = Const1$ . Similarly, angular momentum measured by the observer located in the movable mass  $m$  is:  $L' = m'.r'^2.\omega'$ , which also must be a constant value, by same reason,  $m'.r'^2.\omega' = Const2$ . Replacing by its Lorentz Local transformation (LLT), in the known cases of radius (34) and angular velocity (38), we obtain:

$$L' = m'.r'^2.\omega' = m'.\frac{r^2}{\left(1 - \frac{v^2}{c^2}\right)}.\frac{\omega}{\sqrt{1 - \frac{v^2}{c^2}}} = Const2 \tag{45}$$

When focusing in this last expression and its constituent elements we noticed that the unique way so that the angular momentum is constant, for any value of the speed, present in the Lorentz factors in the denominators, is that the general transformation for the mass cancels out the effect of such factors and then becomes  $m.r^2.\omega = m'.r'^2.\omega'$ . That is to say, a LLT of the mass with the following shape meets such condition:

$$m' = m_0 = \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}.m \quad \Rightarrow \quad m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \tag{46}$$

This it is the same value before obtained in (44) for the local transformation of the mass in a particular case, namely, this result ratifies and generalizes this transformation. It means that the Angular momentum is also invariant under the LLT (the same as the Force).

Since the Lorentz factors influence the physical magnitudes uniformly, the LLT are unique for each magnitude, fact that contrasts with the differences between cross-sectional and longitudinal transformations of the mass, fields, etc. found in the Special Theory of Relativity.

Other local Transformations of Lorentz can be observed in the reference [1].

## V. NEW DEFINITION OF RELATIVISTIC ENERGY

It is important to mention that due to the new definition of mass given in the equation (46), also a new definition of the relativistic kinetic energy variation of a body with mass  $m$  that moves in curvilinear or rectilinear movement at a speed  $v < c$ , different from the one of general equation of Einstein ( $E = m.c^2$ ), emerges [6]:

$$K - K_0 = m.(2.v^2 - c^2) - m_0.(2.V_0^2 - c^2) \quad (47)$$

Being  $m_0$  the mass of the body at speed  $V_0$  which consistently reduces for  $v \ll c$  and  $m_0 \cong m$  to the known relationship:

$$K - K_0 \cong \frac{1}{2}.m.v^2 - \frac{1}{2}.m.V_0^2 \quad (48)$$

From equation (47) it is come off that, for a body like the light photon, with mass  $m$  when moves to the speed  $c$ , but with null mass at rest, its kinetic energy comes given, also consistently, by:

$$K = m.c^2 \quad (49)$$

According to this result the validity of Einstein's general equation of energy is restricted only to the case of the photon of light.

The kinetic energy of a body in movement, starting from the rest ( $V_0 = 0$ ) with rest mass  $M_0$ , will come given by

$$K = m.(2.v^2 - c^2) + M_0.c^2 \quad (50)$$

Reason why the new definition of total energy will be:

$$E = K + M_0.c^2 = m.(2.v^2 - c^2) + M_0.c^2 + M_0.c^2 = 2.M_0.c^2 - m.(c^2 - 2.v^2) \quad (51)$$

For  $v = 0 \Rightarrow m = M_0$ , this expression is reduced consistently to the internal energy,  $E_i = M_0.c^2$ .

## VI. RELATIVISTIC VERSION OF THE SCHRÖDINGER'S EQUATION

As we can observe, the relativistic Energy (51) given by Vectorial Relativity depends direct and explicitly of the speed of the particle, characteristic that also the classic expression of the energy has (basement of the Schrödinger's equation), opposite to the expression of the relativistic energy of Einstein which does not have that explicit dependency. This favorable aspect for the energy expression in Vectorial Relativity, as we will see, allows obtaining a relativistic version of the equation

of Schrödinger more complete than the one of Klein-Gordon and probably that the one of Dirac, because the expression (51) of the energy comes from the correct definition of the relativistic mass.

For example, under Vectorial Relativity kinetic energy of a particle that leaves from the rest ( $K_0 = 0$ ) is given by:

$$K = 2.m.v^2 - m.c^2 + M_0.c^2 = 2.\frac{p^2}{m} - c^2.(m - M_0) \tag{52}$$

Forming the total energy  $E$ , that includes the kinetic energy, the internal and the potential, in order to assure that the total energy preserves constant in any case (including those cases in which part of the energy it turns to any other class of energy or is interchange mass-energy), we will have:

$$E = K + M_0.c^2 + E_p = 2.m.v^2 - m.c^2 + 2.M_0.c^2 + E_p = 2.\frac{p^2}{m} - c^2.(m - 2.M_0) + E_p$$

$$E = 2.\frac{p^2}{m} - c^2.(m - 2.M_0) + E_p \tag{53}$$

This result recalls the classic Hamiltonian: Thus, we can redefine Hamiltonian the relativistic one under Vectorial Relativity like:

$$H_{relativistic} = 2.\frac{\mathbf{p}^2}{m} - c^2.(m - 2.M_0) + E_p(\mathbf{r}) \tag{54}$$

Replacing total energy  $E$  by  $i\hbar\frac{\partial}{\partial t}$  and momentum  $p$  by  $i\hbar\nabla$ , as it is required by the theory of Schrödinger, we obtain the relativistic version of the equation of Schrödinger, depending on time:

$$-\frac{2.\hbar^2}{m}.\frac{\partial^2\psi}{\partial x^2} + [E_p - c^2.(m - 2.M_0)]\psi = j.\hbar.\frac{\partial\psi}{\partial t} \tag{55}$$

A solution to this equation is obtained by means of a function of two separated variables:

$$\psi(w, t) = \xi(x).e^{-\frac{j.E.t}{\hbar}} ; \quad \text{where,} \quad \frac{\partial\psi}{\partial t} = -\frac{j.E}{\hbar}.\xi(x).e^{-\frac{j.E.t}{\hbar}} ; \quad \text{y} \quad \frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\xi}{\partial x^2}.e^{-\frac{j.E.t}{\hbar}} \tag{56}$$

If we replaced these results in (55) we will obtain:

$$-\frac{2.\hbar^2}{m}.\frac{d^2\xi}{dx^2} + [E_p - c^2.(m - 2.M_0)]\xi = E.\xi \tag{57}$$

This is the relativistic expression of the Schrödinger’s equation, independent of the time for a free particle.

### VII. NEW DEFINITION OF GRAVITATIONAL FIELD

In previous works [7] [8] [9] was obtained the expression of the relativistic field of gravitation, exerted by a mass considered fixed and constant  $M$  on other one  $m$ , considered movable and variable according to its tangential speed  $v$ , located one distances between its centers  $r$ , that is to say, the relativistic equivalent of the Gravitational field of Newton:

$$G_{planet} = \frac{2 \cdot \frac{GM}{r^2} \cdot \frac{v}{V_0} - \left( \frac{p_0}{p} - \frac{p}{p_0} \right) \cdot \frac{v \cdot dv}{dr}}{\left( \frac{p}{p_0} + \frac{p_0}{p} \right)}, \text{ which for the photon is reduced to: } G_{photon} = \frac{2 \cdot \frac{GM}{r^2}}{\left( \frac{p}{p_0} + \frac{p_0}{p} \right)} \quad (58)$$

$G$  = Universal Gravitational Constant  
 $p = m \cdot v$  = Momentum

### VIII. CALCULATION OF PLANET ORBITS UNDER VETORIAL RELATIVITY

The result (52) is introduced in the equation of motion of a planet [7] put under the influence of the field of gravitation of a massive body, and it is obtained:

$$\Rightarrow u^2 \cdot \left( \frac{d^2u}{d\theta^2} + u \right) = \frac{m^2}{L^2} \cdot G_{planet} = \frac{m^2}{L^2} \cdot \frac{2 \cdot G \cdot M}{r^2} \cdot \frac{v}{V_0} - v \cdot \frac{dv}{dr} \cdot \left( \frac{p_0}{p} - \frac{p}{p_0} \right) \cdot \frac{1}{\left( \frac{p}{p_0} + \frac{p_0}{p} \right)} \quad (59)$$

For:  $u = \frac{1}{r}$  and  $L = m \cdot r^2 \cdot \omega$ : Angular Momentum y  $\omega$  = Planet Angular Speed

The equation (59) is solved in approximate form using the following results and definitions of the Vectorial Theory of Relativity [7] [9].

(a) The new definition of relativistic mass [6],

$$m = \frac{m_0}{\left( 1 - \frac{v^2}{c^2} \right)^{\frac{3}{2}}} \quad (60)$$

(b) The new definition of Relativistic kinetic energy [2].

$$K - K_0 = \Delta K = m \cdot (2 \cdot v^2 - c^2) - m_0 \cdot (2 \cdot V_0^2 - c^2) \quad (61)$$

(c) The new definition of Tangential speed [3].

$$v^2 = V_0^2 \cdot \frac{m_0^2}{m^2} - 2 \cdot G \cdot M \cdot \left( \frac{1}{r_0} - \frac{1}{r} \right) \cdot \frac{m_0}{m} \quad (62)$$

The equation (approximated) for the planet Movement obtained is:

$$\frac{d^2u}{d\theta^2} + (1 - 6 \cdot \beta) \cdot u = \alpha \quad (63)$$

$$\text{for } \alpha = \frac{M_0^2}{L^2} \cdot \left( GM + 6 \cdot \frac{G^2 M^2}{c^2 \cdot r_0} \right) \text{ and } \beta = \left( \frac{M_0 \cdot GM}{Lc} \right)^2 \Rightarrow \frac{d^2u}{d\theta^2} + (1 - 6 \cdot \beta) \cdot u = \alpha \quad (64)$$

where:  $M_0$ : mass at rest of a planet;  $r_0$ : minimum distance between  $m$  and  $M$ .



The solution of the equation (63) is:

$$u = \frac{\alpha}{1-6.\beta} + \left( u_0 - \frac{\alpha}{1-6.\beta} \right) \cdot \cos(\sqrt{1-6.\beta}.\theta) \tag{65}$$

for,  $u = 1/r$  and  $\Delta = \sqrt{1-6.\beta}$  (66)

By defining  $h' = \frac{\alpha}{1-6.\beta}$ , we obtain the expression of angle:  $\theta = \frac{1}{\Delta} \cdot \arccos \left( \frac{\frac{1}{r} - h'}{\frac{1}{r_0} - h'} \right)$ , (67)

The radius based on the angle would be then:  $r = \frac{1/h'}{1 + \left( \frac{1}{r_0 \cdot h'} - 1 \right) \cdot \cos(\Delta\theta)}$  (68)

With these expressions the orbits of planets can be calculated.

**IX. PRECESSION OF PLANETS PERIHELION CALCULATION UNDER VECTORIAL RELATIVITY**

For a complete revolution of the radius  $r$  the swept total angle is  $\Delta\theta_c = 2\pi$ . So, angle becomes  $\theta_c = \frac{2\pi}{\Delta}$ . Thus the final expression of positive the approximate precession by revolution,  $\Pi$ , is:

$$\begin{aligned} \Pi = \theta_c - 2\pi &= \frac{2\pi}{\Delta} - 2\pi = 2\pi \left( \frac{1}{\sqrt{1-6\beta}} - 1 \right) \cong 2\pi(1+3\beta-1) = 6.\pi.\beta = 6.\pi \cdot \left( \frac{M_0 \cdot GM}{L.c} \right)^2 \\ \Rightarrow \Pi &\cong 6.\pi \cdot \left( \frac{M_0 \cdot GM}{L.c} \right)^2 \end{aligned} \tag{69}$$

The values of the Precession of perihelion of planets (taken from the reference [10]) were:

PLANET	VTR (arc sec)	GTR (arc sec)
MERCURY	43.139	42.9195
VENUS	8.665	8.6186
EARTH	3.8552	3.8345
MARS	1.358	1.3502
JUPITER	0.0624	0.0623
SATURN	0.0137	0.0137
URANUS	0.0024	0.0024
NEPTUNE	0.00077	0.0008
PLUTON	0.00042	0.0004

**Table 1. Precession of perihelion of planets by century given by the VTR and the GTR**

**X. CONCLUSION**

This work, just as previous ones, is conclusive in the demonstration of which the original sin (fault) of the Special Theory of Relativity (SRT) was not to have corrected the error contained in the assumption of the invariance of the cross sectional components to the movement. The complication that it introduced to the SRT prevented its healthy development: the incorrect transformations of physical magnitudes, among them the incorrect definition of the relativistic mass, which entails to a deficient definition of the Energy and the Momentum (and all those of the Dynamic Physics). Additionally, the incomplete conception of the proper time or local, finished complicating the Special Theory of Relativity, in our opinion. We think that the corrections made by Vectorial Relativity in the relativistic concepts will at that time allow to a better understanding of the Theory of Relativity due the three-dimensional approach, and a better understanding between the quantum and relativistic points of view, given that although both fields (theories) are interpretations different of physics, each one is not an excluding characteristic about the other, but both are characteristics of matter. Thus, each particle has a mass and has a wave depending on its speed. These are, definitely and separately general and natural features of matter. Given to the unifying character of Vectorial Relativity by the new definition of energy directly depending on the speed; the easy conception of a mass that varies with the speed; the simplicity of the reasoning within a three-dimensional environment; the uniformity as the speed within the factor of Lorentz contracts the metric measures, expands to the time and the mass, and the fields, in a so logical and specifically so particular manner, that this factor can be used as a dimensional control in calculations; the physical presentation of the formula with their originating variation of the dependency on the relative speed and the agreement of their predictions with experimental results, theoretically make of this an accessible one, much more simple theory and more general than the Special Theory (SR) for being applicable only to rectilinear movements (with dependency of the mass on velocity), and than the General Theory (GR) in which the mass does not vary with speed (!) for being its movement on geodesic.

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## REFERENCES

- [0] A Einstein. *Zur Elektrodynamik bewegter Körper*, Annalen der Physic **17**:891, 1905. English version prepared by John Walker. [On the Electrodynamics of Moving Bodies](#).
- [1] J A Franco R, [Vectorial Lorentz Transformations](#), 2006. EJTP 9 (2006) 35-64.
- [2] J G Quintero D and J A Franco R. [Time as a Vector and Vectorial Lorentz Transformations](#), November 16th 2006. JVR **1** (2006) 1 8-21.
- [3] H Minkowski [Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern](#)
- [4] L D Landau, E M Lifshitz. Vol. 2. *The Classical Theory of Fields*. Fourth edition, English translation (Russian 1973)
- [5] J G Quintero D and J A Franco R. [Local Lorentz Transformations and Vectorial Relativity](#) JVR **1** (2006) 22-32
- [6] J A Franco R. [Energy in Vectorial Relativity:  \$E \approx m.c^2\$](#) . JVR **1** (2006) 43-55.
- [7] J A Franco R. [First Solutions to Gravitation and Orbital Precession under Vectorial Relativity](#), March 16th 2008. JVR **3** (2008) 1 1-13.
- [8] J A Franco R. [Theoretical Result of Deflection of Light Under General Relativity Could Be Wrong](#), June 16 2008. JVR **3** (2008) 2 1-21.
- [9] J G Quintero D and J A Franco R. [Gravitational Forces in Vectorial Relativity](#). Published by JVR on March 16th 2007. JVR **2** (2007) 1 33-42.
- [10] E Valdebenito V: [Estimation of Planetary Orbits via Vectorial Relativity](#), September 16th 2008. JVR **3** (2008) 4 33-41.
- [11] J G Quintero D and J A Franco R. [Quantum Mechanics in Vectorial Relativity](#). Published by JVR on March 16th 2007. JVR **2** (2007) 1 14-21.