the general science journal

Comparison of two translations of Einstein's relativity with commentary

Roger J Anderton
R.J.Anderton@btinternet.com

Comparison of two different translations of Einstein's 1905 paper on Relativity reveals at least two different theories where even the maths is different. My proposal is that Einstein's relativity has not been properly understood because of a bad translation and that a proper understanding of would be that is Boscovich's theory.

After the Copernican revolution Galilo introduced the concept of relativity, between him and 1905-1919 when Einstein became famous for relativity, there were many people writing about relativity who have been mostly forgotten (with a few exceptions: Poincare, Lorentz), of those Boscovich is very important. A proper appreciation of relativity theory needs to come from a historical perspective of its' development, which sadly has not happened hence relativity has been misunderstood by most people in the physics community.

When I look at the original German in which the paper was written I find there are problems with how it has been translated into English.

I have now fortunately got two English translations of the paper to compare, and I find that two totally different theories are being presented as special relativity, even the mathematics is different. It depends on the translator as to whether one goes by theory#1 or theory#2 as special relativity.

It is the aim here to propose that what I will label as theory#2 is the correct version of relativity and that is Boscovich's theory. While theory#1 is the common misunderstanding of relativity. (Of course, one could have misunderstood Einstein's paper and come up with other theories, but for sake of ease of reading will deal with it being just two theories.)

Now dealing with two translations of Einstein's Zur Elektrodynamik bewegter Körper, in Annalen der Physik. 17:891, 1905.

First translation is: Einstein's On the Electrodynamics of Moving Bodies which appeared in the book The Principle of Relativity, published in 1923 by Methuen and Company, Ltd. of London. Most of the papers in that collection are English translations by W. Perrett and G.B. Jeffery from the German Das Relativatsprinzip, 4th ed., published by in 1922 by Tuebner. [1] -– will denote OEM

Second translation is: The electrodynamics of objects in motion, translated by A.F.Kracklauer in book Einstein in English vol. I [2] - will denote AFK.

This paper deals with full comparison of OEM and AFK, while a previous paper dealt only with some. [8]

(1)

What OEM translates as: "It is known that Maxwell's electrodynamics—as usually understood at the present time —when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena."

While AFK translates as: "That Maxwell's electrodynamics— in the form currently cultivated — in the application to moving bodies leads to asymmetries seemingly not in accord with observed phenomena, is well known."

Me: Maxwell's electrodynamics as "usually understood.." VERSUS "currently cultivated"- raises issues as "usually understood" – can that understanding change later; and as "currently cultivated" suggests that cultivation might change.

(2)

OEM: Take, for example, the reciprocal electrodynamic action of a magnet and a conductor.

AFK: Recall the interaction between, for example, a magnet and conductor.

Me: “electrodynamic action” VERSUS no mention of it being “electrodynamic”- I think AFK should have leeway as meaning “electrodynamic”, so both OEM and AFK essentially mean the same.

Both OEM and AFK are often saying essentially the same thing but in different ways; it is not such things as that which I have issue with, rather it is when different things are really being said.

(3)

OEM says: “The observable phenomenon here depends only on the relative motion of the conductor and the magnet whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion.”

AFK says: “The observed phenomenon here depend only on the relative motion of magnet and conductor, while in the usual description, they are distinctly different depending on which one is in motion.”

Me: The issue I have here is that both translations seem confused when dealing with “relative motion”. The phrase “relative motion” is being used without explaining it, presumably when talks of “either the one or the other of these bodies is in motion” - that “motion” is with respect to observer, and observer considers “relative motion” between conductor and magnet.

Insufficient detail is being given in Einstein’s paper about issue of absolute motion versus relative motion. Relative motion and absolute motion in context of Boscovich’s theory is dealt with in talk by Dragoslav Stoiljkovic. [9]

(4)

OEM: “For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated.”

AFK: “If the magnet is in motion while the conductor is stationary then an electric field with a particular energy density is engendered in the vicinity of the magnet, and that, in turn, generates a current in the conductor.”

Me: OEM uses term “energy” while AFK uses term “energy density”; I am just going to excuse this as leeway.

(5)

OEM: “But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. In the conductor however, we find an electromotive force, to which in itself there is no corresponding energy, but which gives rise—assuming equality of relative motion in the two cases discussed—to electric currents of the same path and intensity as those produced by the electric forces in the former case.”

AFK: “But if the magnet is stationary while the conductor moves, then there is no electric field in the vicinity of the magnet, rather there is a force, which in itself corresponds to no energy, but which however, in view of the unity of the two situations, leads to an identical electric current in the conductor.”

Me: OEM says, “electromotive force”, while AFK just says “force”, but what is worrying is the claim of “no corresponding energy” or “no energy” with that force. It needs clarification, but the text does not give it.

(6)

OEM: “Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the “light medium,” suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest.”

AFK: “Similar examples, together with the unsuccessful attempts to discover any motion of the earth relative the “light medium,” lead to the following two propositions. One, that there are no characteristics of natural phenomena corresponding to the notion of absolute motion, not only in mechanics but also in electrodynamics; …”

Me: It is interesting that both use term “light medium”, presumably the same as what is usually called “aether”. It is worrying that one translation mentions “absolute rest” and the other mentions “absolute motion”. I would not classify both those concepts as being the same and think maybe they are getting confused.

(7)

OEM: “They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid just as has been for all frames of reference for which the equations of mechanics hold good.”

Then gives footnote: “The preceding memoir by Lorentz was not at this time known to the author.”

AFK: “Two that in all coordinate systems in which the equations of mechanics are valid, the equations of electrodynamics and optics are also valid just as has been observed, more or less.”

With no footnote.

Me: Putting aside the issue of footnote VERSUS no footnote, we have “hold good” VERSUS “more or less.”

Up to this point we have had problems with understanding what is being said, but now we have mathematical difference, where one translation has it that the equations are EXACT versus they are APPROXIMATE.

Ideally a theory should not be constructed on so much ambiguity. One would hope that as one reads further on in the text that ambiguities are removed, but instead what we have so far is just example of what is to follow, namely – no clarifications.

(8)

OEM: “We will raise this conjecture (the purport of which will hereafter be called the “Principle of Relativity”) to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body.”

AFK: “We will shall take these suppositions, which will be designated below as the “Principle of Relativity”) together with the seemingly incompatible notion, that light in empty space always propagates with a particular velocity c independent of its source, as hypothetical input for a new theory.”

n.b. AFK uses V not c; I have amended above so both translations have as c.

Me: One translation says “conjecture”, the other says “suppositions”; these are not the same thing as far as I am concerned.

As per dictionary [3]: supposition is - A belief held without proof or certain knowledge; an assumption or hypothesis.

While conjecture [4] is: a guess about something based on how it seems and not on proof.

Taking what was meant as “guess”, the difference is that one “guess” is not necessarily based on what it “seems” while the other guess is based on what it “seems”. In other words- belief for principle of relativity (and later mention lightspeed constancy) is based on a “guess” but may or may not be based what it “seems” from experiments. i.e. the difference being that “guess” may or may not be based on empirical evidence. If not going by empirical evidence and just guessing based on believing whatever one likes, then is not empirical science. The confusion generated is thus whether Einstein’s SR is based on empirical evidence or just based on believing whatever one likes.

Then OEM uses word “postulate” which means [5]: postulate suggest or assume the existence, fact, or truth of (something) as a basis for reasoning, discussion, or belief.

So, OEM is using these “guesses” now as basis for reasoning, but AFK does not say that. So, have option of guesses used as basis for reasoning versus guesses not used as basis for reasoning.

Both translations admit that these two guesses don’t seem to fit together, OEM says, “apparently irreconcilable” and AFK says, “seemingly incompatible”.

OEM says: “…that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body.”

AFK says: “… that light in empty space always propagates with a particular velocity c independent of its source, as hypothetical input for a new theory.”

AFK makes out that it is a “new theory” while OEM does not say that. So, the confusion is whether one goes by existing theoretical framework (namely Newtonian physics) or not. I say it should be going by Newtonian framework because not given any clear information that it is otherwise. But OEM versus AFK translation highlights the problem that some people might think it “new” while others not think that.

Also, there is no mention of inertial frames up to this point; inertial frames are a concept that will become important later. But at this stage of mentioning lightspeed constancy, there is no clarifying that idea in the context of inertial frames. If one hopes that it will be clarified later then one hopes in vain. In my view lightspeed constancy is not constant with respect to all inertial frames, but the ambiguity so far presented here in the translations can easily lead many readers of Einstein's relativity astray.

(9)

OEM: "These two postulates suffice to for the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies."

AFK: "These assumptions suffice to arrive at a simple and consistent formulation of the electrodynamics of moving bodies on the basis of Maxwell's theory for stationary bodies."

Me: The difference here is that OEM is talking of a "theory" and AFK is talking of a "formulation"; so, is it just a "formulation" of Maxwell's theory, or is it a "theory"? In the case of "theory" is it the same theory as Maxwell's theory? Is SR just Maxwell's theory formulated in a certain way or is it supposed to be a new theory? Once again there is no clarification.

(10)

OEM: "The introduction of a "luminiferous ether" will prove to be superfluous inasmuch as the view here to be developed will not require an "absolutely stationary space" provided with special properties, nor assign a velocity -vector to a point of the empty space in which electromagnetic processes take place."

AFK: "The necessity of introducing an "aether" for light will turn out to be superfluous; in this new formulation both an "absolutely stationary space" as well as a velocity attached to a point of space are ill conceived notions."

Me: Both translations are deeming the aether will not be needed.

As per Einstein (1920) he admits this was a mistake [6]: "in 1905 I held the opinion, that one was forced to abandon the concept of aether in Physics altogether. This judgment, however, was too radical, as we shall see below, when considering general relativity."

Of course, his reasoning needs further investigation; but in 1920 he reverses his opinion from what he was stating in 1905. An issue I have is that many relativists still deem there is no aether; and hence seem to be going by what Einstein says in 1905 instead of what he says later.

And there are of course other issues, such as: What is supposed to be meant by “absolutely stationary space”.

OEM says: no to “absolutely stationary space” provided with special properties”, does that mean can have “absolutely stationary space” if it does not have special properties, or what?

While AFK says “absolutely stationary space” is an ill-conceived notion (also a velocity attached to a point of space is an ill-conceived notions)- which raises the issue can the notion still be conceived even if ill conceived?

Then what is “a velocity -vector to a point of the empty space in which electromagnetic processes take place” supposed to mean, and what is “a velocity attached to a point of space” supposed to mean. Surely, it is supposed to be about coordinate systems and then when comparing different coordinate systems moving with respect to each other are then dealing with points of those coordinate systems moving. But of course, it is all not properly clarified.

(11)

OEM: “The theory to be developed is based—like all electrodynamics — on the kinematics of the rigid body, since the assertions of any such theory have to do with the relationships between rigid bodies (systems of co-ordinates), clocks, and electromagnetic processes. Insufficient consideration of this circumstance lies at the root of the difficulties which the electrodynamics of moving bodies at present encounters.”

AFK: “The new theory, as are all variants of electrodynamics, is based on the kinematics of rigid bodies as it too concerns the relationships among rigid bodies (co-ordinates systems), clocks, and electromagnetic processes. Insufficient attention to these very factors is the cause of the inadequacies that the theory of the electrodynamics of moving bodies exhibits nowadays.”

Me: Both translations seem to be saying much the same thing, and the conclusion they reach seem to be correct that not sufficient attention has been applied to electrodynamic theory as regard motion of bodies, and Einstein’s text seems to be doing a bad job of it as well.

Next Einstein wants to get onto the issue of Simultaneity, and we have:

(12)

OEM: Kinematical part

AFK: Part 1: Kinematics

(13)

OEM: 1. Definition of simultaneity

AFK: 1. The definition of simultaneity

(14)

OEM: “Let us take a system of co-ordinates in which the equations of Newtonian mechanics hold good.”

And has footnote: “i.e. to the first approximation.”

AFK: “Suppose we have a co-ordinate system in which Newton’s equations hold.”

And has no footnote.

Me: Thus, we have scenario of: Footnote VERSUS no footnote, which boils down to one translation saying equations of Newtonian mechanics are approximations and the other translation saying it isn’t.

That’s just different maths.

The footnote by OEM seems unjustified and is not in the original German paper. Thus, really the equations of Newtonian mechanics hold good without it being an approximation. What I think is lightspeed as constancy as being mere convention, thus is Boscovich’s theory.

As per Karl Svozil [7] points out constancy of lightspeed is a convention and says: “not too much consideration has been given to the possibility that experiments like the one of Michelson and Morley may be a kind of “self-fulfilling prophesy,” a circular, closed tautologic exercise. If the very instruments which should indicate a change in the velocity of light are themselves dilated, then any dilation effect will be effectively nullified. This possibility has already been imagined in the 18th century by Boskovich "

Svozil points out others had same idea in the context of the aether theory. (And as noted – Einstein changed his mind about aether.)

So, the correct version of SR is that it should be Boscovich's theory and any other theory that one can misread into Einstein's paper is not the correct theory.

Theory#1 that can be derived from Einstein's confused relativity paper is the common misunderstanding of relativity. While theory#2 is the correct version of relativity and that is Boscovich's theory.

When the German text of Einstein was first translated into English, then subtleties and nuances should have been sorted out, but they weren't; hence we have the misinterpretation by OEM, and what has been an ongoing mess in relativity theory.

Boscovich's theory is an extension of Newtonian physics, and in that theory Newtonian equations of motion hold good without it being an approximation, and the constancy of lightspeed is imposed on those equations as a convention.

In the maths of theory#1 the equations of SR have Newtonian physics as its approximation when speeds of moving objects are much less than c. While in the maths of theory#2 Newtonian physics is not an approximation of the SR equations, instead SR equations is Newtonian physics with convention of lightspeed constancy imposed on it.

Of course, those who work from a belief in theory#1 then go on to place numerous mistakes piled one on top of another upon their initial mistake of misunderstanding the nature of Newtonian physics.

(15)

OEM: In order to render our presentation more precise and to distinguish this system of co-ordinates verbally from others which will be introduced hereafter, we call it the "stationary system."

AFK: To distinguish this system of co-ordinates from some others to be introduced below, we shall call it the “stationary system.”

Me: So, there is a “stationary system” but really needs to clarify, and presumably is meaning that it is- an inertial observer in a system that is stationary with respect to him.

(16)

OEM: If a material point is at rest relatively to this system of co-ordinates, its position can be defined relatively thereto by the employment of rigid standards of measurement and the methods of Euclidean geometry and can be expressed in Cartesian co-ordinates.

AFK: If a material point is stationary with respect to this coordinate system, then its position relative to this system can be determined using rigid rulers, delineated in terms of Euclidean geometry, and then finally expressed in terms of Cartesian co-ordinates.

Me: A “material point” i.e. a point-particle that has mass, that is Boscovich’ theory; so, building on that; ideally should be referenced what is building upon. Boscovich picked up the idea from Newton. Also, to note: it is “Euclidean geometry” in this 1905 paper (in 1915 Einstein goes to non-Euclidean) that also is in domain of Newtonian physics.

(17)

OEM: If we wish to describe the *motion* of a material point, we give the values of its co-ordinates as functions of the time.

AFK: Usually, if we seek to describe the *motion* of a material point, we give the values of its co-ordinates as functions of the time.

Me: Which is still Newtonian physics with its point-particles and its extension by Boscovich.

(18)

OEM: Now we must bear carefully in mind that a mathematical description of this kind has no physical meaning unless we are quite clear as to what we understand by “time.”

AFK: However, it must be kept foremost in mind that, such a mathematical description can make physical sense only after it is clear just what is to be understood by the term “time.”

Me: So, what is “time”; that is a difficult question to answer. According to Augustine [10]: “What, then, is time? If no one ask of me, I know; if I wish to explain to him who asks, I know not.” Einstein does not seem to be getting the issue any further of what is “time.”

(19)

OEM: We have to take into account that all our judgments in which time plays a part are always judgments of *simultaneous events*.

AFK: We must recognize that, all our judgments in which time plays a role, are judgments on *simultaneous events*.

(20)

OEM: If, for instance, I say, “That train arrives here at 7 o'clock,” I mean something like this: “The pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events.”

Footnote: We shall not here discuss the inexactitude which lurks in the concept of simultaneity of two events at approximately the same place, which can only be removed by an abstraction.

AFK: If one says “the train arrives here at 7,” this is taken to mean something that: “the small hand of my watch points to 7 as the train pulls up.”

Footnote: The imprecision, which is inherent in the concept of simultaneity of two events at (nearly) the same position, which similarly must be put in context abstractly, is not the issue here.

(21)

OEM: It might appear possible to overcome all the difficulties attending the definition of “time” by substituting “the position of the small hand of my watch” for “time.”

AFK: It might seem that, all the difficulties in the definitions of “time” could be overcome simply replacing the term “time” with the position of the hand of a clock.

Me: Seems to tie in with when Einstein said [11]: “Time is that which clocks measure". Several people have looked at what Einstein is doing here, and start talking of Einstein committing a fallacy; he is identifying the operation of measurement of time with “time” itself. Example: Bergson complains about Einstein’s treatment of time. [12]

(22)

OEM: And in fact such a definition is satisfactory when we are concerned with defining a time exclusively for the place where the watch is located; but it is no longer satisfactory when we have to connect in time series of events occurring at different places, or—what comes to the same thing—to evaluate the times of events occurring at places remote from the watch.

AFK: Such a definition would in fact be sufficient, if the issue were to define “time” only at one specific location, e.g. at the place where the watch is located. This definition is insufficient, however, as soon as the issue is the “time” of events at distinct locations that are to be related to each other, or —what amounts to the same thing—the time of events occurring at locations distant from the watch.

(23)

OEM: We might, of course, content ourselves with time values determined by an observer stationed together with the watch at the origin of the co-ordinates, and co-ordinating the corresponding positions of the hands with light signals, given out by every event to be timed, and reaching him through empty space.

AFK: We might try to specify the times of all events in terms of the arrival instants of signal pulses sent from these events to an observer located at the coordinate origin in terms of the positions of the hands of his clock.

(24)

OEM: But this co-ordination has the disadvantage that it is not independent of the standpoint of the observer with the watch or clock, as we know from experience.

AFK: This scheme, however, is unappealing in that it is not independent of the viewpoint of the observer at the origin, as we know from experience.

(25)

OEM: We arrive at a much more practical determination along the following line of thought.

AFK: A much more practical method results from the following.

(26)

OEM: If at the point A of space there is a clock, an observer at A can determine the time values of events in the immediate proximity of A by finding the positions of the hands which are simultaneous with these events.

AFK: Suppose there is a clock at point A in space, an observer located there can assign times to events in the immediate neighbourhood by associating them with the hands of his clock.

(27)

OEM: If there is at the point B of space another clock in all respects resembling the one at A, it is possible for an observer at B to determine the time values of events in the immediate neighbourhood of B.

AFK: If also there is a clock at point B in space – here is meant an exact similar clock – an observer there too can assign times in his neighbourhood.

(28)

OEM: But it is not possible without further assumption to compare, in respect of time, an event at A with an event at B. We have so far defined only an “A time” and a “B time.”

AFK: Thus, so far we have “A” -time and “B” – time, but still no common time for both.

Me: AFK talks of “common time” not in OEM, but mentioned anon.

(29)

OEM: We have not defined a common “time” for A and B, for the latter cannot be defined at all unless we establish *by definition* that the “time” required by light to travel from A to B equals the “time” it requires to travel from B to A.

AFK: This latter, common time can now be defined in terms of the time that light takes to go from A to B, and its equal return, i.e., the time from B to A.

Me: making a definition, which to me seems more of an assumption. I have a lot to say about this see my article “Einstein mistranslated” part 1 [13] and 2. [14]

(30)

OEM: Let a ray of light start at the “A time” t_{A} from A towards B, let it at the “B time” t_{B} be reflected at B in the direction of A, and arrive again at A at the “A time” t'_{A}.

AFK: That is, consider a pulse from A at time t_{A} arriving at B at t_{B} that is then reflected back to A where it arrives at time t'_{A}.

(31)

OEM: In accordance with definition the two clocks synchronize if

$$t_{\mathrm{B}} - t_{\mathrm{A}} = t'_{\mathrm{A}} - t_{\mathrm{B}}.$$

AFK: It can now be said that both clocks run synchronously if

$$t_{\mathrm{B}} - t_{\mathrm{A}} = t'_{\mathrm{A}} - t_{\mathrm{B}}.$$

(32)

OEM: We assume that this definition of synchronism is free from contradictions, and possible for any number of points; and that the following relations are universally valid: —

AFK: We assume that this definition of synchrony is free of contradictions for even arbitrarily many points, i.e., that the following relations hold:

Me: making an assumption instead of showing.

(33)

OEM: 1. If the clock at B synchronizes with the clock at A, the clock at A synchronizes with the clock at B.

AFK: 1. If the B clock is synchronous with the A clock, then A is synchronous with B; and,

(34)

OEM: 2. If the clock at A synchronizes with the clock at B and also with the clock at C, the clocks at B and C also synchronize with each other.

AFK: 2. When the A clock is synchronous with both the B and C clocks, then both the B and C clocks are synchronous with each other.

(35)

OEM: Thus with the help of certain imaginary physical experiments we have settled what is to be understood by synchronous stationary clocks located at different places, and have evidently obtained a definition of "simultaneous," or "synchronous," and of "time."

AFK: Thus, with the aid of gedanken physical experience, we have determined what is to be understood by simultaneity at separated locations, what a stationary clock is, and thereby definitions of the terms "simultaneous" and "time."

Me: This is all being based on thought (gedanken) experiment.

(36)

OEM: The "time" of an event is that which is given simultaneously with the event by a stationary clock located at the place of the event, this clock being synchronous, and indeed synchronous for all time determinations, with a specified stationary clock.

AFK: The "time" of an event is that shown on a collocated and co-moving clock.

(37)

OEM: In agreement with experience we further assume the quantity

$$\frac{2\mathrm{AB}}{t'_A - t_A} = c,$$

to be a universal constant—the velocity of light in empty space.

AFK: Further, we take it that the quantity

$$\frac{2AB}{t'_A - t_A} = c,$$

Is a universal constant (the [two-way] speed of light in empty space).

Me: universal constant velocity of light in empty space VERSUS two-way lightspeed in empty space. AFK decides what is being talked about is two-way without mentioning of that in the German; it should have been said but wasn't.

(38)

OEM: It is essential to have time defined by means of stationary clocks in the stationary system, and the time now defined being appropriate to the stationary system we call it "the time of the stationary system."

AFK: To be stressed here is the fact that, we have defined time by means of a stationary clock in a stationary coordinate system; this "time" in the stationary system is denoted the "time of the stationary system."

Me: But the person who moves with the moving clock might think he is stationary. On issues like this I will pass for now.

(39)

OEM: 2. On the relativity of lengths and times

AFK: 2. On the relativity of lengths and times

(40)

OEM: The following reflexions are based on the principle of relativity and on the principle of the constancy of the velocity of light. These two principles we define as follows: —

AFK: The following considerations are based on the principle of relativity and the constancy of the speed of light; they are delineated as follows:

Me: Now AFK says “constancy of lightspeed” when earlier it was “constancy of two-way lightspeed”. Also still have OEM say “velocity” versus AFK say “speed”.

(41)

OEM: 1. The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform translatory motion.

AFK: 1. The laws according to which the states of physical systems change, are to be independent of which of two uniformly moving coordinate systems to which the physical system is referred.

Me: uniform can apply to acceleration, so uniform equations of motion of Newton.

(42)

OEM: 2. Any ray of light moves in the “stationary” system of co-ordinates with the determined velocity *c*, whether the ray be emitted by a stationary or by a moving body.

AFK: 2. Every light ray moves in a “stationary” coordinate system with the specific velocity c, regardless of whether the ray was emitted from a stationary or moving source.

n.b. Kracklauer uses V not c

Me: It is ambiguous because given a “stationary” of coordinates what is the light being measured relative to; a stationary point in that coordinate system or a moving point in that coordinate system. By Newtonian physics they would not both give c, but because of the ambiguity given here by Einstein many might misunderstand it that way.

(43)

OEM: Hence

$$\text{velocity} = \frac{\text{light path}}{\text{time interval}}$$

where time interval is to be taken in the sense of the definition in § 1.

AFK: In this regard

Speed of light = distance travelled/time taken

Where the "time taken" is in the sense defined in § 1.

Me: AFK back to using "speed" while OEM uses "velocity".

(44)

OEM: Let there be given a stationary rigid rod; and let its length be *l* as measured by a measuring-rod which is also stationary.

AFK: Let us consider a stationary rigid rod for which length has been measured using a likewise rigid ruler and found to be *l*.

(45)

OEM: We now imagine the axis of the rod lying along the axis of *x* of the stationary system of co-ordinates, and that a uniform motion of parallel translation with velocity *v* along the axis of *x* in the direction of increasing *x* is then imparted to the rod.

AFK: Let us take it that the rod is oriented along the x-axis of the stationary coordinate system and that the rod has been given uniform motion with velocity v parallel to this axis.

(46)

OEM: We now inquire as to the length of the moving rod, and imagine its length to be ascertained by the following two operations: —

AFK: Now, we wish to determine the length in the moving system, which we imagine can be done with the following operations.

(47)

OEM: (*a*) The observer moves together with the given measuring-rod and the rod to be measured, and measures the length of the rod directly by superposing the measuring-rod, in just the same way as if all three were at rest.

AFK: (a) The observer arranges to move himself along with the rod and measures with his ruler directly just as if observer, ruler and rod were all together in the stationary system.

(48)

OEM: (*b*) By means of stationary clocks set up in the stationary system and synchronizing in accordance with § 1, the observer ascertains at what points of the stationary system the two ends of the rod to be measured are located at a definite time. The distance between these two points, measured by the measuring-rod already employed, which in this case is at rest, is also a length which may be designated "the length of the rod."

AFK: The observer determines with the use of a clock in the stationary system synchronized as prescribed in § 1, the locations of the end points of the rod at time t. The separation of these two points as determined with the stationary ruler also can be denoted a measurement of "the length of the rod."

(49)

OEM: In accordance with the principle of relativity the length to be discovered by the operation (*a*)—we will call it "the length of the rod in the moving system"—must be equal to the length *l* of the stationary rod.

AFK: Now, according to the principle of relativity, the length determined by procedure (*a*), which we would like to denote as the "length of the rod in the moving system," equals the length *l* of the stationary rod.

(50)

OEM: The length to be discovered by the operation (*b*) we will call "the length of the (moving) rod in the stationary system." This we shall determine on the basis of our two principles, and we shall find that it differs from *l*.

AFK: The length found with operation (*b*) which we would like to denote "the length of a (moving) rod in the stationary system," we shall see as a consequence of the above two principles, differs from *l*.

(51)

OEM: Current kinematics tacitly assumes that the lengths determined by these two operations are precisely equal, or in other words, that a moving rigid body at the epoch *t* may in geometrical respects be perfectly represented by *the same* body *at rest* in a definite position.

AFK: The current formulation of kinematics implicitly assumes that, the lengths measured using these two methods will be identical. In other words, that a moving rigid body at the instant *t,* in all geometrical relations can be replaced by *the same* rod in a stationary condition.

(52)

OEM: We imagine further that at the two ends A and B of the rod, clocks are placed which synchronize with the clocks of the stationary system, that is to say that their indications correspond at any instant to the "time of the stationary system" at the places where they happen to be. These clocks are therefore "synchronous in the stationary system."

AFK: Let us focus on the clocks in the stationary system situated at the ends of the rod and which are synchronous in the stationary system. They being synchronous in the stationary system, give us the "time in the synchronous system" of the ends of the rod in that system.

(53)

OEM: We imagine further that with each clock there is a moving observer, and that these observers apply to both clocks the criterion established in § 1 for the synchronization of two clocks.

AFK: Also let us imagine further that, with each clock there is a collocated observer, who uses the system described in § 1 to synchronize clocks.

(54)

OEM: Let a ray of light depart from A at the time[4] t_{A}, let it be reflected at B at the time t_{B}, and reach A again at the time t'_{A}.

Footnote: "Time" here denotes "time of the stationary system" and also "position of hands of the moving clock situated at the place under discussion."

AFK: At time t_{A} a light pulse is emitted and then received at B at time t_{B}, where it is reflected back toward A and arrives at time t'_{A}.

Footnote: “Time” here means “time in the stationary system” as well as “position of hands of the moving clock at the location which is under discussion.”

(55)

OEM: Taking into consideration the principle of the constancy of the velocity of light we find that

$$t_{\mathrm{B}} - t_{\mathrm{A}} = \frac{r_{\mathrm{AB}}}{c - v} \text{ and } t'_{\mathrm{A}} - t_{\mathrm{B}} = \frac{r_{\mathrm{AB}}}{c + v}$$

where r_{AB} denotes the length of the moving rod—measured in the stationary system.

AFK: Taking the constancy of the speed of light into consideration, leads to

$$t_{\mathrm{B}} - t_{\mathrm{A}} = \frac{r_{\mathrm{AB}}}{c - v} \text{ and } t'_{\mathrm{A}} - t_{\mathrm{B}} = \frac{r_{\mathrm{AB}}}{c + v}$$

where r_{AB} denotes the length of the moving rod—as measured in the stationary coordinate system.

(56)

OEM: Observers moving with the moving rod would thus find that the two clocks were not synchronous, while observers in the stationary system would declare the clocks to be synchronous.

AFK: For the co-moving observers the two clocks are not synchronous, even while the stationary observers would consider them synchronous.

(57)

OEM: So we see that we cannot attach any *absolute* signification to the concept of simultaneity, but that two events which, viewed from a system of co-ordinates, are simultaneous, can no longer be looked upon as simultaneous events when envisaged from a system which is in motion relatively to that system.

AFK: Thus we see that, the notion of simultaneity does not have absolute meaning, rather that two events observed to be synchronous from one coordinate system are not seen as synchronous in a coordinate system uniformly moving with respect to the first.

(58)

OEM: **§ 3. Theory of the Transformation of Co-ordinates and Times from a Stationary System to another System in Uniform Motion of Translation Relatively to the Former**

AFK: § 3. The theory of transformation from a stationary to a moving coordinate system.

(59)

OEM: Let us in "stationary" space take two systems of co-ordinates, i.e. two systems, each of three rigid material lines, perpendicular to one another, and issuing from a point. Let the axes of X of the two systems coincide, and their axes of Y and Z respectively be parallel.

AFK: Consider two stationary coordinate systems oriented such that their x-axes coincide and their y- and z-axes are parallel respectively.

(60)

OEM: Let each system be provided with a rigid measuring-rod and a number of clocks, and let the two measuring-rods, and likewise all the clocks of the two systems, be in all respects alike.

AFK: Each system is equipped with identical rigid rulers and clocks.

(61)

OEM: Now to the origin of one of the two systems (*k*) let a constant velocity *v* be imparted in the direction of the increasing *x* of the other stationary system (K), and let this velocity be communicated to the axes of the co-ordinates, the relevant measuring-rod, and the clocks.

AFK: Now let one of these systems, designated *k,* along with its clocks and observers be set in a uniform motion *v* with respect to the other, designated K, along the common x-axes in the positive direction.

(62)

OEM: To any time of the stationary system K there then will correspond a definite position of the axes of the moving system, and from reasons of symmetry we are entitled to assume that the motion of *k* may be such that the axes of the moving system are at the time *t* (this “*t*” always denotes a time of the stationary system) parallel to the axes of the stationary system.

AFK: Each moment t of the stationary system K corresponds then to a particular position of the moving system. For reasons of symmetry we assume that, the motion of *k* can be so arranged that the axes at the time “*t”* (here “*t*” means time in the stationary system) remain parallel to those of the stationary system.

(63)

OEM: We now imagine space to be measured from the stationary system K by means of the stationary measuring-rod, and also from the moving system *k* by means of the measuring-rod moving with it; and that we thus obtain the co-ordinates *x*, *y*, *z*, and ξ, η, ζ respectively.

AFK: We consider that with use of stationary rulers the stationary system K can be surveyed and given coordinates x,y,z, and the co-moving rulers given the coordinates ξ, η, ζ.

(64)

OEM: Further, let the time *t* of the stationary system be determined for all points thereof at which there are clocks by means of light signals in the manner indicated in § 1; similarly let the time τ of the moving system be determined for all points of the moving system at which there are clocks at rest relatively to that system by applying the method, given in § 1, of light signals between the points at which the latter clocks are located.

AFK: Let all stationary clocks be synchronized with light signals as described in § 1, so that they have common time t, likewise for the moving system so that its clocks have common time τ.

(65)

OEM: To any system of values *x*, *y*, *z*, *t*, which completely defines the place and time of an event in the stationary system, there belongs a system of values ξ, η, ζ, τ, determining that event relatively to the system *k*, and our task is now to find the system of equations connecting these quantities.

AFK: For each set of values for the coordinate system x, *y*, *z*, *t*, specifying the location and time of an event in the stationary system, there is also a set of values with respect to system ξ, η, ζ, τ. Now our task is to find the equations relating these two sets of quantities.

(66)

OEM: In the first place it is clear that the equations must be *linear* on account of the properties of homogeneity which we attribute to space and time.

AFK: From the start it is clear that, the target system of equations, by virtue of the homogeneity of space and time, must be linear.

(67)

OEM: If we place $x'=x-vt$, it is clear that a point at rest in the system *k* must have a system of values *x*', *y*, *z*, independent of time.

AFK: If we set $x'=x-vt$, then it is clear that, in *k* stationary points get particular values *x*', *y*, *z*, independent of time.

Me: This is not very clear at all. A system of points in system k, if x'=x,y'=y,z=z' for all values of time t then they are independent of time, but has x'=x-vt so value of x' changes as time t changes.

So, I think: "system *k* must have a system of values *x*', *y*, *z*, independent of time." – should have really read as "system *k* must have a system of values x, *y*, *z*, independent of time."

What is also interesting is that have points connected as per Newtonian physics equation. If a system has stationary points relative to that system, then it has moving points relative to that system going by Newtonian maths.

This is thus a suitable place to stop because things just get more complicated. The following from (68) -onwards is thus provided with less commentary by me.

(68)

OEM: We first define τ as a function of *x*', *y*, *z*, and *t*. To do this we have to express in equations that τ is nothing else than the summary of the data of clocks at rest in system *k*, which have been synchronized according to the rule given in § 1.

AFK: To determine τ as a function of *x*', *y*, *z*, and *t*, the transformations must reflect the fact that τ is nothing but the readings of k-clocks synchronized per § 1.

(69)

OEM: From the origin of system *k* let a ray be emitted at the time τ_0 along the X-axis to *x*', and at the time τ_1 be reflected thence to the origin of the co-ordinates, arriving there at the time τ_2; we then must have $\frac{1}{2}(\tau_0 + \tau_2) = \tau_1$, or, by inserting the arguments of the function τ and applying the principle of the constancy of the velocity of light in the stationary system:—

$$\frac{1}{2}\left[\tau(0,0,0,t) + \tau\left(0,0,0,t + \frac{x'}{c-v} + \frac{x'}{c+v}\right)\right] = \tau\left(x',0,0,t + \frac{x'}{c-v}\right).$$

AFK: From the origin of k- system let us consider a light ray sent at τ_0 along the x-axis toward *x*' and there reflected at τ_1 back to the origin where it arrives at τ_2; so that we must have:

$$\frac{1}{2}(\tau_0 + \tau_2) = \tau_1,$$

or, where if one writes out the arguments of the function τ and takes into account the constancy of the speed of light in the stationary system, it may be written: —

$$\frac{1}{2}\left[\tau(0,0,0,t) + \tau\left(0,0,0,t + \frac{x'}{c-v} + \frac{x'}{c+v}\right)\right] = \tau\left(x',0,0,t + \frac{x'}{c-v}\right).$$

(70)

OEM: Hence, if *x*' be chosen infinitesimally small,

$$\frac{1}{2}\left(\frac{1}{c-v}+\frac{1}{c+v}\right)\frac{\partial\tau}{\partial t}=\frac{\partial\tau}{\partial x'}+\frac{1}{c-v}\frac{\partial\tau}{\partial t},$$

or

$$\frac{\partial\tau}{\partial x'}+\frac{v}{c^2-v^2}\frac{\partial\tau}{\partial t}=0.$$

AFK: From this equation, if *x*' is chosen infinitesimally small, it follows:

$$\frac{1}{2}\left(\frac{1}{c-v}+\frac{1}{c+v}\right)\frac{\partial\tau}{\partial t}=\frac{\partial\tau}{\partial x'}+\frac{1}{c-v}\frac{\partial\tau}{\partial t},$$

or

$$\frac{\partial\tau}{\partial x'}+\frac{v}{c^2-v^2}\frac{\partial\tau}{\partial t}=0.$$

(71)

OEM: It is to be noted that instead of the origin of the co-ordinates we might have chosen any other point for the point of origin of the ray, and the equation just obtained is therefore valid for all values of *x*', *y*, *z*.

AFK: Note that if, instead of the coordinate origin, we had selected any arbitrary point as the source of the light ray, we would have arrived at the same equation for the values x', *y*, *z*.

(72)

OEM: An analogous consideration—applied to the axes of Y and Z—it being borne in mind that light is always propagated along these axes, when viewed from the stationary system, with the velocity $\sqrt{c^2-v^2}$ gives us

$$\frac{\partial\tau}{\partial y}=0, \frac{\partial\tau}{\partial z}=0.$$

AFK: Analogous considerations applied to the y- z- axes, taking into account that light propagating along these axes as seen from the stationary system moves with the speed $\sqrt{c^2-v^2}$ yields the equations:

$$\frac{\partial \tau}{\partial y} = 0, \frac{\partial \tau}{\partial z} = 0.$$

(73)

OEM: Since τ is a *linear* function, it follows from these equations that

$$\tau = a\left(t - \frac{v}{c^2 - v^2}x'\right)$$

where *a* is a function $\phi(v)$ at present unknown, and where for brevity it is assumed that at the origin of *k*, $\tau = 0$, when *t*=0.

AFK: From these equations, insofar as τ is a *linear* function, follows:

$$\tau = a\left(t - \frac{v}{c^2 - v^2}x'\right)$$

where *a* , provisionally, is an unknown function $\phi(v)$, and is for sake of brevity taken such,

that at the origin of *k*, it satisfies $\tau = 0$, t = 0.

(74)

OEM: With the help of this result we easily determine the quantities ξ, η, ζ by expressing in equations that light (as required by the principle of the constancy of the velocity of light, in combination with the principle of relativity) is also propagated with velocity *c* when measured in the moving system

AFK: With the aid of these results it is easy to determine the magnitudes of ξ, η, ζ by expressing with equations the fact that light (as required by the principle of the constancy of its speed) is measured to have velocity c.

n.b. Kracklauer uses V instead of c; he also now says “velocity” when previously been saying “speed.” The German word can be translated as “velocity” or “speed” and Kracklauer has been up to now translating as “speed”, I think he should have continued to do so, and not used “velocity”.

(75)

OEM: For a ray of light emitted at the time $\tau = 0$ in the direction of the increasing ξ

$$\xi = c\tau \text{ or } \xi = ac\left(t - \frac{v}{c^2 - v^2}x'\right).$$

AFK: For a light ray emitted at the time $\tau = 0$ in the positive ξ - direction, one has:

$$\xi = c\tau \text{ or } \xi = ac\left(t - \frac{v}{c^2 - v^2}x'\right).$$

(76)

OEM: But the ray moves relatively to the initial point of *k*, when measured in the stationary system, with the velocity *c*-*v*, so that

$$\frac{x'}{c - v} = t.$$

AFK: But, if the light ray from the origin of system k has the velocity c-*v as* measured in the stationary system, then we have:

$$\frac{x'}{c - v} = t.$$

(77)

OEM: If we insert this value of *t* in the equation for ξ, we obtain

$$\xi = a\frac{c^2}{c^2 - v^2}x'.$$

AFK: Putting this value into the equation for ξ gives:

$$\xi = a\frac{c^2}{c^2 - v^2}x'.$$

(78)

OEM: In an analogous manner we find, by considering rays moving along the two other axes, that

$$\eta = c\tau = ac\left(t - \frac{v}{c^2 - v^2}x'\right)$$

when

$$\frac{y}{\sqrt{c^2 - v^2}} = t,\ \ x' = 0.$$

AFK: Analogously we find by considering light rays along the other two axes

$$\eta = c\tau = ac\left(t - \frac{v}{c^2 - v^2}x'\right)$$

where

$$\frac{y}{\sqrt{c^2 - v^2}} = t,\ \ x' = 0.$$

(79)

OEM: Thus

$$\eta = a\frac{c}{\sqrt{c^2 - v^2}}y \text{ and } \zeta = a\frac{c}{\sqrt{c^2 - v^2}}z.$$

AFK: So that

$$\eta = a\frac{c}{\sqrt{c^2 - v^2}}y \text{ and } \zeta = a\frac{c}{\sqrt{c^2 - v^2}}z.$$

(80)

OEM: Substituting for *x*' its value, we obtain

$$\begin{aligned} \tau &= \phi(v)\beta(t - vx/c^2), \\ \xi &= \phi(v)\beta(x - vt), \\ \eta &= \phi(v)y, \\ \zeta &= \phi(v)z, \end{aligned}$$

where

$$\beta = \frac{1}{\sqrt{1 - v^2/c^2}},$$

and ϕ is an as yet unknown function of *v*.

AFK: Replacing x' by its values gives

$$\begin{aligned} \tau &= \phi(v)\beta(t - vx/c^2), \\ \xi &= \phi(v)\beta(x - vt), \\ \eta &= \phi(v)y, \\ \zeta &= \phi(v)z, \end{aligned}$$

where

$$\beta = \frac{1}{\sqrt{1 - v^2/c^2}},$$

and ϕ is provisionally an unknown function.

(81)

OEM: If no assumption whatever be made as to the initial position of the moving system and as to the zero point of τ, an additive constant is to be placed on the right side of each of these equations.

AFK: If one makes no assumptions regarding the initial position and time, then there must be constants added to the right side.

(82)

OEM: We now have to prove that any ray of light, measured in the moving system, is propagated with the velocity *c*, if, as we have assumed, this is the case in the stationary system; for we have not as yet furnished the proof that the principle of the constancy of the velocity of light is compatible with the principle of relativity.

AFK: Now we must show that every ray measured in the moving system propagates with velocity *c*, if this is true, as we have assumed it to be, in the stationary system; indeed, thus far we have not demonstrated that this assumption is compatible with the principle of relativity.

(83)

OEM: At the time $t = \tau = 0$, when the origin of the co-ordinates is common to the two systems, let a spherical wave be emitted therefrom, and be propagated with the velocity *c* in system K. If (*x*, *y*, *z*) be a point just attained by this wave, then

$x^2+y^2+z^2=c^2t^2$.

AFK: Suppose that at time $t = \tau = 0$, when the origins of both systems coincide, a spherical wave is launched which expands in K at velocity c. If (*x*, *y*, *z*) is a point on this sphere, then:

$x^2+y^2+z^2=c^2t^2$.

(84)

OEM: Transforming this equation with the aid of our equations of transformation we obtain after a simple calculation

$$\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2.$$

AFK: After simply transforming this equation with the transformations derived above, one gets:

$$\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2.$$

(85)

OEM: The wave under consideration is therefore no less a spherical wave with velocity of propagation *c* when viewed in the moving system. This shows that our two fundamental principles are compatible.

Footnote: The equations of the Lorentz transformation may be more simply deduced directly from the condition that in virtue of those equations the relation $x^2+y^2+z^2=c^2t^2$ shall have as its consequence the second relation $\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2$.

AFK: That is, the considered wave is identical in k. That shows that these two principles are fully compatible.

No footnote

(86)

OEM: In the equations of transformation which have been developed there enters an unknown function Φ of *v*, which we will now determine.

AFK: In the transformations as derived above, there is still an undetermined function Φ (*v)*, which shall now be fixed.

(87)

OEM: For this purpose we introduce a third system of co-ordinates K', which relatively to the system *k* is in a state of parallel translatory motion parallel to the axis of Ξ,[*1] such that the origin of co-ordinates of system K', moves with velocity -*v* on the axis of Ξ.

TRANSLATOR NOTE: In Einstein's original paper, the symbols (Ξ, H, Z) for the co-ordinates of the moving system *k* were introduced without explicitly defining them. In the 1923 English translation, (X, Y, Z) were used, creating an ambiguity between X co-ordinates in the fixed system K and the parallel axis in moving system *k*. Here and in subsequent references we use Ξ when referring to the axis of system *k* along which the system is translating with respect to K. In addition, the reference to system K', later in this sentence was incorrectly given as "*k*" in the 1923 English translation.

AFK: For this purpose a third system of co-ordinate system is introduced, which, relative to k, is in parallel translational motion along the Ξ -axis but with velocity -*v.*

N.B. confused about notation

(88)

OEM: At the time *t*=0 let all three origins coincide, and when *t*=*x*=*y*=*z*=0 let the time *t*' of the system K'be zero.

AFK: At t=0 the origins of all three systems coincide, and for t=*x*=*y*=*z*=0, let t' =0 in K' .

(89)

OEM: We call the co-ordinates, measured in the system K', *x*', *y*', *z*', and by a twofold application of our equations of transformation we obtain

$$\begin{aligned} t' &= \phi(-v)\beta(-v)(\tau + v\xi/c^2) &&= \phi(v)\phi(-v)t, \\ x' &= \phi(-v)\beta(-v)(\xi + v\tau) &&= \phi(v)\phi(-v)x, \\ y' &= \phi(-v)\eta &&= \phi(v)\phi(-v)y, \\ z' &= \phi(-v)\zeta &&= \phi(v)\phi(-v)z. \end{aligned}$$

AFK: We call x', *y*', *z'* the coordinates as measured in K' and get with double application of our transformations

$$\begin{aligned} t' &= \phi(-v)\beta(-v)(\tau + v\xi/c^2) &&= \phi(v)\phi(-v)t, \\ x' &= \phi(-v)\beta(-v)(\xi + v\tau) &&= \phi(v)\phi(-v)x, \\ y' &= \phi(-v)\eta &&= \phi(v)\phi(-v)y, \\ z' &= \phi(-v)\zeta &&= \phi(v)\phi(-v)z. \end{aligned}$$

(90)

OEM: Since the relations between *x*', *y*', *z*' and *x*, *y*, *z* do not contain the time *t*, the systems K and K' are at rest with respect to one another, and it is clear that the transformation from K to K' must be the identical transformation. Thus

$$\phi(v)\phi(-v) = 1.$$

AFK: Insofar as the relationships between *x*', *y*', *z*' and *x*, *y*, *z* do not involve t, a transformation from K to K' must be the identity; therefore we get:

$$\phi(v)\phi(-v) = 1.$$

(91)

OEM: We now inquire into the signification of Φ(v).

AFK: The meaning of the functions Φ(v) remain an interesting question.

(92)

OEM: We give our attention to that part of the axis of Y of system *k* which lies between $\xi = 0, \eta = 0, \zeta = 0$ and $\xi = 0, \eta = l, \zeta = 0$.

AFK: Consider a portion of the η - axis in k between (0,0,0) and (0,l,0).

n.b. letter l not number 1

(93)

OEM: This part of the axis of Y is a rod moving perpendicularly to its axis with velocity *v* relatively to system K. Its ends possess in K the co-ordinates

$$x_1 = vt,\ y_1 = \frac{l}{\phi(v)},\ z_1 = 0$$

and

$$x_2 = vt,\ y_2 = 0,\ z_2 = 0.$$

AFK: This line segment is orthogonal to the direction of translation with velocity v; and, its end points in K are given by

$$x_1 = vt,\ y_1 = \frac{l}{\phi(v)},\ z_1 = 0$$

and

$$x_2 = vt,\ y_2 = 0,\ z_2 = 0.$$

(94)

OEM: The length of the rod measured in K is therefore $l/\phi(v)$; and this gives us the meaning of the function $\phi(v)$.

AFK: In K this segment has length $l/\phi(v)$; which specifies the meaning of $\phi(v)$.

(95)

OEM: From reasons of symmetry it is now evident that the length of a given rod moving perpendicularly to its axis, measured in the stationary system, must depend only on the velocity and not on the direction and the sense of the motion.

AFK: Symmetry considerations illuminate the consequences in that here it is seen that the length of the segment depends only on velocity and not on direction and the sense of the motion.

(96)

OEM: The length of the moving rod measured in the stationary system does not change, therefore, if *v* and -*v* are interchanged. Hence follows that $l/\phi(v) = l/\phi(-v)$, or

$$\phi(v) = \phi(-v).$$

AFK: This implies that the length is unaltered when the velocity is reversed, or

$l/\phi(v) = l/\phi(-v)$, or

$$\phi(v) = \phi(-v).$$

(97)

OEM: It follows from this relation and the one previously found that $\phi(v) = 1$, so that the transformation equations which have been found become

$$\begin{aligned} \tau &= \beta(t - vx/c^2), \\ \xi &= \beta(x - vt), \\ \eta &= y, \\ \zeta &= z, \end{aligned}$$

where

$$\beta = 1/\sqrt{1 - v^2/c^2}.$$

AFK: This, together with the preceding relations all indicate that,

$\phi(v) = 1$; thereby revealing the fundamental transformations to be of the

$$\begin{aligned} \tau &= \beta(t - vx/c^2), \\ \xi &= \beta(x - vt), \\ \eta &= y, \\ \zeta &= z, \end{aligned}$$

form

where

$$\beta = 1/\sqrt{1 - v^2/c^2}.$$

(98)

OEM: **§ 4. Physical Meaning of the Equations Obtained in Respect to Moving Rigid Bodies and Moving Clocks**

AFK: § 4. The physical significance of the above transformations for Rigid Bodies and Clocks

(99)

OEM: We envisage a rigid sphere of radius R, at rest relatively to the moving system *k*, and with its centre at the origin of co-ordinates of *k*.

FOOTNOTE: That is, a body possessing spherical form when examined at rest.

AFK: Consider a rigid sphere of radius R at rest in a moving system *k* and centred at its origin.

FOOTNOTE: That is, an object which has shape when examined at rest.

(100)

OEM: The equation of the surface of this sphere moving relatively to the system K with velocity *v* is

$$\xi^2 + \eta^2 + \zeta^2 = \mathrm{R}^2.$$

AFK: The equation for the surface of this sphere relative to K, which is moving with velocity *v,* is

$$\xi^2 + \eta^2 + \zeta^2 = \mathrm{R}^2.$$

(101)

OEM: The equation of this surface expressed in *x*, *y*, *z* at the time *t*=0 is

$$\frac{x^2}{(\sqrt{1-v^2/c^2})^2} + y^2 + z^2 = \mathrm{R}^2.$$

AFK: The equation for this same surface in the coordinates x, *y*, *z* is

$$\frac{x^2}{(\sqrt{1-v^2/c^2})^2} + y^2 + z^2 = \mathrm{R}^2.$$

(102)

OEM: A rigid body which, measured in a state of rest, has the form of a sphere, therefore has in a state of motion—viewed from the stationary system—the form of an ellipsoid of revolution with the axes

$$R\sqrt{1-v^2/c^2},\ R,\ R.$$

AFK: A rigid body which at rest has the discernible form of a sphere, in motion – as seen from the system at rest — has the form of an ellipsoid of revolution with axes

$$R\sqrt{1-v^2/c^2},\ R,\ R.$$

(103)

OEM: Thus, whereas the Y and Z dimensions of the sphere (and therefore of every rigid body of no matter what form) do not appear modified by the motion, the X dimension appears shortened

in the ratio $1:\sqrt{1-v^2/c^2}$, i.e. the greater the value of *v*, the greater the shortening.

AFK: While the y- and z- axes of the sphere (or any other shape) of a rigid body remain unmodified by the motion, the x – axis appears contracted

in the ratio $1:\sqrt{1-v^2/c^2}$, that is, the greater v is, the stronger the contraction.

(104)

OEM: For *v*=*c* all moving objects—viewed from the “stationary” system—shrivel up into plane figures.

TRANSLATOR NOTE: In the original 1923 English edition, this phrase was erroneously translated as “plain figures”. I have used the correct “plane figures” in this document.

AFK: For *v*=*c* all moving objects viewed from the rest system appear to be flattened completely.

(105)

OEM: For velocities greater than that of light our deliberations become meaningless; we shall, however, find in what follows, that the velocity of light in our theory plays the part, physically, of an infinitely great velocity.

AFK: At superluminal velocities these considerations become nonsense; we shall see below that this theory the speed of light plays the role of an infinite velocity.

(106)

OEM: It is clear that the same results hold good of bodies at rest in the "stationary" system, viewed from a system in uniform motion.

AFK: Clearly, the same results are obtained for bodies at rest in a system viewed from a uniformly moving system.

Me: So, talking about uniform motion, in context of Newtonian physics that would mean both uniform speed/velocity and uniform acceleration. So, the way it is written the uniform equations of Newton apply. i.e. SR is not just about constant speed/velocity it is supposed to be true all for constant acceleration, and equations of motion it obeys for that is Newtonian. Hence another reason for Boscovich-Newtonian physics.

In other places the translation is not so clear about it being uniform motion.

(107)

OEM: Further, we imagine one of the clocks which are qualified to mark the time *t* when at rest relatively to the stationary system, and the time τ when at rest relatively to the moving system, to be located at the origin of the co-ordinates of *k*, and so adjusted that it marks the time τ. What is the rate of this clock, when viewed from the stationary system?

AFK: A clock, showing the rest time *t* in a system at rest relative to a moving system shows the time τ, as is in fact reflected in the transformations from K to k. What is the tempo of this clock as read from K?

(108)

OEM: Between the quantities x, t, and τ, which refer to the position of the clock, we have, evidently, *x*=*vt* and

$$\tau = \frac{1}{\sqrt{1 - v^2/c^2}}(t - vx/c^2).$$

AFK: Among the variables x, t, and τ, pertaining to this clock at this location, one has the equation

$$\tau = \frac{1}{\sqrt{1 - v^2/c^2}}(t - vx/c^2).$$

And x = vt.

(109)

OEM: Therefore,

$$\tau = t\sqrt{1 - v^2/c^2} = t - (1 - \sqrt{1 - v^2/c^2})t$$

whence it follows that the time marked by the clock (viewed in the stationary system) is slow

by $1 - \sqrt{1 - v^2/c^2}$ seconds per second, or—neglecting magnitudes of fourth and higher order—by $\frac{1}{2}v^2/c^2$.

AFK: Thus, one gets

$$\tau = t\sqrt{1 - v^2/c^2} = t - (1 - \sqrt{1 - v^2/c^2})t$$

From which it follows that, the readings from the clock as made from the rest system are delayed per second by the amount

$1 - \sqrt{1 - v^2/c^2}$ sec., i.e. up to the fourth order by an amount $\frac{1}{2}v^2/c^2$ sec.

(110)

OEM: From this there ensues the following peculiar consequence. If at the points A and B of K there are stationary clocks which, viewed in the stationary system, are synchronous; and if the clock at A is moved with the velocity *v* along the line AB to B, then on its arrival at B the two clocks no longer synchronize, but the clock moved from A to B lags behind the other which has remained at B by $\frac{1}{2}tv^2/c^2$ (up to magnitudes of fourth and higher order), *t* being the time occupied in the journey from A to B.

AFK: This leads to the following peculiar circumstance. If two clocks at points A and B both at rest in K are synchronized as viewed from the rest system k, then if the clock at A is moved with speed *v* on the line joining it to B, it will upon arrival at B be seen to be no longer synchronized; but rather the clock A relative to the one at B is seen to be $\frac{1}{2}tv^2/c^2$ seconds retarded (up to the fourth order), where *t* is the time interval for traversing the separation.

(111)

OEM: It is at once apparent that this result still holds good if the clock moves from A to B in any polygonal line, and also when the points A and B coincide.

AFK: This circumstance also obtains if the clock in K is transported from A to B over an arbitrary path, and even when A and B coincide.

(112)

OEM: If we assume that the result proved for a polygonal line is also valid for a continuously curved line, we arrive at this result: If one of two synchronous clocks at A is moved in a closed curve with constant velocity until it returns to A, the journey lasting *t* seconds, then by the clock which has remained at rest the travelled clock on its arrival at A will be $\frac{1}{2}tv^2/c^2$ second slow.

AFK: If one assumes that in the limit what pertains for a broken polygonal line is also true of a smooth curve, one might say: if of two synchronized clocks first located at A, one is moved in t seconds at a constant velocity around a curved path back to A, then this clock will be delayed with respect to B by $\frac{1}{2}tv^2/c^2$ seconds.

(113)

OEM: Thence we conclude that a balance-clock[7] at the equator must go more slowly, by a very small amount, than a precisely similar clock situated at one of the poles under otherwise identical conditions.

FOOTNOTE: Not a pendulum-clock, which is physically a system to which the Earth belongs. This case had to be excluded.

AFK: From this one should expect a clock at the earth's equator to run slower than an identical clock at a pole.

NO FOOTNOTE

N.B. a lot written on this, will PASS for now.

(114)

OEM: **§ 5. The Composition of Velocities**

AFK: A velocity addition theorem

(115)

OEM: In the system *k* moving along the axis of X of the system K with velocity *v*, let a point move in accordance with the equations

$$\xi = w_\xi \tau, \eta = w_\eta \tau, \zeta = 0,$$

where w_ξ and w_η denote constants.

AFK: Suppose a point object moves with velocity w with respect to the system *k,* which itself is moving with velocity *v* with respect to K, so that the equations of motion of the object in k are

$$\xi = w_\xi \tau, \eta = w_\eta \tau, \zeta = 0,$$

where w_ξ and w_η are constants.

(116)

OEM: Required: the motion of the point relatively to the system K. If with the help of the equations of transformation developed in § 3 we introduce the quantities *x*, *y*, *z*, *t* into the equations of motion of the point, we obtain

$$x = \frac{w_\xi + v}{1 + v w_\xi / c^2} t,$$
$$y = \frac{\sqrt{1 - v^2/c^2}}{1 + v w_\xi / c^2} w_\eta t,$$
$$z = 0.$$

AFK: We seek to express the motion of this object with respect to K. By expressing the equations of motion of the object in terms of *x*, *y*, *z*, *t* using the transformations developed above in § 3 gives

$$x = \frac{w_\xi + v}{1 + v w_\xi / c^2} t,$$
$$y = \frac{\sqrt{1 - v^2/c^2}}{1 + v w_\xi / c^2} w_\eta t,$$
$$z = 0.$$

(117)

OEM: Thus the law of the parallelogram of velocities is valid according to our theory only to a first approximation. We set

$$V^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2,$$
$$w^2 = w_\xi^2 + w_\eta^2,$$
$$a = \tan^{-1} w_\eta / w_\xi,$$

a is then to be looked upon as the angle between the velocities *v* and *w*.

TRANSLATOR NOTE: This equation was incorrectly given in Einstein's original paper and the 1923 English translation as $a = \tan^{-1} w_y / w_x$.

AFK: The parallelogram law for velocities in our theory holds only in a first approximation. Set

$$
\begin{aligned}
V^2 &= \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2, \\
w^2 &= w_\xi^2 + w_\eta^2, \\
a &= \tan^{-1} w_\eta / w_\xi,
\end{aligned}
$$

Where *a* is then the angle between v and *w*.

(118)

OEM: After a simple calculation we obtain

$$V = \frac{\sqrt{(v^2 + w^2 + 2vw\cos a) - (vw\sin a/c)^2}}{1 + vw\cos a/c^2}.$$

TRANSLATOR NOTE: The exponent of *c* in the denominator of the sine term of this equation was erroneously given as 2 in the 1923 edition of this paper. It has been corrected to unity here.

AFK: After some algebra one gets:

$$V = \frac{\sqrt{(v^2 + w^2 + 2vw\cos a) - (vw\sin a/c)^2}}{1 + vw\cos a/c^2}.$$

(119)

OEM: It is worthy of remark that *v* and *w* enter into the expression for the resultant velocity in a symmetrical manner. If *w* also has the direction of the axis of X, we get

$$V = \frac{v + w}{1 + vw/c^2}.$$

AFK: It is noteworthy that *v* and *w* enter into the expression symmetrically. If *w* parallel to v, i.e. along the x – axis (SYMBOL-axis). Then one gets:

$$V = \frac{v + w}{1 + vw/c^2}.$$

(120)

OEM: It follows from this equation that from a composition of two velocities which are less than *c*, there always results a velocity less than *c*. For if we set $v = c - \kappa, w = c - \lambda$, κ and λ being positive and less than *c*, then

$$V = c\frac{2c - \kappa - \lambda}{2c - \kappa - \lambda + \kappa\lambda/c} < c.$$

AFK: From this equation it follows that, for the composition of two velocities, each less than *c*, the result is also less than *c*. For example if we set

$v = c - \kappa, w = c - \lambda$, where κ and λ are less than *c*, then

$$V = c\frac{2c - \kappa - \lambda}{2c - \kappa - \lambda + \kappa\lambda/c} < c.$$

n.b. different notation

(121)

OEM: It follows, further, that the velocity of light *c* cannot be altered by composition with a velocity less than that of light. For this case we obtain

$$V = \frac{c + w}{1 + w/c} = c.$$

AFK: Moreover, it can be seen that composing the velocity of light *c* with any subliminal velocity, results in no change. For this case one gets

$$V = \frac{c + w}{1 + w/c} = c.$$

(122)

OEM: We might also have obtained the formula for V, for the case when *v* and *w* have the same direction, by compounding two transformations in accordance with § 3. If in addition to the systems K and *k* figuring in § 3 we introduce still another system of co-ordinates *k*' moving parallel to *k*, its initial point moving on the axis of Ξ[*5] with the velocity *w*, we obtain equations between the quantities *x*, *y*, *z*, *t* and the corresponding quantities of *k*', which differ from the equations found in § 3 only in that the place of “*v*” is taken by the quantity

$$\frac{v+w}{1+vw/c^2};$$

from which we see that such parallel transformations—necessarily—form a group.

TRANSLATOR NOTE: “X” in the 1923 English translation.

AFK: We would have gotten the formula for V as the composition, if they are parallel, of v and *w* using the transformations from § 3 twice. By introducing yet another systems, k’, in addition to K and *k,* again in motion parallel to the motion of k but with velocity w, then in the transformation from K to k’, instead of the velocity v, one finds the expression

$$\frac{v+w}{1+vw/c^2};$$

And one sees thereby that such parallel transformations constitute a group.

(123)

OEM: We have now deduced the requisite laws of the theory of kinematics corresponding to our two principles, and we proceed to show their application to electrodynamics.

AFK: Hence, we have derived the kinematics resulting from our two principles, and so, we now proceed to develop their application in electrodynamics.

(124)

OEM: **II. ELECTRODYNAMICAL PART**

AFK: II: Electrodynamics

(126)

OEM: § 6. Transformation of the Maxwell-Hertz Equations for Empty Space. On the Nature of the Electromotive Forces Occurring in a Magnetic Field During Motion

AFK: § 6. The transformations of the Maxwell-Hertz Equations in free Space. On the Nature of the Electromotive Forces from motion in Magnetic Fields

(127)

OEM: Let the Maxwell-Hertz equations for empty space hold good for the stationary system K, so that we have

$$\begin{aligned}
\frac{1}{c}\frac{\partial X}{\partial t} &= \frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}, & \frac{1}{c}\frac{\partial L}{\partial t} &= \frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}, \\
\frac{1}{c}\frac{\partial Y}{\partial t} &= \frac{\partial L}{\partial z} - \frac{\partial N}{\partial x}, & \frac{1}{c}\frac{\partial M}{\partial t} &= \frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z}, \\
\frac{1}{c}\frac{\partial Z}{\partial t} &= \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}, & \frac{1}{c}\frac{\partial N}{\partial t} &= \frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x},
\end{aligned}$$

where (X, Y, Z) denotes the vector of the electric force, and (L, M, N) that of the magnetic force.

AFK: The Maxwell-Hertz equations in free space and valid in the rest system K are:

$$\begin{aligned}
\frac{1}{c}\frac{\partial X}{\partial t} &= \frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}, & \frac{1}{c}\frac{\partial L}{\partial t} &= \frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}, \\
\frac{1}{c}\frac{\partial Y}{\partial t} &= \frac{\partial L}{\partial z} - \frac{\partial N}{\partial x}, & \frac{1}{c}\frac{\partial M}{\partial t} &= \frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z}, \\
\frac{1}{c}\frac{\partial Z}{\partial t} &= \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}, & \frac{1}{c}\frac{\partial N}{\partial t} &= \frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x},
\end{aligned}$$

where (X, Y, Z) represents the vector of electric force and (L, M, N) the vector of magnetic force.

(128)

OEM: If we apply to these equations the transformation developed in § 3, by referring the electromagnetic processes to the system of co-ordinates there introduced, moving with the velocity *v*, we obtain the equations[*6]

$$\begin{aligned}
\frac{1}{c}\frac{\partial X}{\partial \tau} &= \frac{\partial}{\partial \eta}\left\{\beta\left(N - \frac{v}{c}Y\right)\right\} & &-\frac{\partial}{\partial \zeta}\left\{\beta\left(M + \frac{v}{c}Z\right)\right\}, \\
\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(Y - \frac{v}{c}N\right)\right\} &= \frac{\partial L}{\partial \zeta} & &-\frac{\partial}{\partial \xi}\left\{\beta\left(N - \frac{v}{c}Y\right)\right\}, \\
\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(Z + \frac{v}{c}M\right)\right\} &= \frac{\partial}{\partial \xi}\left\{\beta\left(M + \frac{v}{c}Z\right)\right\} & &-\frac{\partial L}{\partial \eta}, \\
\frac{1}{c}\frac{\partial L}{\partial \tau} &= \frac{\partial}{\partial \zeta}\left\{\beta\left(Y - \frac{v}{c}N\right)\right\} & &-\frac{\partial}{\partial \eta}\left\{\beta\left(Z + \frac{v}{c}M\right)\right\}, \\
\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(M + \frac{v}{c}Z\right)\right\} &= \frac{\partial}{\partial \xi}\left\{\beta\left(Z + \frac{v}{c}M\right)\right\} & &-\frac{\partial X}{\partial \zeta}, \\
\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(N - \frac{v}{c}Y\right)\right\} &= \frac{\partial X}{\partial \eta} & &-\frac{\partial}{\partial \xi}\left\{\beta\left(Y - \frac{v}{c}N\right)\right\},
\end{aligned}$$

where

$\beta = 1/\sqrt{1 - v^2/c^2}$.

TRANSLATOR NOTE:

In the 1923 English translation, the quantities "ζ" and "ξ" were interchanged in the second equation. They were given correctly in the original 1905 paper.

AFK: If we use the transformations from § 3 to transform these electromagnetic processes to the frame moving with velocity *v*, we get

$$\begin{aligned}
\frac{1}{c}\frac{\partial \mathrm{X}}{\partial \tau} &= \frac{\partial}{\partial \eta}\left\{\beta\left(\mathrm{N} - \frac{v}{c}\mathrm{Y}\right)\right\} - \frac{\partial}{\partial \zeta}\left\{\beta\left(\mathrm{M} + \frac{v}{c}\mathrm{Z}\right)\right\}, \\
\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(\mathrm{Y} - \frac{v}{c}\mathrm{N}\right)\right\} &= \frac{\partial \mathrm{L}}{\partial \zeta} - \frac{\partial}{\partial \xi}\left\{\beta\left(\mathrm{N} - \frac{v}{c}\mathrm{Y}\right)\right\}, \\
\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(\mathrm{Z} + \frac{v}{c}\mathrm{M}\right)\right\} &= \frac{\partial}{\partial \xi}\left\{\beta\left(\mathrm{M} + \frac{v}{c}\mathrm{Z}\right)\right\} - \frac{\partial \mathrm{L}}{\partial \eta}, \\
\frac{1}{c}\frac{\partial \mathrm{L}}{\partial \tau} &= \frac{\partial}{\partial \zeta}\left\{\beta\left(\mathrm{Y} - \frac{v}{c}\mathrm{N}\right)\right\} - \frac{\partial}{\partial \eta}\left\{\beta\left(\mathrm{Z} + \frac{v}{c}\mathrm{M}\right)\right\}, \\
\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(\mathrm{M} + \frac{v}{c}\mathrm{Z}\right)\right\} &= \frac{\partial}{\partial \xi}\left\{\beta\left(\mathrm{Z} + \frac{v}{c}\mathrm{M}\right)\right\} - \frac{\partial \mathrm{X}}{\partial \zeta}, \\
\frac{1}{c}\frac{\partial}{\partial \tau}\left\{\beta\left(\mathrm{N} - \frac{v}{c}\mathrm{Y}\right)\right\} &= \frac{\partial \mathrm{X}}{\partial \eta} - \frac{\partial}{\partial \xi}\left\{\beta\left(\mathrm{Y} - \frac{v}{c}\mathrm{N}\right)\right\},
\end{aligned}$$

Where, again,

$\beta = 1/\sqrt{1 - v^2/c^2}$.

(129)

OEM: Now the principle of relativity requires that if the Maxwell-Hertz equations for empty space hold good in system K, they also hold good in system *k*; that is to say that the vectors of the electric and the magnetic force—(X', Y', Z') and (L', M', N')—of the moving system *k*, which are defined by their ponderomotive effects on electric or magnetic masses respectively, satisfy the following equations:—

$$\begin{aligned}
\frac{1}{c}\frac{\partial \mathrm{X}'}{\partial \tau} &= \frac{\partial \mathrm{N}'}{\partial \eta} - \frac{\partial \mathrm{M}'}{\partial \zeta}, & \frac{1}{c}\frac{\partial \mathrm{L}'}{\partial \tau} &= \frac{\partial \mathrm{Y}'}{\partial \zeta} - \frac{\partial \mathrm{Z}'}{\partial \eta}, \\
\frac{1}{c}\frac{\partial \mathrm{Y}'}{\partial \tau} &= \frac{\partial \mathrm{L}'}{\partial \zeta} - \frac{\partial \mathrm{N}'}{\partial \xi}, & \frac{1}{c}\frac{\partial \mathrm{M}'}{\partial \tau} &= \frac{\partial \mathrm{Z}'}{\partial \xi} - \frac{\partial \mathrm{X}'}{\partial \zeta}, \\
\frac{1}{c}\frac{\partial \mathrm{Z}'}{\partial \tau} &= \frac{\partial \mathrm{M}'}{\partial \xi} - \frac{\partial \mathrm{L}'}{\partial \eta}, & \frac{1}{c}\frac{\partial \mathrm{N}'}{\partial \tau} &= \frac{\partial \mathrm{X}'}{\partial \eta} - \frac{\partial \mathrm{Y}'}{\partial \xi}.
\end{aligned}$$

AFK: The principle of relativity demands that, the Maxwell-Hertz equations for free space, if they are valid in K, shall be valid also in k; that is, the expressions for the ponderomotive effect of electric and magnetic forces expressed in the k’ system in terms of (X', Y', Z') and (L', M', N') respectively, satisfy the equations:

$$\begin{aligned}
\frac{1}{c}\frac{\partial X'}{\partial \tau} &= \frac{\partial N'}{\partial \eta} - \frac{\partial M'}{\partial \zeta}, & \frac{1}{c}\frac{\partial L'}{\partial \tau} &= \frac{\partial Y'}{\partial \zeta} - \frac{\partial Z'}{\partial \eta},\\
\frac{1}{c}\frac{\partial Y'}{\partial \tau} &= \frac{\partial L'}{\partial \zeta} - \frac{\partial N'}{\partial \xi}, & \frac{1}{c}\frac{\partial M'}{\partial \tau} &= \frac{\partial Z'}{\partial \xi} - \frac{\partial X'}{\partial \zeta},\\
\frac{1}{c}\frac{\partial Z'}{\partial \tau} &= \frac{\partial M'}{\partial \xi} - \frac{\partial L'}{\partial \eta}, & \frac{1}{c}\frac{\partial N'}{\partial \tau} &= \frac{\partial X'}{\partial \eta} - \frac{\partial Y'}{\partial \xi}.
\end{aligned}$$

(130)

OEM: Evidently the two systems of equations found for system *k* must express exactly the same thing, since both systems of equations are equivalent to the Maxwell-Hertz equations for system K.

AFK: Obviously, both sets of equations for k must express the same reality, as both sets are equivalent Maxwell-Hertz systems.

(131)

OEM: Since, further, the equations of the two systems agree, with the exception of the symbols for the vectors, it follows that the functions occurring in the systems of equations at corresponding places must agree, with the exception of a factor $\psi(v)$, which is common for all functions of the one system of equations, and is independent of ξ, η, ζ and τ but depends upon *v*.

AFK: In that the equations in both systems agree with each other up to the vector symbols, it follows that, the functions in these equations at corresponding positions must be equal, with possible exception of a common function independent of ξ, η, ζ and τ but maybe not of *v*.

(132)

OEM: Thus we have the relations

$$\begin{aligned}
X' &= \psi(v)X, & L' &= \psi(v)L,\\
Y' &= \psi(v)\beta\left(Y - \tfrac{v}{c}N\right), & M' &= \psi(v)\beta\left(M + \tfrac{v}{c}Z\right),\\
Z' &= \psi(v)\beta\left(Z + \tfrac{v}{c}M\right), & N' &= \psi(v)\beta\left(N - \tfrac{v}{c}Y\right).
\end{aligned}$$

AFK: That is, it may be written:

$$
\begin{aligned}
X' &= \psi(v)X, & L' &= \psi(v)L, \\
Y' &= \psi(v)\beta\left(Y - \tfrac{v}{c}N\right), & M' &= \psi(v)\beta\left(M + \tfrac{v}{c}Z\right), \\
Z' &= \psi(v)\beta\left(Z + \tfrac{v}{c}M\right), & N' &= \psi(v)\beta\left(N - \tfrac{v}{c}Y\right).
\end{aligned}
$$

(133)

OEM: If we now form the reciprocal of this system of equations, firstly by solving the equations just obtained, and secondly by applying the equations to the inverse transformation (from *k* to K), which is characterized by the velocity -*v*, it follows, when we consider that the two systems of equations thus obtained must be identical, that $\psi(v)\psi(-v) = 1$.

AFK: If one now constructs the inverse equation, firstly by solving these equations and secondly by use of the equations for the inverse transformation, i.e., from *k* to K, which are characterized by the velocity -*v*, then it follows that the resulting sets of equations must be identical: $\psi(v)\psi(-v) = 1$.

(134)

OEM: Further, from reasons of symmetry[8] and therefore

$$\psi(v) = 1,$$

and our equations assume the form

$$
\begin{aligned}
X' &= X, & L' &= L, \\
Y' &= \beta\left(Y - \tfrac{v}{c}N\right), & M' &= \beta\left(M + \tfrac{v}{c}Z\right), \\
Z' &= \beta\left(Z + \tfrac{v}{c}M\right), & N' &= \beta\left(N - \tfrac{v}{c}Y\right).
\end{aligned}
$$

FOOTNOTE: If, for example, X=Y=Z=L=M=0, and N $\neq$ 0, then from reasons of symmetry it is clear that when *v* changes sign without changing its numerical value, Y' must also change sign without changing its numerical value.

AFK: Further, it follows from reasons of symmetry:

ψ (v) = ψ(-v)

thus

$$\psi(v) = 1,$$

and the equation set above becomes:

$$\begin{aligned} X' &= X, & L' &= L, \\ Y' &= \beta\left(Y - \frac{v}{c}N\right), & M' &= \beta\left(M + \frac{v}{c}Z\right), \\ Z' &= \beta\left(Z + \frac{v}{c}M\right), & N' &= \beta\left(N - \frac{v}{c}Y\right). \end{aligned}$$

FOOTNOTE: If, for example, X=Y=Z=L=M=0, and N $\neq$ 0, then from symmetry it is clear that, a change of sign without changing numerical value, implies then that Y' too must change its sign without changing its numerical value.

ME: the equation ψ (v) = ψ(-v) was missed out in the first translation; but don't think it significant; it is in "A Stubborn persistent illusion: the essential scientific works of Albert Einstein", by Stephen Hawking, isbn-13 978-0-7624-3564-7 p.20

(135)

OEM: As to the interpretation of these equations we make the following remarks: Let a point charge of electricity have the magnitude "one" when measured in the stationary system K, i.e. let it when at rest in the stationary system exert a force of one dyne upon an equal quantity of electricity at a distance of one cm.

AFK: To interpret these equations, we first note the following. Suppose there is a point charge in K of magnitude "1" unit of charge, i.e., it experiences 1 dyne of force when 1 cm from an identical charge.

(136)

OEM: By the principle of relativity this electric charge is also of the magnitude "one" when measured in the moving system.

AFK: According to the principle of relativity, this charge will also be seen to be of magnitude 1 when measured from K, although located in k, i.e., moving system with velocity v.

(137)

OEM: If this quantity of electricity is at rest relatively to the stationary system, then by definition the vector (X, Y, Z) is equal to the force acting upon it.

AFK: If this charge is at rest in the rest system, then by definition the vector (X, Y, Z) is equal to the force acting on it.

(138)

OEM: If the quantity of electricity is at rest relatively to the moving system (at least at the relevant instant), then the force acting upon it, measured in the moving system, is equal to the vector (X', Y', Z').

AFK: If, however, the charge is at rest in the moving system (at least at the moment), then the force in k acting on it is expressed with the vector (X', Y', Z').

(139)

OEM: Consequently the first three equations above allow themselves to be clothed in words in the two following ways: —

AFK: The first three of the equations can be described with words in the following two possibilities:

(140)

OEM: 1.If a unit electric point charge is in motion in an electromagnetic field, there acts upon it, in addition to the electric force, an "electromotive force" which, if we neglect the terms multiplied by the second and higher powers of *v/c*, is equal to the vector-product of the velocity of the charge and the magnetic force, divided by the velocity of light. (Old manner of expression.)

AFK: 1. If a point charge moves in an electromagnetic field, then, in addition to an electric force there was an "electromotive force," which, when neglecting effects described by terms of order greater than two in (*v/c)*, is equal to the vector product of its velocity with the magnetic force divided by c (old notation).

(141)

OEM: 2. If a unit electric point charge is in motion in an electromagnetic field, the force acting upon it is equal to the electric force which is present at the locality of the charge, and which we ascertain by transformation of the field to a system of co-ordinates at rest relatively to the electrical charge. (New manner of expression.)

AFK: 2. If a point charge moves in an electromagnetic field, the force on it equals the local electric force, which one gets by transforming the field to the charge's rest system (new notation).

(142)

OEM: The analogy holds with "magnetomotive forces." We see that electromotive force plays in the developed theory merely the part of an auxiliary concept, which owes its introduction to the circumstance that electric and magnetic forces do not exist independently of the state of motion of the system of co-ordinates.

AFK: Analogous arguments pertain to "magnetic forces." One sees that in the new theory electromotive force only play a role as aids, which are introduced thanks to the circumstance that electric and magnetic forces themselves are not independent of the state of coordinate system's motion.

(143)

OEM: Furthermore it is clear that the asymmetry mentioned in the introduction as arising when we consider the currents produced by the relative motion of a magnet and a conductor, now disappears. Moreover, questions as to the "seat" of electrodynamic electromotive forces (unipolar machines) now have no point.

AFK: Moreover it is clear, that the asymmetry between magnets and conductors with respect to relative motion vanishes. Also, issues regarding the venue of electrodynamical electromotive forces (e.g., in unipolar machines), are meaningless.

(144)

OEM: **§ 7. Theory of Doppler's Principle and of Aberration**

AFK: 7. Theory of the Doppler effect and aberration

(145)

OEM: In the system K, very far from the origin of co-ordinates, let there be a source of electrodynamic waves, which in a part of space containing the origin of co-ordinates may be represented to a sufficient degree of approximation by the equations

$$\begin{aligned} X &= X_0 \sin\Phi, & L &= L_0 \sin\Phi, \\ Y &= Y_0 \sin\Phi, & M &= M_0 \sin\Phi, \\ Z &= Z_0 \sin\Phi, & N &= N_0 \sin\Phi, \end{aligned}$$

where

$$\Phi = \omega\left\{t - \frac{1}{c}(lx + my + nz)\right\}.$$

Here (X_0, Y_0, Z_0) and (L_0, M_0, N_0) are the vectors defining the amplitude of the

wave-train, and *l*, *m*, *n* the direction-cosines of the wave-normals.

AFK: Suppose that in the system K very far from the origin there be a source of electrodynamic waves, which in a region around the coordinate origin can be expressed to a sufficient accuracy as follows

$$\begin{aligned} X &= X_0 \sin\Phi, & L &= L_0 \sin\Phi, \\ Y &= Y_0 \sin\Phi, & M &= M_0 \sin\Phi, \\ Z &= Z_0 \sin\Phi, & N &= N_0 \sin\Phi, \end{aligned}$$

where

$$\Phi = \omega\left\{t - \frac{1}{c}(lx + my + nz)\right\}.$$

and (X_0, Y_0, Z_0) , (L_0, M_0, N_0) are the wave's electric and magnetic field vectors respectively and l,m,n are direction cosines of the wave vector.

N.B. Kracklauer uses different notation.

(146)

OEM: We wish to know the constitution of these waves, when they are examined by an observer at rest in the moving system *k*.

AFK: Now we investigate the characteristics of this waves as seen by an observer at rest in a moving system *k*.

(147)

OEM: Applying the equations of transformation found in § 6 for electric and magnetic forces, and those found in § 3 for the co-ordinates and the time, we obtain directly

$$\begin{array}{ll} \mathrm{X}' = \mathrm{X}_0 \sin \Phi', & \mathrm{L}' = \mathrm{L}_0 \sin \Phi', \\ \mathrm{Y}' = \beta(\mathrm{Y}_0 - v\mathrm{N}_0/c) \sin \Phi', & \mathrm{M}' = \beta(\mathrm{M}_0 + v\mathrm{Z}_0/c) \sin \Phi', \\ \mathrm{Z}' = \beta(\mathrm{Z}_0 + v\mathrm{M}_0/c) \sin \Phi', & \mathrm{N}' = \beta(\mathrm{N}_0 - v\mathrm{Y}_0/c) \sin \Phi', \end{array}$$
$$\Phi' = \omega' \left\{ \tau - \tfrac{1}{c}(l'\xi + m'\eta + n'\zeta) \right\}$$

where

$$\begin{aligned} \omega' &= \omega\beta(1 - lv/c), \\ l' &= \frac{l - v/c}{1 - lv/c}, \\ m' &= \frac{m}{\beta(1 - lv/c)}, \\ n' &= \frac{n}{\beta(1 - lv/c)}. \end{aligned}$$

AFK: By means of the transformations found in § 6 for the coordinates and time, we get

$$\begin{array}{ll} \mathrm{X}' = \mathrm{X}_0 \sin \Phi', & \mathrm{L}' = \mathrm{L}_0 \sin \Phi', \\ \mathrm{Y}' = \beta(\mathrm{Y}_0 - v\mathrm{N}_0/c) \sin \Phi', & \mathrm{M}' = \beta(\mathrm{M}_0 + v\mathrm{Z}_0/c) \sin \Phi', \\ \mathrm{Z}' = \beta(\mathrm{Z}_0 + v\mathrm{M}_0/c) \sin \Phi', & \mathrm{N}' = \beta(\mathrm{N}_0 - v\mathrm{Y}_0/c) \sin \Phi', \end{array}$$
$$\Phi' = \omega' \left\{ \tau - \tfrac{1}{c}(l'\xi + m'\eta + n'\zeta) \right\}$$

and

$$\begin{aligned} \omega' &= \omega\beta(1 - lv/c), \\ l' &= \frac{l - v/c}{1 - lv/c}, \\ m' &= \frac{m}{\beta(1 - lv/c)}, \\ n' &= \frac{n}{\beta(1 - lv/c)}. \end{aligned}$$

n.b. AFK uses different notation

(148)

OEM: From the equation for ω'it follows that if an observer is moving with velocity *v* relatively to an infinitely distant source of light of frequency ν, in such a way that the connecting line “source-observer” makes the angle ϕ with the velocity of the observer referred to a system of co-ordinates which is at rest relatively to the source of light, the frequency ν' of the light perceived by the observer is given by the equation

$$\nu' = \nu\frac{1-\cos\phi\cdot v/c}{\sqrt{1-v^2/c^2}}.$$

AFK: From the equation for ω' follows: If an observer of an infinitely distant light source emitting radiation of frequency ν, such that his velocity vector makes the angle ϕ with the line of sight as expressed in the source’s rest frame, then this observer sees light with frequency ν' given by

$$\nu' = \nu\frac{1-\cos\phi\cdot v/c}{\sqrt{1-v^2/c^2}}.$$

(149)

OEM: This is Doppler's principle for any velocities whatever. When $\phi = 0$ the equation assumes the perspicuous form

$$\nu' = \nu\sqrt{\frac{1-v/c}{1+v/c}}.$$

AFK: This is the Doppler principle for arbitrary velocity. For $\phi = 0$ this formula takes on the well known form

$$\nu' = \nu\sqrt{\frac{1-v/c}{1+v/c}}.$$

(150)

OEM: We see that, in contrast with the customary view, when $v = -c, \nu' = \infty$.

AFK: Here one sees, in contrast to the usual formulation, that for $v = -c, \nu' = \infty$.

(151)

OEM: If we call the angle between the wave-normal (direction of the ray) in the moving system and the connecting line “source-observer” ϕ' , the equation for ϕ' *7 assumes the form

$$\cos\phi' = \frac{\cos\phi - v/c}{1 - \cos\phi \cdot v/c}.$$

Translator note: Erroneously given as *l′* in the 1923 English translation, propagating an error, despite a change in symbols, from the original 1905 paper.

AFK: If one calls ϕ' the angle between the wave vector in the moving system and the direction of motion, then the equation for l’ takes the form

$$\cos\phi' = \frac{\cos\phi - v/c}{1 - \cos\phi \cdot v/c}.$$

n.b. AFK uses different notation uses a’ not l’. BUT check with footnote above

(152)

OEM: This equation expresses the law of aberration in its most general form. If $\phi = \frac{1}{2}\pi$, the equation becomes simply

$$\cos\phi' = -v/c.$$

AFK: This equation gives aberration law in its most general form. When $\phi = \frac{1}{2}\pi$, it can be simplified:

$$\cos\phi' = -v/c.$$

(153)

OEM: We still have to find the amplitude of the waves, as it appears in the moving system. If we call the amplitude of the electric or magnetic force A or A' respectively, accordingly as it is measured in the stationary system or in the moving system, we obtain

$$\mathrm{A}'^2 = \mathrm{A}^2 \frac{(1-\cos\phi \cdot v/c)^2}{1-v^2/c^2}$$

which equation, if $\phi = 0$, simplifies into

$$\mathrm{A}'^2 = \mathrm{A}^2 \frac{1-v/c}{1+v/c}.$$

AFK: Now we still must find the amplitude of the wave as it is seen in the moving system. If A or A' are the amplitude of the electric or magnetic forces in the rest or moving system respectively, one gets

$$\mathrm{A}'^2 = \mathrm{A}^2 \frac{(1-\cos\phi \cdot v/c)^2}{1-v^2/c^2}$$

which for $\phi = 0$ takes the simplified form

$$\mathrm{A}'^2 = \mathrm{A}^2 \frac{1-v/c}{1+v/c}.$$

(154)

OEM: It follows from these results that to an observer approaching a source of light with the velocity *c*, this source of light must appear of infinite intensity.

AFK: It follows from these equations that, for an observer moving with velocity approaching that of light, *c*, the light intensity must approach infinity.

(155)

OEM: § 8. Transformation of the Energy of Light Rays. Theory of the Pressure of Radiation Exerted on Perfect Reflectors

AFK: Transforming light ray energy. The theory of light pressure on a total reflector

(156)

OEM:

Since $A^2/8\pi$ equals the energy of light per unit of volume, we have to regard

$A'^2/8\pi$, by the principle of relativity, as the energy of light in the moving system.

AFK:

As $A^2/8\pi$ equals the energy density of light, according to the principle of relativity,

$A'^2/8\pi$ must be regarded as seen from the moving system.

(157)

OEM: Thus

A'^2/A^2 would be the ratio of the “measured in motion” to the “measured at rest” energy of

a given light complex, if the volume of a light complex were the same, whether measured in K or in *k*.

AFK: Thus,

A'^2/A^2 should be the ratio of “moving to stationary” energy of a particular light-complex if the volumes as measured in k and K respectively are equal.

(158)

OEM: But this is not the case. If *l*, *m*, *n* are the direction-cosines of the wave-normals of the light in the stationary system, no energy passes through the surface elements of a spherical surface moving with the velocity of light: —

$$(x - lct)^2 + (y - mct)^2 + (z - nct)^2 = R^2.$$

We may therefore say that this surface permanently encloses the same light complex.

AFK: But, this is not the case. If *l*, *m*, *n* are the direction cosines of the wave vector in the rest system, then no energy penetrates the surface of the expanding sphere given by:

$$(x - lct)^2 + (y - mct)^2 + (z - nct)^2 = \mathrm{R}^2.$$

we can say, therefore, that this surface always encompasses the light-complex.

n.b. AFK different notation

(159)

OEM: We inquire as to the quantity of energy enclosed by this surface, viewed in system *k*, that is, as to the energy of the light complex relatively to the system *k*.

AFK: We now ask what surface in *k* contains this energy density relative to *k*.

(160)

OEM: The spherical surface—viewed in the moving system—is an ellipsoidal surface, the equation for which, at the time $\tau = 0$, is

$$(\beta\xi - l\beta\xi v/c)^2 + (\eta - m\beta\xi v/c)^2 + (\zeta - n\beta\xi v/c)^2 = \mathrm{R}^2.$$

AFK: The spherical surface as seen in the moving system is an ellipsoidal of revolution, which, at e time $\tau = 0$, satisfies the equation

$$(\beta\xi - l\beta\xi v/c)^2 + (\eta - m\beta\xi v/c)^2 + (\zeta - n\beta\xi v/c)^2 = \mathrm{R}^2.$$

(161)

OEM: If S is the volume of the sphere, and S' that of this ellipsoid, then by a simple calculation

$$\frac{\mathrm{S}'}{\mathrm{S}} = \frac{\sqrt{1 - v^2/c^2}}{1 - \cos\phi \cdot v/c}.$$

AFK: Calling the volume of the sphere S and that of this ellipsoid S’, then by a simple calculations shows

$$\frac{S'}{S} = \frac{\sqrt{1 - v^2/c^2}}{1 - \cos\phi \cdot v/c}.$$

(162)

OEM: Thus, if we call the light energy enclosed by this surface E when it is measured in the stationary system, and E' when measured in the moving system, we obtain

$$\frac{E'}{E} = \frac{A'^2 S'}{A^2 S} = \frac{1 - \cos\phi \cdot v/c}{\sqrt{1 - v^2/c^2}},$$

and this formula, $\phi = 0$ when , simplifies into

$$\frac{E'}{E} = \sqrt{\frac{1 - v/c}{1 + v/c}}.$$

AFK: Calling E and E’ the measured energy in the rest moving systems respectively, one gets:

$$\frac{E'}{E} = \frac{A'^2 S'}{A^2 S} = \frac{1 - \cos\phi \cdot v/c}{\sqrt{1 - v^2/c^2}},$$

Which for $\phi = 0$

Can be simplified to

$$\frac{E'}{E} = \sqrt{\frac{1 - v/c}{1 + v/c}}.$$

(163)

OEM: It is remarkable that the energy and the frequency of a light complex vary with the state of motion of the observer in accordance with the same law.

AFK: It is noteworthy that, the energy and frequency of a light complex changes according to the same laws as those governing the state of motion of the observer.

(164)

OEM: Now let the co- $\xi = 0$ ordinate plane be a perfectly reflecting surface, at which the

plane waves considered in § 7 are reflected. We seek for the pressure of light exerted on the reflecting surface, and for the direction, frequency, and intensity of the light after reflexion.

AFK: Suppose that at $\xi = 0$

There is a fully reflecting plane on which the wave considered in the last paragraph is reflected.

(165)

OEM: We seek for the pressure of light exerted on the reflecting surface, and for the direction, frequency, and intensity of the light after reflexion.

AFK: Let us determine the pressure exercised by reflection of the wave on the plane, as well as the direction, frequency and intensity of the reflected wave,

(166)

OEM: Let the incidental light be defined by the $\cos\phi$ quantities A, , ν(referred to system K).

AFK: The incoming light is specified by the amplitude A,

$\cos\phi$

And ν (with respect to K).

(167)

OEM: Viewed from *k* the corresponding quantities are

$$
\begin{aligned}
\mathrm{A}' &= \mathrm{A}\frac{1-\cos\phi\cdot v/c}{\sqrt{1-v^2/c^2}},\\
\cos\phi' &= \frac{\cos\phi - v/c}{1-\cos\phi\cdot v/c},\\
\nu' &= \nu\frac{1-\cos\phi\cdot v/c}{\sqrt{1-v^2/c^2}}.
\end{aligned}
$$

AFK: As seen in k, these parameters are:

$$
\begin{aligned}
\mathrm{A}' &= \mathrm{A}\frac{1-\cos\phi\cdot v/c}{\sqrt{1-v^2/c^2}},\\
\cos\phi' &= \frac{\cos\phi - v/c}{1-\cos\phi\cdot v/c},\\
\nu' &= \nu\frac{1-\cos\phi\cdot v/c}{\sqrt{1-v^2/c^2}}.
\end{aligned}
$$

(168)

OEM: For the reflected light, referring the process to system *k*, we obtain

$$
\begin{aligned}
\mathbf{A}'' &= \mathbf{A}'\\
\cos\phi'' &= -\cos\phi'\\
\nu'' &= \nu'
\end{aligned}
$$

AFK: For the reflected light we get

$$
\begin{aligned}
\mathbf{A}'' &= \mathbf{A}'\\
\cos\phi'' &= -\cos\phi'\\
\nu'' &= \nu'
\end{aligned}
$$

(169)

OEM: Finally, by transforming back to the stationary system K, we obtain for the reflected light

$$\begin{aligned}
A''' &= A''\frac{1+cos\phi''\cdot v/c}{\sqrt{1-v^2/c^2}} = A\frac{1-2\cos\phi\cdot v/c+v^2/c^2}{1-v^2/c^2}, \\
\cos\phi''' &= \frac{\cos\phi''+v/c}{1+\cos\phi''\cdot v/c} = -\frac{(1+v^2/c^2)\cos\phi-2v/c}{1-2\cos\phi\cdot v/c+v^2/c^2}, \\
\nu''' &= \nu''\frac{1+\cos\phi''\cdot v/c}{\sqrt{1-v^2/c^2}} = \nu\frac{1-2\cos\phi\cdot v/c+v^2/c^2}{1-v^2/c^2}.
\end{aligned}$$

AFK: Finally one gets with the reverse transforms to the rest system K of the reflected light

$$\begin{aligned}
A''' &= A''\frac{1+cos\phi''\cdot v/c}{\sqrt{1-v^2/c^2}} = A\frac{1-2\cos\phi\cdot v/c+v^2/c^2}{1-v^2/c^2}, \\
\cos\phi''' &= \frac{\cos\phi''+v/c}{1+\cos\phi''\cdot v/c} = -\frac{(1+v^2/c^2)\cos\phi-2v/c}{1-2\cos\phi\cdot v/c+v^2/c^2}, \\
\nu''' &= \nu''\frac{1+\cos\phi''\cdot v/c}{\sqrt{1-v^2/c^2}} = \nu\frac{1-2\cos\phi\cdot v/c+v^2/c^2}{1-v^2/c^2}.
\end{aligned}$$

(170)

OEM: The energy (measured in the stationary system) which is incident

upon unit area of the mirror in unit time is $A^2(c\cos\phi - v)/8\pi$ evidently .

AFK: The energy falling on the mirror per unit of area

(as measured in the rest system) is obviously

$$A^2(c\cos\phi - v)/8\pi$$

(171)

OEM: The energy leaving the unit of surface of the

mirror in the unit of time $A'''^2(-c\cos\phi''' + v)/8\pi$ is .

AFK: The energy reflected from the

mirror per unit area and unit time

is . $A'''^2(-c\cos\phi''' + v)/8\pi$

(172)

OEM: The difference of these two expressions is, by the principle of energy, the work done by the pressure of light in the unit of time.

AFK: The difference between these two expressions, according to conservation of energy, is the work done by the light per unit of time.

(173)

OEM: If we set down this work as equal to the product P*v*, where P is the pressure of light, we obtain

$$P = 2 \cdot \frac{A^2}{8\pi} \frac{(\cos\phi - v/c)^2}{1 - v^2/c^2}.$$

AFK: If one sets this latter quantity equal to the product P*v*, where P is the pressure of light, one gets

$$P = 2 \cdot \frac{A^2}{8\pi} \frac{(\cos\phi - v/c)^2}{1 - v^2/c^2}.$$

(174)

OEM: In agreement with experiment and with other theories, we obtain to a first approximation

$$P = 2 \cdot \frac{A^2}{8\pi} \cos^2\phi.$$

AFK: To first approximation, this agrees with the expression from other methods, namely

$$P = 2 \cdot \frac{A^2}{8\pi} \cos^2\phi.$$

(175)

OEM: All problems in the optics of moving bodies can be solved by the method here employed. What is essential is, that the electric and magnetic force of the light which is influenced by a moving body, be transformed into a system of co-ordinates at rest relatively to the body. By this means all problems in the optics of moving bodies will be reduced to a series of problems in the optics of stationary bodies.

AFK: Using these methods all problems involving the optics of moving bodies can be solved. The basic tactic is to express the electric and magnetic fields acting on a moving body in a system at rest with respect to moving body in a system at rest with respect to the moving body so as to reformulate the optics of moving bodies as if it were the optics of stationary bodies.

(176)

OEM: § 9. Transformation of the Maxwell-Hertz Equations when Convection-Currents are Taken into Account

AFK: The transformation of the Maxwell-Hertz equations including convention current

(177)

OEM: We start from the equations

$$\begin{aligned} \frac{1}{c}\left\{\frac{\partial X}{\partial t}+u_x\rho\right\} &= \frac{\partial N}{\partial y}-\frac{\partial M}{\partial z}, & \frac{1}{c}\frac{\partial L}{\partial t} &= \frac{\partial Y}{\partial z}-\frac{\partial Z}{\partial y}, \\ \frac{1}{c}\left\{\frac{\partial Y}{\partial t}+u_y\rho\right\} &= \frac{\partial L}{\partial z}-\frac{\partial N}{\partial x}, & \frac{1}{c}\frac{\partial M}{\partial t} &= \frac{\partial Z}{\partial x}-\frac{\partial X}{\partial z}, \\ \frac{1}{c}\left\{\frac{\partial Z}{\partial t}+u_z\rho\right\} &= \frac{\partial M}{\partial x}-\frac{\partial L}{\partial y}, & \frac{1}{c}\frac{\partial N}{\partial t} &= \frac{\partial X}{\partial y}-\frac{\partial Y}{\partial x}, \end{aligned}$$

where

$$\rho=\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}$$

denotes 4π times the density of electricity, and (u_x,u_y,u_z) the velocity-vector of the charge.

AFK: We begin with the equations

$$\begin{aligned} \frac{1}{c}\left\{\frac{\partial X}{\partial t}+u_x\rho\right\} &= \frac{\partial N}{\partial y}-\frac{\partial M}{\partial z}, & \frac{1}{c}\frac{\partial L}{\partial t} &= \frac{\partial Y}{\partial z}-\frac{\partial Z}{\partial y}, \\ \frac{1}{c}\left\{\frac{\partial Y}{\partial t}+u_y\rho\right\} &= \frac{\partial L}{\partial z}-\frac{\partial N}{\partial x}, & \frac{1}{c}\frac{\partial M}{\partial t} &= \frac{\partial Z}{\partial x}-\frac{\partial X}{\partial z}, \\ \frac{1}{c}\left\{\frac{\partial Z}{\partial t}+u_z\rho\right\} &= \frac{\partial M}{\partial x}-\frac{\partial L}{\partial y}, & \frac{1}{c}\frac{\partial N}{\partial t} &= \frac{\partial X}{\partial y}-\frac{\partial Y}{\partial x}, \end{aligned}$$

where

$$\rho=\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}$$

Is the charge density times 4π and (u_x,u_y,u_z) are the components of the charge's velocity.

(178)

OEM: If we imagine the electric charges to be invariably coupled to small rigid bodies (ions, electrons), these equations are the electromagnetic basis of the Lorentzian electrodynamics and optics of moving bodies.

AFK: If we consider the charges to be bound on small, ridged bodies (ions, electrons), then these equations are the electromagnetic basis for Lorentz's electrodynamics and optics of moving bodies.

(179)

OEM: Let these equations be valid in the system K, and transform them, with the assistance of the equations of transformation given in §§ 3 and 6, to the system *k*. We then obtain the equations

$$\begin{aligned}
\frac{1}{c}\left\{\frac{\partial X'}{\partial\tau}+u_\xi\rho'\right\} &= \frac{\partial N'}{\partial\eta}-\frac{\partial M'}{\partial\zeta}, & \frac{1}{c}\frac{\partial L'}{\partial\tau} &= \frac{\partial Y'}{\partial\zeta}-\frac{\partial Z'}{\partial\eta},\\
\frac{1}{c}\left\{\frac{\partial Y'}{\partial\tau}+u_\eta\rho'\right\} &= \frac{\partial L'}{\partial\zeta}-\frac{\partial N'}{\partial\xi}, & \frac{1}{c}\frac{\partial M'}{\partial\tau} &= \frac{\partial Z'}{\partial\xi}-\frac{\partial X'}{\partial\zeta},\\
\frac{1}{c}\left\{\frac{\partial Z'}{\partial\tau}+u_\zeta\rho'\right\} &= \frac{\partial M'}{\partial\xi}-\frac{\partial L'}{\partial\eta}, & \frac{1}{c}\frac{\partial N'}{\partial\tau} &= \frac{\partial X'}{\partial\eta}-\frac{\partial Y'}{\partial\xi},
\end{aligned}$$

where

$$\begin{aligned}
u_\xi &= \frac{u_x-v}{1-u_xv/c^2}\\
u_\eta &= \frac{u_y}{\beta(1-u_xv/c^2)}\\
u_\zeta &= \frac{u_z}{\beta(1-u_xv/c^2)},
\end{aligned}$$

and

$$\begin{aligned}
\rho' &= \frac{\partial X'}{\partial\xi}+\frac{\partial Y'}{\partial\eta}+\frac{\partial Z'}{\partial\zeta}\\
&= \beta(1-u_xv/c^2)\rho.
\end{aligned}$$

AFK: By transforming these equations with the aid of the transformations from §§ 3 and 6, so as to valid in K, one gets:

$$\begin{aligned}
\frac{1}{c}\left\{\frac{\partial X'}{\partial\tau}+u_\xi\rho'\right\} &= \frac{\partial N'}{\partial\eta}-\frac{\partial M'}{\partial\zeta}, & \frac{1}{c}\frac{\partial L'}{\partial\tau} &= \frac{\partial Y'}{\partial\zeta}-\frac{\partial Z'}{\partial\eta},\\
\frac{1}{c}\left\{\frac{\partial Y'}{\partial\tau}+u_\eta\rho'\right\} &= \frac{\partial L'}{\partial\zeta}-\frac{\partial N'}{\partial\xi}, & \frac{1}{c}\frac{\partial M'}{\partial\tau} &= \frac{\partial Z'}{\partial\xi}-\frac{\partial X'}{\partial\zeta},\\
\frac{1}{c}\left\{\frac{\partial Z'}{\partial\tau}+u_\zeta\rho'\right\} &= \frac{\partial M'}{\partial\xi}-\frac{\partial L'}{\partial\eta}, & \frac{1}{c}\frac{\partial N'}{\partial\tau} &= \frac{\partial X'}{\partial\eta}-\frac{\partial Y'}{\partial\xi},
\end{aligned}$$

where

$$\begin{aligned}
u_\xi &= \frac{u_x-v}{1-u_xv/c^2}\\
u_\eta &= \frac{u_y}{\beta(1-u_xv/c^2)}\\
u_\zeta &= \frac{u_z}{\beta(1-u_xv/c^2)},
\end{aligned}$$

and

$$\begin{aligned}\rho' &= \frac{\partial X'}{\partial \xi} + \frac{\partial Y'}{\partial \eta} + \frac{\partial Z'}{\partial \zeta} \\ &= \beta(1 - u_x v/c^2)\rho.\end{aligned}$$

(180)

OEM: Since—as follows from the theorem of addition of velocities (§ 5)—the vector is (u_ξ, u_η, u_ζ) nothing else than the velocity of the electric charge, measured in the system *k*, we have the proof that, on the basis of our kinematical principles, the electrodynamic foundation of Lorentz's theory of the electrodynamics of moving bodies is in agreement with the principle of relativity.

AFK: In that the vector

$$(u_\xi, u_\eta, u_\zeta)$$

-as follows from the velocity addition theorem – is nothing other than the velocity of the electric particles as seen in k, this follows that on the basis of our kinematical principles, the Lorentzian theory of the electrodynamicly moved bodies conforms to the principle of relativity.

(181)

OEM: In addition I may briefly remark that the following important law may easily be deduced from the developed equations: If an electrically charged body is in motion anywhere in space without altering its charge when regarded from a system of co-ordinates moving with the body, its charge also remains—when regarded from the “stationary” system K—constant.

AFK: Note that the development of these equations admits the following conclusion: A charged body moving in space without altering its charge, will appear both from its rest frame and from the stationary system to have an invariant charge.

(182)

OEM: § 10. Dynamics of the Slowly Accelerated Electron

AFK: The dynamics of a (slowly accelerated) electron

(183)

OEM: Let there be in motion in an electromagnetic field an electrically charged particle (in the sequel called an “electron”), for the law of motion of which we assume as follows: —

If the electron is at rest at a given epoch, the motion of the electron ensues in the next instant of time according to the equations

$$
\begin{aligned}
m\frac{d^2x}{dt^2} &= \epsilon X \\
m\frac{d^2y}{dt^2} &= \epsilon Y \\
m\frac{d^2z}{dt^2} &= \epsilon Z
\end{aligned}
$$

where *x*, *y*, *z* denote the co-ordinates of the electron, and *m* the mass of the electron, as long as its motion is slow.

AFK: Suppose that an electromagnetic field acts on a point particle with charge epsilon symbol (hereinafter denoted an “electron”) for which the following laws governing its motion are:

$$
\begin{aligned}
m\frac{d^2x}{dt^2} &= \epsilon X \\
m\frac{d^2y}{dt^2} &= \epsilon Y \\
m\frac{d^2z}{dt^2} &= \epsilon Z
\end{aligned}
$$

where *x*, *y*, *z* are the coordinates of the electron, and *m* its mass – so long, at least, as it moves slowly.

n.b. AFK uses mu instead of m

Me: AFK uses term “point particle” that would mean Boscovich-Newtonian physics; main theory on point particles being dealt with by Boscovich.

The German is: In einem elektromagnetischen Felde bewege sich ein punktförmiges, mit einer elektrischen Ladung versehenes Teilchen (im folgenden „Elektron" genannt), über dessen Bewegungsgesetz wir nur folgendes annehmen:

Which translates as: In an electromagnetic field move a point-shaped particle provided with an electric charge (in the following called "electron"), about whose law of motion we assume only the following:

The relevant word is punktförmiges which probably translates best as pointlike.

Teilchen is particle.

OEM: Now, secondly, let the velocity of the electron at a given epoch be *v*. We seek the law of motion of the electron in the immediately ensuing instants of time.

AFK: Let the electron have velocity v during a certain epoch. We seek now to determine the law governing its immediately subsequent motion.

(185)

OEM: Without affecting the general character of our considerations, we may and will assume that the electron, at the moment when we give it our attention, is at the origin of the co-ordinates, and moves with the velocity *v* along the axis of X of the system K. It is then clear that at the given moment (t=0) the electron is at rest relatively to a system of co-ordinates which is in parallel motion with velocity *v* along the axis of X.

AFK: We may, with no loss of generality, take it that at the moment of interest the electron has velocity *v* at the origin along the x-axis. Clearly, at this instant, this electron can be observed from a co-moving frame.

(186)

OEM: From the above assumption, in combination with the principle of relativity, it is clear that in the immediately ensuing time (for small values of t) the electron, viewed from the system k, moves in accordance with the equations

$$\begin{aligned} m\frac{d^2\xi}{d\tau^2} &= \epsilon X', \\ m\frac{d^2\eta}{d\tau^2} &= \epsilon Y', \\ m\frac{d^2\zeta}{d\tau^2} &= \epsilon Z', \end{aligned}$$

in which the symbols ξ, η, ζ, X', Y', Z' refer to the system k.

AFK: In view of the assumptions made above, together with the principle of relativity, it is clear that, the electron in the immediately following interval (i.e., for small values of t) in k has the equations of motion:

$$\begin{aligned} m\frac{d^2\xi}{d\tau^2} &= \epsilon X', \\ m\frac{d^2\eta}{d\tau^2} &= \epsilon Y', \\ m\frac{d^2\zeta}{d\tau^2} &= \epsilon Z', \end{aligned}$$

Where ξ, η, ζ, X', Y', Z' pertain to *k*.

(187)

OEM: If, further, we decide that

when *t*=*x*=*y*=*z*=0 then $\tau = \xi = \eta = \zeta = 0$, the transformation equations of §§ 3 and 6 hold good, so that we have

$$\xi = \beta(x - vt), \eta = y, \zeta = z, \tau = \beta(t - vx/c^2),$$
$$X' = X, Y' = \beta(Y - vN/c), Z' = \beta(Z + vM/c).$$

AFK: Now, by setting *t*=*x*=*y*=*z*=0, it then follows that $\tau = \xi = \eta = \zeta = 0$, so that the transformations from §3 and § 6 become

$$\xi = \beta(x - vt), \eta = y, \zeta = z, \tau = \beta(t - vx/c^2),$$
$$X' = X, Y' = \beta(Y - vN/c), Z' = \beta(Z + vM/c).$$

(188)

OEM: With the help of these equations we transform the above equations of motion from system *k* to system K, and obtain

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= \frac{\epsilon}{m\beta^3}X \\ \frac{d^2y}{dt^2} &= \frac{\epsilon}{m\beta}\left(Y - \frac{v}{c}N\right) \\ \frac{d^2z}{dt^2} &= \frac{\epsilon}{m\beta}\left(Z + \frac{v}{c}M\right) \end{aligned}\right\} \quad \cdot \cdot \cdot \quad \textbf{(A)}$$

AFK: With these equations we transform the above equations of motion from system *k* to system K to get:

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= \frac{\epsilon}{m\beta^3}X \\ \frac{d^2y}{dt^2} &= \frac{\epsilon}{m\beta}\left(Y - \frac{v}{c}N\right) \\ \frac{d^2z}{dt^2} &= \frac{\epsilon}{m\beta}\left(Z + \frac{v}{c}M\right) \end{aligned}\right\} \quad \cdot \cdot \cdot \quad \textbf{(A)}$$

(189)

OEM: Taking the ordinary point of view we now inquire as to the “longitudinal” and the “transverse” mass of the moving electron. We write the equations **(A)** in the form

$$
\begin{aligned}
m\beta^3 \frac{d^2x}{dt^2} &= \epsilon \mathrm{X} &&= \epsilon \mathrm{X}', \\
m\beta^2 \frac{d^2y}{dt^2} &= \epsilon\beta\left(\mathrm{Y} - \tfrac{v}{c}\mathrm{N}\right) &&= \epsilon \mathrm{Y}', \\
m\beta^2 \frac{d^2z}{dt^2} &= \epsilon\beta\left(\mathrm{Z} + \tfrac{v}{c}\mathrm{M}\right) &&= \epsilon \mathrm{Z}',
\end{aligned}
$$

and remark firstly that $\epsilon\mathrm{X}'$, $\epsilon\mathrm{Y}'$, $\epsilon\mathrm{Z}'$ are the components of the ponderomotive force acting upon the electron, and are so indeed as viewed in a system moving at the moment with the electron, with the same velocity as the electron.

AFK: Now, from the usual viewpoint, we ask what the “longitudinal” and “transverse” masses of the electron are. Let us write equations **(A)** in the form

$$
\begin{aligned}
m\beta^3 \frac{d^2x}{dt^2} &= \epsilon \mathrm{X} &&= \epsilon \mathrm{X}', \\
m\beta^2 \frac{d^2y}{dt^2} &= \epsilon\beta\left(\mathrm{Y} - \tfrac{v}{c}\mathrm{N}\right) &&= \epsilon \mathrm{Y}', \\
m\beta^2 \frac{d^2z}{dt^2} &= \epsilon\beta\left(\mathrm{Z} + \tfrac{v}{c}\mathrm{M}\right) &&= \epsilon \mathrm{Z}',
\end{aligned}
$$

and to begin note, that $\epsilon\mathrm{X}'$, $\epsilon\mathrm{Y}'$, $\epsilon\mathrm{Z}'$ are the components of the ponderomotive force acting on the electron, as seen from a system co-moving with the electron.

(190)

OEM: (This force might be measured, for example, by a spring balance at rest in the last-mentioned system.) Now if we call this force simply “the force acting upon the electron,”[9] and maintain the equation—mass × acceleration = force—and if we also decide that the accelerations are to be measured in the stationary system K, we derive from the above equations

$$
\begin{aligned}
\text{Longitudinal mass} &= \frac{m}{(\sqrt{1-v^2/c^2})^3}. \\
\text{Transverse mass} &= \frac{m}{1-v^2/c^2}.
\end{aligned}
$$

FOOTNOTE: The definition of force here given is not advantageous, as was first shown by M. Planck. It is more to the point to define force in such a way that the laws of momentum and energy assume the simplest form.

AFK: (Such a force could be measured, for example, with a spring scale at rest in the same system.) If we call this force simply the “force acting on this electron,”[9] and maintain the validity of the equation mass × acceleration = force, and then require that acceleration to be measured in the rest system, we get:

$$
\begin{aligned}
\text{Longitudinal mass} &= \frac{m}{(\sqrt{1-v^2/c^2})^3}. \\
\text{Transverse mass} &= \frac{m}{1-v^2/c^2}.
\end{aligned}
$$

FOOTNOTE: As noted by Planck, this definition of “force” is suboptimal; it is better to so define it that the conservation laws of momentum and energy take the simplest form.

Me: Einstein sent his paper to a journal reviewed by Planck, the footnote referring to Planck must mean that Planck had made comment to original draft of the paper, and that comment was added to the draft that was published.

(191)

OEM: With a different definition of force and acceleration we should naturally obtain other values for the masses. This shows us that in comparing different theories of the motion of the electron we must proceed very cautiously.

AFK: Naturally, with other definitions of force and acceleration one gets different values for these masses; therefore one sees that, one must proceed very carefully when comparing various theories of the motion of the electrons.

(192)

OEM: We remark that these results as to the mass are also valid for ponderable material points, because a ponderable material point can be made into an electron (in our sense of the word) by the addition of an electric charge, *no matter how small*.

AFK: We note that, these results hold also for the mass of ponderable material particles, as they can be converted by the attachment of arbitrarily small electrical charges to electrons (in our sense).

Me: OEM refers to “material points” and AFK to “material particles”

The German is: Wir bemerken, daß diese Resultate über die Masse auch für die ponderabeln materiellen Punkte gilt; denn ein ponderabler materieller Punkt kann durch Zufügen einer beliebig kleinen elektrischen Ladung zu einem Elektron (in unserem Sinne) gemacht werden.

Translates as: We note that these results on mass also apply to the ponderable material points; for a ponderable material point can be made into an electron by adding an arbitrarily small electrical charge (in our sense).

materiellen Punkte is material points

So, once again point particle theory of Boscovich.

(193)

OEM: We will now determine the kinetic energy of the electron. If an electron moves from rest at the origin of co-ordinates of the system K along the axis of X under the action of an electrostatic force X, it is clear that the energy withdrawn from the electrostatic field has the value $\int \epsilon \mathrm{X}\, dx$.

AFK: Let us determine the kinetic energy of an electron. If it is moving from the origin of K with an initial velocity of 0 along the x-axis of X under the influence of an electric force, it is clear that, the energy extracted from the field equals $\int \epsilon \mathrm{X}\, dx$.

(194)

OEM: As the electron is to be slowly accelerated, and consequently may not give off any energy in the form of radiation, the energy withdrawn from the electrostatic field must be put down as equal to the energy of motion W of the electron. Bearing in mind that during the whole process of motion which we are considering, the first of the equations **(A)** applies, we therefore obtain

$$\begin{aligned} \mathrm{W} &= \int \epsilon \mathrm{X}\, dx = m \int_0^v \beta^3 v\, dv \\ &= mc^2 \left\{ \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right\}. \end{aligned}$$

Thus, when v=c, W becomes infinite. Velocities greater than that of light have—as in our previous results—no possibility of existence.

AFK: As the electron shall be slowly accelerated so as not to emit radiation, the energy extracted from the field must equal the kinetic energy of motion W of the electron. Thus, in accord with the first of the equations **(A):**

$$\begin{aligned} \mathrm{W} &= \int \epsilon \mathrm{X}\, dx = m \int_0^v \beta^3 v\, dv \\ &= mc^2 \left\{ \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right\}. \end{aligned}$$

For v=c then, W will be infinite. Superluminal velocities cannot be real – as was shown above also.

(195)

OEM: This expression for the kinetic energy must also, by virtue of the argument stated above, apply to ponderable masses as well.

AFK: We will now enumerate the properties of the motion of the electron which result from the system of equations **(A)**, and are accessible to experiment.

(196)

OEM: This expression must also pertain to ponderable matter.

AFK: Let us enumerate the possible experimental consequences of equations **(A):**

(197)

OEM: 1. From the second equation of the system **(A)** it follows that an electric force Y and a magnetic force N have an equally strong deflective action on an electron moving with the velocity *v*, when $\mathrm{Y} = \mathrm{N}v/c$. Thus we see that it is possible by our theory to determine the velocity of the electron from the ratio of the magnetic power of deflexion A_m to the electric power of deflexion A_e, for any velocity, by applying the law

$$\frac{\mathrm{A}_m}{\mathrm{A}_e} = \frac{v}{c}.$$

This relationship may be tested experimentally, since the velocity of the electron can be directly measured, e.g. by means of rapidly oscillating electric and magnetic fields.

AFK: 1. From the second of equations **(A)** it follows that, an electric force Y and a magnetic force N will be equally effective at diverting an electron when $\mathrm{Y} = \mathrm{N}v/c$. Thus one sees that, from the work of magnetic diversion, A_m, and the work of electric diversion, A_e, according to our theory, the determination of the velocity of the electron is possible for an arbitrary velocity by using the law

$$\frac{\mathrm{A}_m}{\mathrm{A}_e} = \frac{v}{c}.$$

This relationship can be tested experimentally because the velocity of the electron can be directly measured, for example by means of rapidly oscillating electric and magnetic fields.

(198)

OEM: 2, From the deduction for the kinetic energy of the electron it follows that between the potential difference, P, traversed and the acquired velocity *v* of the electron there must be the relationship

$$P = \int X dx = \frac{m}{\epsilon} c^2 \left\{ \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right\}.$$

AFK: 2. From the derivation of the expression for the kinetic energy of the electron it follows that, between the velocity and the traversed potential difference the following equation must hold:

$$P = \int X dx = \frac{m}{\epsilon} c^2 \left\{ \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right\}.$$

(199)

OEM: 3. We calculate the radius of curvature of the path of the electron when a magnetic force N is present (as the only deflective force), acting perpendicularly to the velocity of the electron. From the second of the equations **(A)** we obtain

$$-\frac{d^2 y}{dt^2} = \frac{v^2}{R} = \frac{\epsilon}{m} \frac{v}{c} N \sqrt{1 - \frac{v^2}{c^2}}$$

or

$$R = \frac{mc^2}{\epsilon} \cdot \frac{v/c}{\sqrt{1 - v^2/c^2}} \cdot \frac{1}{N}.$$

These three relationships are a complete expression for the laws according to which, by the theory here advanced, the electron must move.

AFK: 3. Let us calculate the radius of curvature of the orbit of an electron subject only to a magnetic force N perpendicular to its velocity. From the second of the equations **(A)** we get

$$-\frac{d^2 y}{dt^2} = \frac{v^2}{R} = \frac{\epsilon}{m} \frac{v}{c} N \sqrt{1 - \frac{v^2}{c^2}}$$

or

$$R = \frac{mc^2}{\epsilon} \cdot \frac{v/c}{\sqrt{1 - v^2/c^2}} \cdot \frac{1}{N}.$$

These three relations constitute a complete determination of the laws, according to which, in our theory, an electron's motion is governed.

n.b. AFK makes mistake says equation (1) not (A), amended here

(200)

OEM: In conclusion I wish to say that in working at the problem here dealt with I have had the loyal assistance of my friend and colleague M. Besso, and that I am indebted to him for several valuable suggestions.

AFK: In concluding, I wish to note that, work on these problems was supported by my friend and colleague M. Besso, whom I wish to thank for much valuable stimulation.

References

[1] https://www.fourmilab.ch/etexts/einstein/specrel/www/#tex2html1

[2] A.F.Kracklauer in book Einstein in English vol. I, 2013 isbn 978-1-304-65941-5 p.71-89

[3] https://en.oxforddictionaries.com/definition/supposition : supposition : A belief held without proof or certain knowledge; an assumption or hypothesis. E.g. 'they were working on the supposition that his death was murder', 'their outrage was based on supposition and hearsay' Origin: Late Middle English (as a term in scholastic logic): from Old French, or from late Latin suppositio(n-) (translating Greek hupothesis 'hypothesis'), from the verb supponere (see suppose).

[4] https://dictionary.cambridge.org/dictionary/english/conjectur : conjecture : a guess about something based on how it seems and not on proof: e.g. There's been a lot of conjecture in the media recently about the marriage.

[5] https://en.oxforddictionaries.com/definition/postulate postulate : Suggest or assume the existence, fact, or truth of (something) as a basis for reasoning, discussion, or belief.

[6] Einstein in English vol. II revised edition, trans. A F Kracklauer 2013, ISBN 978-304-65942-2 p. 755 Einstein: "The development of the foundations of relativity: concepts and methods"

[7] Conventions in relativity theory and quantum mechanics, Karl Svozil, Institut fur Theoretische Physik, University of Technology Vienna. https://arxiv.org/pdf/quant-ph/0110054.pdf

For further information on Boscovich see my other papers and videos etc.

e.g. From Boscovich's theory to modern quantum theory: Prof Dragoslav Stoiljkovic
https://www.youtube.com/watch?v=w1vi0yk7BvU

[8] Einstein's Relativity understood in the context of Boscovich's theory, R Anderton http://gsjournal.net/Science-Journals/Research%20Papers-Relativity%20Theory/Download/7561

[9] Dragoslav Stoiljkovic and Roger Anderton: "Boscovich's comprehensions of relative movement" presented at Serbian Astronomical Conference October 2017 https://www.youtube.com/watch?v=pfoLk6ZYNyI

[10] **The Confessions of St. Augustine**
Bishop of Hippo BOOK XI.
http://www.leaderu.com/cyber/books/augconfessions/bk11.html

[11] A Brief History of Timelessness, Matthew Marsden https://sites.google.com/site/abriefhistoryoftimelessness/mathematics/what-do-clocks-measure

[12] This Philosopher Helped Ensure There Was No Nobel for Relativity: Henri Bergson's debate with Albert Einstein reached and swayed the 1921 Nobel committee. By Jimena Canales
http://nautil.us/issue/35/boundaries/this-philosopher-helped-ensure-there-was-no-nobel-for-relativity

[13] Einstein mistranslated part 1: https://www.gsjournal.net/Science-Journals/Research%20Papers-Relativity%20Theory/Download/7671

[14] And part 2: https://www.gsjournal.net/Science-Journals/Research%20Papers-Relativity%20Theory/Download/7676

c.updated 19May2019 – mainly forgot to add and mention references [13] and [14], and to add comment before (68): “The following from (68) -onwards is thus provided with less commentary by me.”