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# RELATIVISTIC AND NON-RELATIVISTIC EXPLANATIONS OF THE RESULT OF FIZEAU'S EXPERIMENT

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## Abstract

*Fizeau's experiment showed that the speed of light is affected by the motion of the medium in which it is propagated. This effect is regarded as a consequence of relativistic addition of velocities, which gives Fresnel's law. A correction is proposed to Fresnel's law for the speed of light in a moving medium, on the basis of addition of velocities in classical mechanics, to give a non-relativistic explanation of the experiment.*

**Keywords**: *Classical mechanics, experiment, light, medium, special relativity, speed.*

## 1. Introduction

Fizeau's experiment was conducted in the 1850s [1] to measure the speed of light, at normal incidence, in moving water. The theory of special relativity gives the speed of light $w$ propagated at normal incidence in a medium of refractive index $\mu$ moving with speed $v$ in a vacuum as:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right) \tag{1}$$

A non-relativistic formula is given in here, for the speed of light in a moving medium, as:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right) \tag{2}$$

Equation (1) is Fresnel's law. This law and equation (2) are used to respectively give relativistic and non-relativistic explanations of the result of Fizeau's experiment.

## 2. Laws of Refraction of Light

Figure 1 depicts a ray of light **SP**, emitted with velocity **c** from a stationary source S, incident at a point P on the boundary of two media, *1* and *2*. **OPN** is the normal to the surface at **P**. A reflected ray **PR** is one propagated with velocity **u** from the boundary in the same medium *(1)* as the incident ray **SP**. A refracted ray **PT** is one transmitted and propagated with velocity **w** in the second medium *(2).* The angle of incidence is $\iota$, the angle of reflection $\rho$ and the angle of refraction is $\tau$.

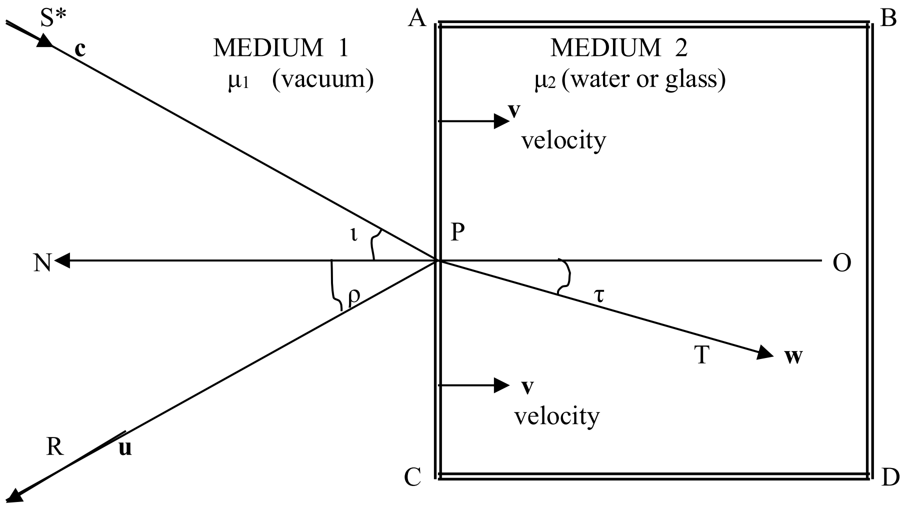


Figure 1 Reflection and refraction of a ray of light emitted from a stationary source S with velocity **c** in medium *1 (vacuum)*, incident at a point P on the surface of medium *2* moving with velocity **v**. The ray **PR** is reflected with velocity **u** and the refracted ray **PT** transmitted in medium 2 (ABCD) with velocity **w**.

The laws of refraction of light, for a stationary medium, are:

(i) The incident ray **SP**, the refracted ray **PT** and the normal **OPN**, at the point of incidence P, are coplanar.

(ii) The ratio of the sine of angle of refraction to the sine of angle of incidence is equal to the ratio of the speeds of light $w$ and $c$ (Snell's law).

The relative indices of refraction $\mu_1$ and $\mu_2$ of *media 1* and *2* respectively, are defined, for a stationary medium, as the ratio of the speed $w$ and $s$:

$$\frac{w}{s} = \frac{\sin\tau}{\sin\iota} = \frac{\mu_1}{\mu_2} \tag{3}$$

## 3. Speed of light propagated in a moving medium

In Figure 1, let *medium 2* (ABCD) move in *medium 1* (vacuum) with velocity **v** along the normal. The ray, from a stationary source, is transmitted through the medium and propagated with velocity **w** at the angle of refraction $\tau$. The refractive indices are defined in terms of the ratio of magnitudes of relative velocities of the refracted ray and the incident ray, with respect to the moving medium and in accordance with the Galilean (classical) relativity. The ratio of magnitudes of the relative velocities, given by (**w** – **v**) and (**c** – **v**), is obtained as the ratio of modulus, thus:

$$\frac{|\mathbf{w}-\mathbf{v}|}{|\mathbf{c}-\mathbf{v}|}=\frac{\mu_1}{\mu_2} \tag{4}$$

With reference to Figure 1, and with $\mu_1 = 1$ for a vacuum and $\mu_2 = \mu$ is the absolute refractive index, equation (4) becomes:

$$\frac{\sqrt{w^2+v^2-2wv\cos\tau}}{\sqrt{c^2+v^2-2cv\cos\iota}}=\frac{1}{\mu} \tag{5}$$

At normal incidence, $\iota = \tau = 0$ and we obtain:

$$\frac{w-v}{c-v}=\frac{1}{\mu}$$

$$w\mu - v\mu = c - v$$

$$w=\frac{c}{\mu}+v\left(1-\frac{1}{\mu}\right) \tag{6}$$

Equation (6) is used to give a non-relativistic explanation of the result of Fizeau's experiment, which measured the speed of light in moving water.

## 4. Fizeau's experiment

A schematic diagram of the apparatus of Fizeau's experiment [1] is shown in Figure 2. Carried out in the 1850s, it is one of the most remarkable experiments in physics. Light from a source was sent in two opposite directions through transmission and reflection by four half-silvered mirrors $M_1$ – $M_4$. One beam travelled downstream (from $M_1$) through moving water and the other travelled upstream (from $M_4$) in the same water. By an ingenious arrangement of the mirrors the two beams were made to recombine and be observed in an interferometer. An interference pattern, as observed in the interferometer, resulted from the difference in the time taken for the two beams to travel the same path, partly in moving water.

Various explanations have been given for the result of Fizeau's experiment. One accepted explanation is that the velocity of light in the moving water was increased or decreased in accordance with the *relativistic rule for addition of velocities* based on constancy of the speed of light relative to an observer or object that is stationary or moving. A new explanation is proposed in this paper, on the basis of equation (6), outside the theory of special relativity [2, 3].

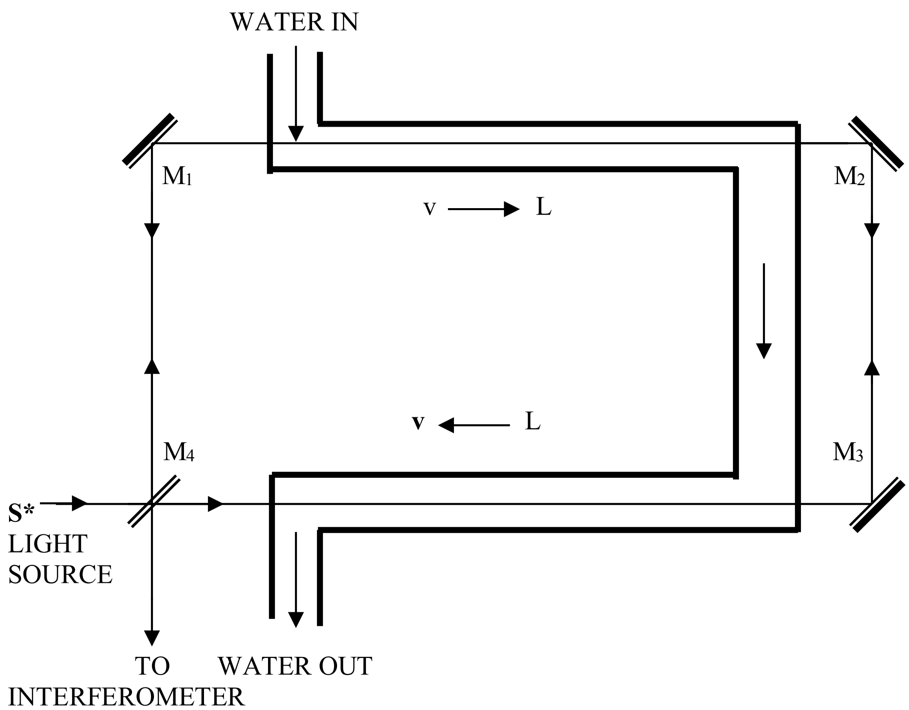


Figure 2. Schematic diagram of the apparatus of Fizeau's experiment

## 5. Relativistic explanation of the result of Fizeau's experiment

According to Einstein's *relativistic velocity addition rule,* if you move with velocity **u** relative to a medium moving with velocity **v** (as light propagated in water moving with velocity **v**), the magnitude of your velocity s, relative to an observer, is:

$$s = \frac{u + v}{1 + \frac{uv}{c^2}} \tag{7}$$

where $c$ is the speed of light in a vacuum and $u$ and $v$ are the magnitudes of the respective velocities. For speeds much less than $c$, or if $c$ is infinitely large, Einstein's relativistic formula (equation 7) reduces to the Galilean (classical) relativity, $s = u + v$.

According to the theory of special relativity, the velocity of light, relative to a medium moving in a vacuum with velocity **v**, remains as a constant **c**. The velocity of light within the medium of refractive index $\mu$ is $\mathbf{c}/\mu$. Einstein's *velocity addition rule* (equation 7), with $u = c/\mu$, gives magnitude of velocity **w** in the moving medium (with respect to an observer), as:

$$w = \frac{\frac{c}{\mu} + v}{1 + \frac{cv}{\mu c^2}} = \frac{\frac{c}{\mu} + v}{1 + \frac{v}{\mu c}} \approx \frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right)$$

$$w \approx \frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right) \tag{8}$$

Equation (8) is Fresnel's law obtained for $v << c$. This equation is used to obtain the transit time difference between the two beams in Fizeau's experiment, and thereby explain the result from the point of view of relativity theory.

Transit time $t_1$ of beam going downstream, with speed $v$ very small compared to $c$, is obtained from equation (8) as:

$$t_1 = \frac{2L}{\frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu^2}\right)\right\}$$

$$t_1 \approx \frac{2L\mu}{c}\left(1 + \frac{v}{\mu c} - \frac{\mu v}{c}\right)$$

The transit time $t_2$ of the beam going upstream with speed $-v$, is:

$$t_2 = \frac{2L}{\frac{c}{\mu} - v\left(1 - \frac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu^2}\right)\right\}$$

$$t_2 \approx \frac{2L\mu}{c}\left(1 - \frac{v}{\mu c} + \frac{\mu v}{c}\right)$$

The time difference between the beam going downstream and the other beam going upstream is obtained as:

$$\Delta t = t_2 - t_1 \approx \frac{4Lv\mu^2}{c^2}\left(1 - \frac{1}{\mu^2}\right) \tag{9}$$

The fringe shift $\delta_y$, according to special relativity, for light of wavelength $\lambda$, is obtained as:

$$\delta_y = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c}\left(1 - \frac{1}{\mu^2}\right) \tag{10}$$

This fringe shift is easily measurable with the interferometer.

## 6. Non-relativistic explanation of the result of Fizeau's experiment

According to the Galilean-Newtonian relativity of classical mechanics, the speed of light in a medium that is moving in a vacuum with velocity **v**, in the opposite direction of the normal, is given in terms of the refractive index $\mu$ of the medium, by equation (6). This gives the speed $w$ of transmission and propagation of light (downstream). If the medium is water of index of refraction $\mu$ moving with velocity $v$ in a vacuum (Figure 2), the time $t_1$ taken for the beam that is going downstream to cover the distance $2L$ (where the magnitude $v$ is very small compared with the speed of light $c$ and $v^2/c^2$ can be neglected), is obtained as given by:

$$t_1 = \frac{2L}{\frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right)} \approx \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} \tag{11}$$

For the beam going upstream, with velocity **–v**, the longer transit time $t_2$ is:

$$t_2 = \frac{2L}{\frac{c}{\mu} - v\left(1 - \frac{1}{\mu}\right)} \approx \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} \tag{12}$$

The time difference $(\Delta t) = (t_2 - t_1)$ between the two beams in moving water, is

$$t_2 - t_1 = \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} - \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\}$$

$$\Delta t = \frac{4Lv\mu^2}{c^2}\left(1 - \frac{1}{\mu}\right) \tag{13}$$

The fringe shift $\delta_x$, for light of wavelength $\lambda$, is obtained as:

$$\delta_x = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c}\left(1 - \frac{1}{\mu}\right) \tag{14}$$

As $\mu > 1 < 2$, $\delta_x$ in equation (14) is smaller than the fringe shift $\delta_y$ as given by equation (10).

In the experiments performed by Fizeau [1], $L = 3\ m$, $v = 7\ m/sec.$, $\lambda = 6 \times 10^{-7}\ m$ (yellow light), $c = 3 \times 10^8\ m/sec$ and $\mu = 4/3$ (for water). The fringe shift $\delta_x$ or $\delta_y$ is obtained as $\approx 0.2$, which was easily observable and measurable in the interferometer. Fizeau's experiment, however, could not determine the correct explanation – relativistic or non-relativistic.

## 7. Conclusion

Michelson and Morley in 1886 and later P. Zeeman and associates in 1915 repeated Fizeau's experiment with greater precision in which the interferometer could measure a fringe shift as low as *0.01*. The influence of the motion of the medium on the propagation of light has thus been verified. As to which is the correct explanation, equation (10) for $\delta_y$ in accordance with the theory of relativity or equation (14) for $\delta_x$ according to the Galilean-Newtonian relativity, remains to be seen.

The treatment of refraction of light in this paper, clearly demonstrates the relativity of the velocity of light with respect to a moving medium, in accordance with Galileo's relativity of classical mechanics. The result of Fizeau's experiment [1] need not be a direct consequence of the relativistic *velocity addition rule* but is more likely to the effect of motion of the transmission medium on the speed of light as shown in equation (6). It should be noted that equation (6) is not an approximation and it applies for all values of speed $v$ up that of light $c$ or $-c$, in contrast to equation (8) obtained for $v << c$.

For $v = 0$, equations (6) and (8) give the speed of light in the medium $w = c/\mu$, as expected. For $\mu = 1$ both equations give $w = c$, also as expected. For $v = c$, equation (8) is not applicable but equation (6) gives $w = c$, again as expected. For $v = -c$, equation (8) is not applicable but equation (6) gives $w = c(2/\mu - 1)$, which is reasonable, particularly if $\mu = 1$.

Equation (10) is another good example of Beckmann's *correspondence theory* [4], whereby the desired result is obtained mathematically but based on the wrong underlying principles. Fizeau's experiment might as well have verified the fringe shift in equation (14), rather than the relativistic equation (10), for the transmission of light in a moving medium. Curt Renshaw [5] analysed the results of several experiments conducted to measure the effect of the speed of a medium on the speed of transmission of light in the medium and he concluded that the results could be explained without invoking special relativity.

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