

A REVIEW OF RELATIVISTIC AND NON-RELATIVISTIC EXPLANATIONS OF THE RESULT OF FIZEAU'S EXPERIMENT

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Fizeau's experiment showed that the speed of light is affected by the motion of the medium in which it is propagated at normal incidence. This effect is regarded as a consequence of relativistic addition of velocities, which gives Fresnel's law. A correction is proposed to Fresnel's law for the speed of light in a moving medium, on the basis of addition of velocities in classical mechanics, to give a non-relativistic explanation of result of the experiment.

Keywords: Classical mechanics, experiment, light, medium, refractive index, special relativity, speed.

1. INTRODUCTION

The French physicist, Hippolyte Fizeau's experiment was conducted in 1851 [1, 2, 3] to measure the speed of light, at normal incidence, in moving water. According to the velocity addition rule of the theory of special relativity (4, 5), the speed of light w propagated at normal incidence in a medium of refractive index μ moving with speed v , in a vacuum, is given by:

$$w = \frac{c}{\mu} + v \left(1 - \frac{1}{\mu^2} \right) \quad (1)$$

Equation (1) is Fresnel's law, which Fizeau's experiment seemed to have confirmed.

In this paper a non-relativistic formula is given, for the speed of light, transmitted at normal incidence, in a moving medium, as:

$$w = \frac{c}{\mu} + v \left(1 - \frac{1}{\mu} \right) \quad (2)$$

Equation (1) and equation (2) are used to respectively give relativistic and non-relativistic explanations of the result of Fizeau's experiment for the speed of light in moving water.

2. LAWS OF REFRACTION OF LIGHT

Figure 1 depicts a ray of light SP, emitted with velocity c from a stationary source S, incident at a point P on the boundary of two media, 1 and 2. OPN is the normal to the surface at P. A reflected ray PR is one propagated with velocity u from the boundary in the same medium (1) as the incident ray SP. A refracted ray PT is one transmitted and propagated with velocity w in the second medium (2). The angle of incidence is ι , the angle of reflection ρ and the angle of refraction is τ .

The laws of refraction of light, for a stationary medium, are:

- (i) The incident ray SP, the reflected ray PR, the refracted ray PT and the normal OPN, at the point of incidence P, are coplanar.
- (ii) The ratio of the sine of angle of refraction to the sine of angle of incidence is equal to the ratio of the speeds of light w and c (Snell's law).

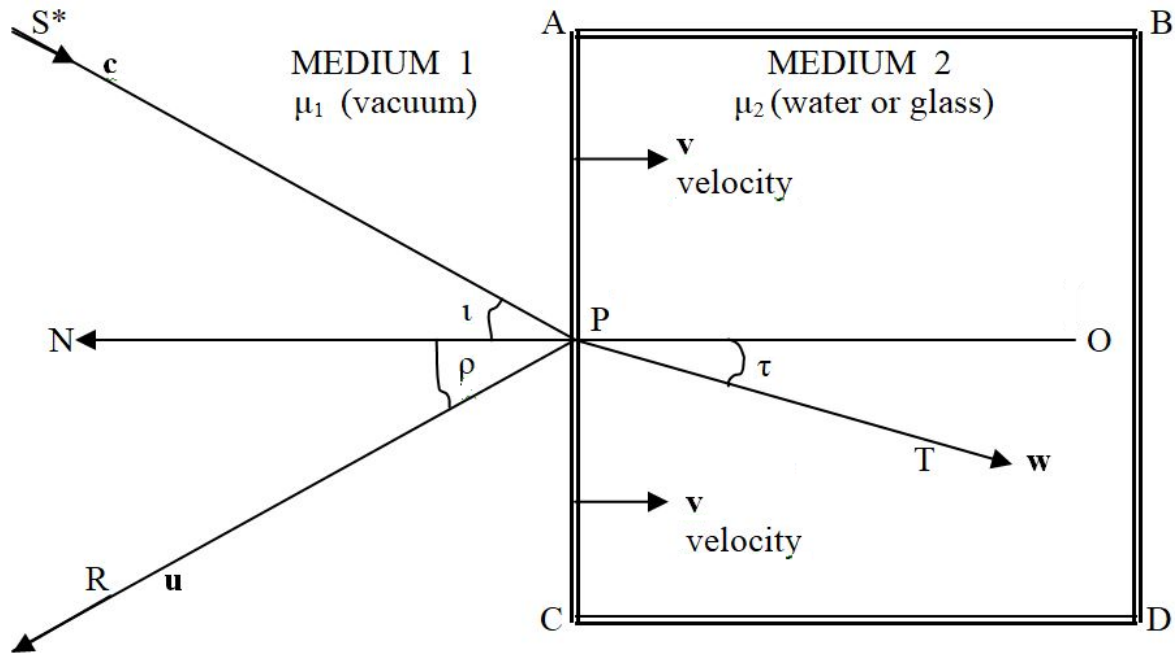


Figure 1: Reflection and refraction of a ray of light emitted from a stationary source S with velocity c in medium 1 (*vacuum*), incident at a point P on the surface of medium 2 moving with velocity v . The reflected ray PR is transmitted in medium 1 with velocity u and the refracted ray PT transmitted in medium 2 ($ABCD$) with velocity w .

The relative indices of refraction μ_1 and μ_2 of *media* 1 and 2 respectively, are defined, for a stationary medium, as the ratio of the speed w and, thus:

$$\frac{w}{s} = \frac{\sin \tau}{\sin i} = \frac{\mu_1}{\mu_2} \quad (3)$$

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3. SPEED OF LIGHT PROPAGATED IN A MOVING MEDIUM AT NORMAL INCIDENCE

In Figure 1, let the *medium* 2 ($ABCD$) move in *medium* 1 (*vacuum*) with velocity v along the normal. The ray, from a stationary source, is transmitted through the medium and propagated with velocity w at the angle of refraction τ . For a moving medium, the second law of refraction and the refractive indices are defined in terms of the ratio of magnitudes of relative velocities of the refracted ray and the incident ray, with respect to the moving medium and in accordance with

the Galilean (classical) relativity. The ratio of magnitudes of the relative velocities, given by the vectors $(\mathbf{w} - \mathbf{v})$ and $(\mathbf{c} - \mathbf{v})$, is obtained as the ratio of moduli, thus:

$$\frac{|\mathbf{w} - \mathbf{v}|}{|\mathbf{c} - \mathbf{v}|} = \frac{\mu_1}{\mu_2} \quad (4)$$

With reference to Figure 1, and with $\mu_1 = 1$ for a vacuum and $\mu_2 = \mu$ is the absolute refractive index, equation (4) becomes:

$$\frac{\sqrt{w^2 + v^2 - 2wv \cos \tau}}{\sqrt{c^2 + v^2 - 2cv \cos \iota}} = \frac{1}{\mu} \quad (5)$$

At normal incidence, $\iota = \tau = 0$ and we obtain:

$$\begin{aligned} \frac{w - v}{c - v} &= \frac{1}{\mu} \\ w\mu - v\mu &= c - v \\ w &= \frac{c}{\mu} + v \left(1 - \frac{1}{\mu}\right) \end{aligned} \quad (6)$$

Equation (6) is used to give a non-relativistic explanation of the result of Fizeau's experiment, which measured the speed of light in moving water.

4. FIZEAU'S EXPERIMENT

A schematic diagram of the apparatus of Fizeau's experiment [1] is shown in Figure 2.

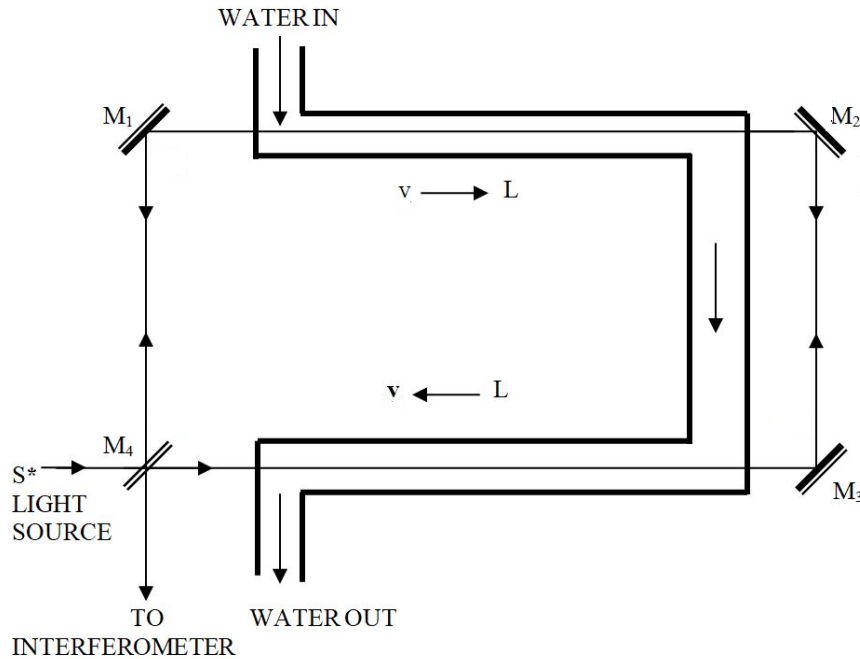


Figure 2: Schematic diagram of the apparatus of Fizeau's experiment

Carried out in 1851, it is one of the most remarkable experiments in physics. Light from a source was sent in two opposite directions through transmission and reflection by four half-silvered mirrors $M_1 - M_4$. One beam travelled downstream (from M_1) through moving water and the other travelled upstream (from M_4) in the same water. By an ingenious arrangement of the mirrors the

two beams were made to recombine and be observed in an interferometer. An interference pattern, as observed in the interferometer, resulted from the difference in the time taken for the two beams to travel the same path, partly in moving water.

Various explanations have been given for the result of Fizeau's experiment. One accepted explanation is that the velocity of light in the moving water was increased or decreased in accordance with the *relativistic rule for addition of velocities* based on constancy of the speed of light relative to an observer or object that is stationary or moving. A new explanation is proposed in this paper, on the basis of equation (6), outside the theory of special relativity [2, 3].

5. RELATIVISTIC EXPLANATION OF THE RESULT OF FIZEAU'S EXPERIMENT

According to Einstein's *relativistic velocity addition rule*, if you move with velocity \mathbf{u} relative to a medium moving with velocity \mathbf{v} (as light propagated in water moving with velocity \mathbf{v}), the magnitude of your velocity s , relative to an observer, is:

$$s = \frac{u + v}{1 + \frac{uv}{c^2}} \quad (7)$$

where c is the speed of light in a vacuum and u and v are the magnitudes of the respective velocities. For speeds much less than c , or if c is infinitely large, Einstein's relativistic formula (equation 7) reduces to the Galilean (classical) relativity, $s = u + v$.

According to the theory of special relativity, the speed of light, relative to a medium moving in a vacuum with speed v , remains a constant c . The speed of light within the medium of refractive index μ is c/μ . Einstein's *velocity addition rule* (equation 7), with $u = c/\mu$, gives the speed w in the moving medium (with respect to an observer), as:

$$w = \frac{\frac{c}{\mu} + v}{1 + \frac{cv}{\mu c^2}} = \frac{\frac{c}{\mu} + v}{1 + \frac{v}{\mu c}} \approx \frac{c}{\mu} + v \left(1 - \frac{1}{\mu^2}\right)$$

$$w \approx \frac{c}{\mu} + v \left(1 - \frac{1}{\mu^2}\right) \quad (8)$$

Equation (8) is Fresnel's law obtained for $v \ll c$. This equation is used to obtain the transit time difference between the two beams in Fizeau's experiment, and thereby explain the result from the point of view of relativity theory.

Transit time t_1 of beam going downstream, with speed v very small compared to c , is obtained from equation (8) as:

$$t_1 = \frac{2L}{\frac{c}{\mu} + v \left(1 - \frac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c} \left\{1 - \frac{\mu v}{c} \left(1 - \frac{1}{\mu^2}\right)\right\}$$

$$t_1 \approx \frac{2L\mu}{c} \left(1 + \frac{v}{\mu c} - \frac{\mu v}{c}\right)$$

Transit time as t_2 of the beam going upstream, with speed of $-v$, is:

$$t_2 = \frac{2L}{\frac{c}{\mu} - v \left(1 - \frac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c} \left\{1 + \frac{\mu v}{c} \left(1 - \frac{1}{\mu^2}\right)\right\}$$

$$t_2 \approx \frac{2L\mu}{c} \left(1 - \frac{v}{\mu c} + \frac{\mu v}{c} \right)$$

Time difference between the beam going downstream and the other beam going upstream is obtained as:

$$\Delta t = t_2 - t_1 \approx \frac{4Lv\mu^2}{c^2} \left(1 - \frac{1}{\mu^2} \right) \quad (9)$$

The fringe shift δ_y , according to special relativity, for light of wavelength λ , is obtained as:

$$\delta_y = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c} \left(1 - \frac{1}{\mu^2} \right) \quad (10)$$

This fringe shift was easily measurable with the interferometer.

6. NON-RELATIVISTIC EXPLANATION OF THE RESULT OF FIZEAU'S EXPERIMENT

According to the Galilean-Newtonian relativity of classical mechanics, the speed of light in a medium that is moving in a vacuum with velocity \mathbf{v} , in the opposite direction of the normal, is given in terms of the refractive index μ of the medium, by equation (6). This gives the speed w of transmission and propagation of light (downstream). If the medium is water of index of refraction μ moving with velocity v in a vacuum (Figure 2), the time t_1 taken for the beam that is going downstream to cover the distance $2L$ (where the magnitude v is very small compared with the speed of light c and v^2/c^2 can be neglected), is obtained as given by:

$$t_1 = \frac{2L}{\frac{c}{\mu} + v \left(1 - \frac{1}{\mu} \right)} \approx \frac{2\mu L}{c} \left\{ 1 - \frac{\mu v}{c} \left(1 - \frac{1}{\mu} \right) \right\} \quad (11)$$

For the beam going upstream, with velocity $-v$, the longer transit time t_2 is:

$$t_2 = \frac{2L}{\frac{c}{\mu} - v \left(1 - \frac{1}{\mu} \right)} \approx \frac{2\mu L}{c} \left\{ 1 + \frac{\mu v}{c} \left(1 - \frac{1}{\mu} \right) \right\} \quad (12)$$

The time difference (Δt) = ($t_2 - t_1$) between the two beams in moving water, is:

$$t_2 - t_1 = \frac{2\mu L}{c} \left\{ 1 + \frac{\mu v}{c} \left(1 - \frac{1}{\mu} \right) \right\} - \frac{2\mu L}{c} \left\{ 1 - \frac{\mu v}{c} \left(1 - \frac{1}{\mu} \right) \right\}$$

$$\Delta t = \frac{4Lv\mu^2}{c^2} \left(1 - \frac{1}{\mu} \right) \quad (13)$$

The fringe shift δ_x , for light of wavelength λ , is obtained as:

$$\delta_x = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c} \left(1 - \frac{1}{\mu} \right) \quad (14)$$

As $\mu > 1 < 2$, δ_x in equation (14) is smaller than the fringe shift δ_y as given by equation (10).

In the experiments performed by Fizeau [1], $L = 3 \text{ m}$, $v = 7 \text{ m/sec.}$, $\lambda = 6 \times 10^{-7} \text{ m}$ (yellow light), $c = 3 \times 10^8 \text{ m/sec}$ and $\mu = 4/3$ (for water). The fringe shift δ_x or δ_y is obtained as ≈ 0.2 , which was easily observable and measurable in the interferometer. Fizeau's experiment, however,

could not determine the correct explanation to be relativistic or non-relativistic. But a close examination of equation (6) vis-à-vis equation (8) may reveal something.

7. RESULTS AND DISCUSSION

Michelson and Morley in 1886 and later P. Zeeman and associates in 1915 repeated Fizeau's experiment with greater precision in which the interferometer could measure a fringe shift as low as 0.01 . The influence of the motion of the medium on the propagation of light has thus been verified. As to which is the correct explanation, equation (10) for δ_y in accordance with the theory of relativity or equation (14) for δ_x according to the Galilean-Newtonian relativity, remains to be seen.

The treatment of refraction of light in this paper, clearly demonstrates the relativity of the speed of light with respect to a moving medium, in accordance with Galileo's relativity of classical mechanics. The result of Fizeau's experiment need not be a direct consequence of the relativistic *velocity addition rule* but is more likely to be the effect of motion of the transmission medium on the speed of light in equation (6). It should be noted that equation (6) is not an approximation as it applies for all values of speed v up to that of light c or $-c$, in contrast to equation (8) applicable for $v \ll c$.

For $v = 0$, equations (6) and (8) give the speed of light in the medium $w = c/\mu$, as expected. For $\mu = 1$ both equations give $w = c$, also as expected. For $v = c$, equation (8) is not applicable but equation (6) gives $w = c$, again as expected. For $v = -c$, equation (8) is not applicable but equation (6) gives $w = c/(2\mu - 1)$, which is reasonable, particularly if $\mu = 1$.

Equation (10) is a good example of Beckmann's *correspondence theory* [6], whereby the desired or expected result is obtained mathematically but based on the wrong underlying principles. Fizeau's experiment might as well have verified the fringe shift in equation (14), rather than the relativistic equation (10), for the transmission of light in a moving medium. Curt Renshaw [7] analyzed the results of several experiments conducted to measure the effect of the speed of a medium on the speed of transmission of light in the medium and he concluded that the results could be explained without invoking special relativity.

7. CONCLUSION

Fresnel's law as expressed in equation (8), giving the speed of light w at normal incidence, in a medium of refractive index μ moving with speed v , may not be correct even for $v \ll c$. The correct formula should be equation (6), which is applicable for values of v up to c or $-c$.

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