P1. Intrinsic Units
Mass-Energy Equivalence Relation

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Abstract

The equivalence of inertial frames as sites of one physical world implies that a system of intrinsic units of length, time, mass and charge should be defined in terms of basic constituent physical blocks that have the same identity in all inertial frames. It was shown in earlier papers that the consistency of the concept of time with motion in space requires time and distance to be of the same dimension, and thus measured by the same unit. The arising reduced system of units reveals that mass and energy are only different facets of one entity, and results in the well-known mass-energy equivalence formula as a natural consequence.

1.Introduction

Consider an arbitrary inertial frame $S \equiv OXYZ$. The coordinates system is assumed to be already calibrated using a given unit of length, say LS. The existence of the Cartesian system of coordinates *OXYZ* in S presupposes the geometry of the space is Euclidean [1]. The geometric distance between two points $A \in S$ and $B \in S$, or geometric length of a rod *AB* stationary in *S*, refers to the result L_{AB} obtained by laying the unit of length LS in S along the rod repeatedly from one end till reaching the other by multiples and fractions of LS. This can be done rationally (or assumed to be already done) and takes no time. In other words, *a coordinate system, and accordingly geometric distances in S are already given data*.

The unit of time, though arbitrary, is chosen as the duration, say “second”, between two consecutive ticks (or readings) of identical clocks that run at synchrony with each other. Contemplating in the latter statement, we may be astounded by the fact that we really have defined nothing concerns the world outside the clocks. In order that a unit of time, say a second, bears a meaning as far as motion in *S* is concerned, *it should be correlated to what can happen during “a second” in the world outside the clocks*, and more precisely, it should quantify the amount of the spatial displacement intrinsic to some *reference physical phenomenon*, such as the propagation of light from an arbitrary point in *S*. *A “second” must thus be corresponded with (and actually could be measured by) a certain distance traveled by light within (inside) S during a corresponding duration. Time measurements therefore must be reducible to specific types of spatial displacement’s measurements.*

2. Time Measurement by Spatial Displacements –Global Time

In the theory of universal space and time (UST) which will be presented in subsequent parts, the frame independent entity, namely, “the optical time”, or the duration of a light trip, is defined in terms of spatial displacements and can be measured by length units.

Global timing in an inertial frame S is set up by synchronization with an arbitrary observer $O \in S$ employing light's signals. *The concept of global time emerges through envisaging a "linear" correspondence between each instant of time T read by the timer O and the compound event: (the wave front of the pulse that was emitted from O at $T = T_0$ occupies at the instant T points at equal distances R from O).* Through this correspondence, time duration ΔT, is essentially measured by distance R, i.e.

$$\Delta T \equiv (T - T_0) = a\, R. \tag{2.1}$$

The proportionality constant defines a constant velocity c of light by $c = 1/a = R/\Delta T$. Because of the space is geometrically homogeneous (also physically, and in particular for the propagation of light), *time durations* defined in this way are independent of the master timer's position O. The homogeneity of time follows also from the homogeneity of space. The latter statements are valid in an almost empty space, The above correspondence is only one step short of synchronization, which is achieved by each S observer at $(R, \emptyset, \theta)$ taking note of his distance R from O and the instant of time T_0 at which the pulse emanated from O, and thus setting his timer at $T = T_0 + aR$ when he receives the pulse. By its way of construction, the global timing is unique up of course to an arbitrary choice of a time unit and of zeroing, i.e., up to a transformation of the form $T' = \alpha T + T'_0$, where $\alpha > 0$ and T'_0 are arbitrary numbers.

Time intervals in the past may be quantified by $\Delta T \equiv (T - T_0) = -a\, R$ which should be understood as follows: Light that is received by O at T_0 has already taken a duration $(-\Delta T) = aR$, where R is the distance of its source from O. Thus the instant of time in the past at which light had emanated from a source at a distance R from O and arrived at O at T_0 was

$$T = T_0 - aR. \tag{2.2}$$

The further the location of the event the deeper in the past it happened. All past events(R, T) that satisfy the latter relation are detected by the observer O at the same instant T_0. Out of the latter set, subsets which have the same R were simultaneous. If each event (R, T) of the set that satisfy (2.2) continues to happen then the same set of events will all be detected at any later instant $T'_0 > T_0$ by O. Simultaneous events, corresponding to an instant T, are detected by O at instants $T_R = T + aR$, that depend on R. The latter relation is true whether T refers to an instant in past, present, or future. Two events taking place at R_1 and R_2 and detectable by O at $T_{R_1}$and T_{R_2} are simultaneous if $T_{R_1} - aR_1 = T_{R_2} - aR_2$.

By the latter paragraph, it is justified to imagine a system of synchronized timers that supplements any system of coordinates in S. This means that, in the same way we envisage rationally the assignment of a triplet $(R, \emptyset, \theta)$ to each point B in S, we can also imagine that a timer can be placed at each point $B \in S$ which is synchronized with $O \in S$ and runs uniformly at the same rate as the master timer, and accordingly at synchrony with all other timers. Thus a *global timing* in S can be practically established, with the notion of an "instant T_0" has a global meaning in S, in the sense that if an event takes place at $B_0(R_0, \emptyset_0, \theta_0)$ at T_0 then it will be detected at $B(R, \emptyset, \theta)$ through a light signal emanating from B_0 and arriving at B at the instant $T = T_0 + a|\boldsymbol{R}_0 - \boldsymbol{R}|$. Thus every S observer B assigns to the event of light's emission the same instant $T_0 = T - ar$, where r is his spatial separation from B_0 and T is the time read at the clock B when light is received. The concept of time arrow (past, present, and future) has therefore a global meaning in S, and any two or more S observers have the same temporal ordering of all events monitored by them. In particular, the notions of simultaneity and non-simultaneity are well-defined global concepts in S.

An inertial frame S endowed with a *global time as described above is said to be timed* (*or synchronous*). As far as one observer O is concerned, no synchronization in the real sense needs to be done in order an inertial frame becomes timed. In fact, the existence of a coordinate system as well as a timer at O is sufficient to determine the duration of any light's trip with given ends, whether O was one of the ends or not, provided the spatial coordinates (or radial distance) of each event is messaged instantly to O by a light signal. Also, the duration Δt of any event at any point in S, say $B \in S$, is observed form O to have the same duration. The arbitrary observer O, if he wishes, can replace his timer by a universal one [2], since his own timer should be equivalent to the universal one. The latter legitimate replacement is applicable to any observer, and hence a universal timer will do for all observers in a coordinated frame S with finite extension.

In Newton's mechanics, time and geometric length are independent absolute entities [3], with distance is measured by a calibrated ruler, and Newton's global time was assumed to be readable at each point of space. The synchrony of all point-wise timers was partially circumvented through appealing to a *universal timer* [2] formed by the fixed stars in the firmament, which generalizes the approximately uniform global time set up in the region from which almost all our observations are conducted, namely the earth surface.

3. The Reduced System of Units

Since time durations have to be defined in terms of spatial displacements, geometric lengths and time durations must have the same dimension; both are measured by the same unit. If the unit of time TS in S is defined as the duration taken by light to cross the unit of distance LS (a given rod stationary in S) from one end to another, say $LS = 1meter\ (1m)$, we may designate the unit of time also by "*meter*", to mean the time required by a light's signal to cross this distance.

In terms of a system of units of time, length, and mass

$$RSUI \equiv \{TS = LS = m, MS = kg\}, \quad (3.1)$$

the dimensions of some mechanical observables are listed in the table:

$$[velocity] = LS.TS^{-1} = 1\ \ (dimensionless)$$

$[a$

$$[momentum] = kg,\ [force] = kg.m^{-1}$$

$$[energy] = kg.LS^2.TS^{-2} = kg = [mass]$$

$$[angular\ momentum] = m.kg,\ [torque] = \ kg.$$

The velocity $\boldsymbol{v} = \Delta\boldsymbol{R}/\Delta t$ in *RSUI* is a dimensionless 3-vector, and the speed of light in vacuum is 1 regardless of the chosen unit of length LS, provided we choose $TS = LS$. Mass and energy have the same unit, "*kilogram*". In practical applications however, it is convenient to take $LS = 1\,meter,$ and adopt a multiple of the unit $TS = 1\,meter$, namely, "$second$". The latter is defined by the period taken by light to travel a distance of $c\,meters = 3 \times 10^8 m$, so that, $1\,second = c\,meters$. In the reduced system of units

$$RSUII \equiv \{LS = m, second \equiv c.m, kg\}, \qquad (3.2)$$
$$[velocity] = m.(c.m)^{-1} = \mathrm{c}^{-1},$$
$$[acceleration] = (c\,sec)^{-1}, [momentum] = \mathrm{c}^{-1}kg,$$
$$[force] = kg.m.sec^{-2} = kg.(c\,\mathrm{sec})^{-1} \equiv Newton,$$
$$[energy] = [work] = kg.m^2.sec^{-2} = kg.c^{-2} \equiv Joule,$$
$$[angular\ momentum] = c^{-1}\,m.kg = Joule.sec = [action]$$
$$[torque] = c^{-2}\,kg \equiv Joule.$$

The reduced systems of units, I or II, suggest that observables which are measurable by the same unit are of the same nature, although they may manifest themselves in different facets. Mass and energy for instance are both scalar quantities and both are measurable in *RSUI* by kg. This means that 1 kg of mass is equal to 1 kg of energy, and that, under suitable circumstances either quantity may be transformed to the other. In the *RSUII*, $1kg = c^2(c^{-2}kg) = c^2 Joule$, and

$$m(kg) = mc^2 Joule.$$

The latter relation holds for any type of energy.

If a vector observable $\boldsymbol{A}$ and a scalar observable B are of the same dimension then their squares, A^2 and B^2, are of the same nature and in principle are transformable to each other.

It is noted that the reduced system of units I and II are the system *MKS* with the unit of time is taken as the unit of length itself in *RSUI*, and defined in terms of the unit of length, with $sec = c.m$ for *RSUII*. Symbolically, $RSUI \equiv MKM\ and\ RSUII \equiv MKcM$.

4. Remarks

1. Clocks. The common perception of a "second" as the duration between two consecutive ticks of a clock becomes identical to our previous conception only if we demand that light travels rectilinearly c meters in S during a second. The latter definition conditions the clocks to conform to the reference criterion, which is light propagation. We have thus on one hand the measuring instruments, "the clocks" that can be synchronized and distributed (hypothetically) everywhere in the space, and on the other hand the happenings in the outside world, namely the distance that light travels during the period read by the clocks. Note that the clocks which are indeed indispensable for measuring time *are no more than instruments that can be manipulated to conform to the reference criterion defining time, and their performance should certainly be rejected when they do not.* A clock measuring time at one point is in accord with the set criteria if the periods they read agree with distance covered by a return light's trip from the clock's location to a reflector at rest in S and back to the clock location.

2. **Geometric Time**: Employing synchronized clocks and a light signal, distance between two points in a timed inertial frame can be measured, and it is certainly equal to the *geometric distance* as measured by meters (or seconds) where no light signals and clocks are involved. The geometric distance between two points in S may obviously be measured by the time period taken by a light signal to go from one point to another; it is called the *geometric time* between the two points.

Being proportional, the geometric distance and geometric time pertaining to two points in S may be looked at as being the same, and both may be measured by the same unit, second or meter. The latter statements hold good only when the two points in consideration are at rest in S.

3. **Optical Time**: The *geometric distance* between a source of light b that is moving in the timed inertial frame S and an observer $O \in S,$ at an instant t_0, is the distance between the point $B \in S$ occupied by b at t_0 and the observer O,

$$d(b, O \in S\ at\ t_0) = d(B \in S, O \in S) = R\ (meters) = (R/c)\ (seconds) \equiv T\ seconds.$$

If the moving source of light b emits when at $B \in S$ a pulse of light that arrives at $O \in S$ after t seconds, then its *time* distance, from O at the instant of emission t_0 is

$$d(b\ at\ B, O \in S) = t\ seconds.$$

The optical distance becomes identical to the geometric distance in the inertial frame in which the source of light and the observer are both stationary; otherwise (as we shall see) the two entities are different.

References

[1] Eisenhart Luther Pfahler, *Riemannian Geometry*, Princeton University Press (1964).

[2] C. P. Viazminsky and P. Vizminiska, The Scaling Theory - XII: Universal Timer - The Zeroth Law of Motion, February 12, 2010

[3] Adams Steve, *Relativity, An Introduction to Space-Time Physics*, Taylor and Francis Ltd, London, (1997).