On the Gravitational Redshift

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Abstract

We show in this note that the gravitational redshift as predicted by general relativity can also be deduced by a simple argument based on the conservation of energy of photons in a gravitational field.

Redshift Equation

Imagine a particle of a mass m moving in the gravitational field

$$\Phi(r) = -GM/r, \tag{1}$$

where M is the mass of some star (or a planet), G is the Newtonian gravitational constant, and $(r, \emptyset, \theta)$ is a system of spherical coordinates with origin at the star's center. The quantity $\Phi(r)$ is the potential of the field at a distance r $(r \geq R)$ of the star's center, where R is the star's radius. The energy of the particle is constant at a value E_0 throughout the motion; it is equal to the sum of its potential energy $V(r) = m\Phi(r)$ and kinetic energy $E_0 - V(r)$. The total energy of the particle is also equal to its kinetic energy at infinity because $\Phi(r)$ vanishes there.

Consider a monochromatic beam of light passing through the given gravitational field. If ν is the frequency of the beam at a point $(r, \emptyset, \theta)$ then the energy of a photon of the beam at that point is

$$h\nu + \frac{h\nu}{c^2}\Phi(r) = h\nu(1 + \frac{\Phi(r)}{c^2}) \tag{2}$$

The first term is just the Planck's formula for the photon's energy and the second is the additional potential energy in the gravitational field. Indeed, a photon of energy $h\nu$ is equivalent to a mass $h\nu/c^2$ which in turn gives rise to the potential energy given by the second term. The total energy of the photon in the field is conserved. If ν_0 is the frequency of the beam at a distant point $(r = \infty)$, then

$$h\nu\left[1 + \frac{\Phi(r)}{c^2}\right] = h\nu_0\left[1 + \frac{\Phi(\infty)}{c^2}\right] = h\nu_0 \tag{3}$$

It follows that the frequency ν of the photon at a distance r $(r \geq R)$ of the center of the field is related to its frequency ν_0 outside the field (at $r = \infty$) by the relation

$$\nu = \frac{\nu_0}{1 + \Phi(\mathrm{r})/c^2} \tag{4}$$

Because Φ is negative and decreases with r, the denominator decreases with r and hence the frequency increases with descending to the field center. The latter equation makes sense as long as the denominator is positive which implies fulfillment of the following condition

$$r > GM/c^2. \tag{5}$$

In particular, the radius R of the star must satisfy the latter inequality.

It is clear that the same equation (4) is obtained if the light trip reverses direction, in the sense that, the beam is generated at a point $(r, \emptyset, \theta)$ and heads to infinity. If ν_1 and ν_2 are the frequencies of the beam at distances r_1 and r_2 respectively from the field's center, then by (4)

$$\nu_1[1 + \Phi(\mathrm{r}_1)/c^2] = \nu_2[1 + \Phi(\mathrm{r}_2)/c^2] = \nu_0 \tag{6}$$

The latter equation quantifies the gravitational redshift in full agreement with that given by relativity theory [`1]. For $\mathrm{r}_1 < \mathrm{r}_2$ we have $\Phi(\mathrm{r}_1) < \Phi(\mathrm{r}_2)$ and $\nu_1 > \nu_2$ which again asserts that the closer the beam approaches the field center the greater becomes its frequency. Setting $r_1 = R$ and $r_2 = r$ in equations (6) we obtain for $r > R$,

$$\nu_r = \nu_R \frac{1+\Phi(\mathrm{R})/c^2}{1+\Phi(\mathrm{r})/c^2} < \nu_R \tag{7}$$

The last equation gives the frequency of the beam, ν_r, at a distance r in terms of its frequency ν_R on the star surface.

References

1. Landau L. D. and Lifshitz E. M., The Classical Theory of Fields, Pergamon Press (1980).