

## Lorentz Transformations

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**Abstract:** Lorentz studied the transformations of coordinates from a stationary frame to an inertial frame. We can arbitrarily choose a multitude of stationary frames and employ them in the derivation of the Lorentz transformations. For each of them, the Lorentz transformations generate a different set of data for the quantities of space and time that describes the law of physics in the inertial frame. Therefore, the Lorentz transformations offer a law for the propagation of the light not uniquely defined in an inertial frame, and consequently, we cannot accept them.

### 1 Introduction

Section 2 presents a summary of the derivation of the Lorentz transformations. It offers the equations for the propagation of light in the stationary and inertial frames and then the result of the transformations of coordinates. The mathematical derivation of the transformations of coordinates is not a subject for this paper.

Section 3 brings to our attention that we can arbitrarily choose a multitude of stationary frames and employ them in the derivation of the Lorentz transformations.

Section 4 concludes that the Lorentz transformations are not uniquely defined in an inertial frame, and consequently they cannot be accepted and employed in the interpretation of physics phenomena.

### 2 Lorentz transformations

Figure 1 depicts the illustration of the stationary frame and inertial frame used in the derivation of the Lorentz transformations. The stationary frame is an inertial frame considered at relative rest for the inertial frame.

Lorentz derived the laws of physics in the inertial frame  $O'X'Y'Z'$  through the stationary frame  $OXYZ$ . Both frames are moving in the same direction of their common axes  $OX$  and  $O'X'$ . Axes  $OY$  and  $O'Y'$  have the same direction and axes  $OZ$  and  $O'Z'$  as well. The inertial frame is moving away from the stationary frame along the axis  $OX$  with the relative speed  $v$ .

At the initial instance, when the origin  $O$  of the stationary frame and the origin  $O'$  of the inertial frame coincide, a wavefront of light starts to spread out in space from this common point.

Lorentz considered that in the stationary frame, the wavefront of light propagates on a sphere with its center at the origin  $O$  of this frame. The center of the sphere is understood as in geometry and is referred to as the geometric center. Furthermore, he considered that in the inertial frame, the wavefront of light propagates on the same sphere with its non-geometric center at the origin  $O'$  of this frame.

The speed of light is the constant  $c$ . Therefore, space and time have to undergo contractions and dilations in the inertial frame to satisfy the propagation of the wavefront of light on this sphere with the center different from the geometric one, as the rays from the origins  $O$  and  $O'$  to point  $P$  suggests.

Lorentz exemplified the transformations of coordinates for the ray of light that travels along the common direction of axes  $OX$  and  $O'X'$ . Figure 1 displays the ray of light that, after a time  $t$ , arrives at the common points  $x$  of the stationary frame and  $x'$  of the inertial frame.

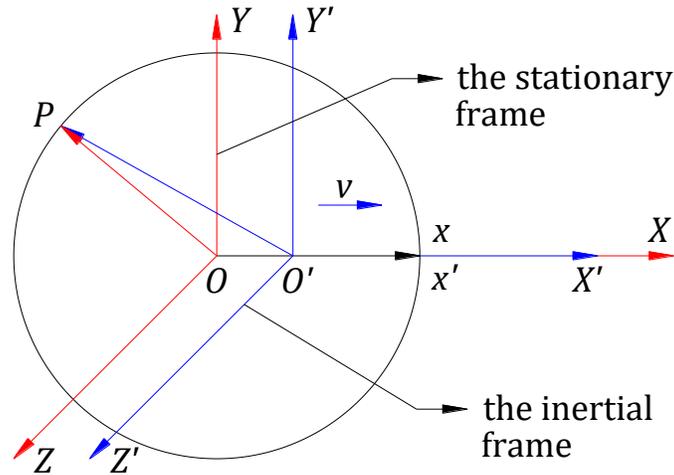


Figure 1. Illustration for the derivation of the Lorentz transformations.

The law of propagation of the wavefront of light in the stationary frame is the equation of a spherical wave:

$$x^2 + y^2 + z^2 = r^2.$$

The radius  $r = ct$  and the equation is defined by the quantities  $x, y, z, t$ , and  $c$ .

The law of propagation of the wavefront of light in the inertial frame must also be the equation of a spherical wave:

$$x'^2 + y'^2 + z'^2 = r'^2.$$

The radius  $r' = c't' = ct'$ , and the equation is defined by the quantities  $x', y', z', t'$ , and  $c$ .

The identity of the two equations,

$$x^2 + y^2 + z^2 - r^2 \equiv x'^2 + y'^2 + z'^2 - r'^2,$$

gives the transformations of coordinates, [1-3], from the stationary frame to the inertial frame for the ray of light along the common direction of axes  $OX$  and  $O'X'$ , with the formulas of transformation for the quantities  $x', y', z'$  and  $t'$ , as follows:

$$x' = \frac{1}{\alpha}(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{1}{\alpha} \left( t - \frac{vx}{c^2} \right)$$

where  $\alpha = \sqrt{1 - v^2/c^2}$ .

### 3 Multiple results generated by the Lorentz transformations for the same inertial frame

At the initial instance, a multitude of stationary frames similar to the stationary frame  $OXYZ$  can have their origin at the common points  $O$  and  $O'$ . We can accomplish the transformations of coordinates for the ray of light from each of these stationary frames to the inertial frame  $O'X'Y'Z'$ . Figure 2 illustrates two of these stationary frames with their relative speeds  $v_1$  and  $v_2$ , at the same time  $t$ .

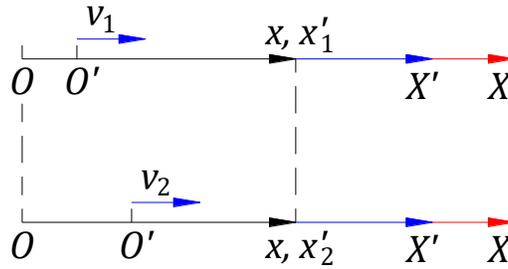


Figure 2. Illustration for the derivation of the Lorentz transformations for two stationary frames.

In the inertial frame, the quantities  $x'_1, y'_1, z'_1, t'_1$  and  $x'_2, y'_2, z'_2, t'_2$  depend on the quantities  $x, y, z$ , and  $t$  that have the same magnitudes for both stationary frames, and on the relative speeds  $v_1$  and  $v_2$  that have different magnitudes. In the inertial frame, the law of propagation of the ray of light for these stationary frames has the same form  $x'_1 = ct'_1$  and  $x'_2 = ct'_2$  but the set of data for the quantities  $x'_1, y'_1, z'_1, t'_1$  and the set of data for the quantities  $x'_2, y'_2, z'_2, t'_2$  are different due to different values of the relative speeds  $v_1$  and  $v_2$ , as follows:

$$x'_1 = ct'_1 \qquad x'_2 = ct'_2$$

$$x'_1 = \frac{1}{\alpha_1} (x - v_1 t) \qquad x'_2 = \frac{1}{\alpha_2} (x - v_2 t)$$

$$y'_1 = y \qquad y'_2 = y$$

$$z'_1 = z \qquad z'_2 = z$$

$$t'_1 = \frac{1}{\alpha_1} \left( t - \frac{v_1 x}{c^2} \right) \qquad t'_2 = \frac{1}{\alpha_2} \left( t - \frac{v_2 x}{c^2} \right).$$

We can do another study in the opposite direction when the inertial frame is a stationary

frame in which the wavefront of light is on a sphere with a geometric center, and the stationary frame is an inertial frame in which the wavefront of light is on the same sphere with a non-geometric center. In this case, each frame ends up with two sets of data: one set of data for the quantities  $x, y, z$ , and  $t$ , and another set of data for the quantities  $x', y', z'$ , and  $t'$ .

We can conclude that the Lorentz transformations of coordinates give a law for the propagation of the wavefront of light not uniquely defined in the inertial frame  $O'X'Y'Z'$ .

#### 4 Conclusions

According to section 3, the Lorentz transformations are not uniquely defined in an inertial frame, and because of this, they cannot be accepted and employed in the interpretation of physics phenomena.

The stationary frame is also an inertial frame, but it gets special treatment. Lorentz considered that in the stationary frame the wavefront of light is at all times on a sphere with a geometric center. With no reason, he chose to treat the stationary frame in this way instead of the inertial frame.

The conclusion that the speed of light is the constant  $c$  in any inertial frame arises from the direct experimental observations in Earth's inertial frame. Instead, to study the propagation of light directly in an inertial frame, Lorentz chose to do the study through the transformations of coordinates from another inertial frame, which he considered stationary.

Lorentz, as well as Einstein, disregarded the fixed frames. Without fixed frames, there are no frames in which the light propagates on a sphere with its center at the geometric one. Thus, we can disregard the Lorentz transformations from the start.

The speed of light has the same constant value  $c$  in any inertial frame, behavior that we do not understand yet. A reasonable explanation is that the speed of light depends on the velocity of its source and the source is at rest in the inertial frame under study.

#### References:

- [1] H. A. Lorentz, "The Relative Motion of the Earth and the Aether", *Zittingsverlag Akad. V. Wet.*, (1), 74–79. (1892).
- [2] A. Einstein (1905), *On the Electrodynamics of Moving Bodies*, June 30, 1905. (English translation from "Das Relativitätsprinzip", 4th ed. 1922, 1923).
- [3] N. Bărbulescu, *Bazele Fizice ale Relativității Einsteiniene* (București: Editura Științifică și Enciclopedică, Enciclopedia de Buzunar, 1975).
- [4] F. Dambi, "Theory of the Inertial Frames Relativity", *The General Science Journal*, (2019). <http://gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/7622>