Are Black Holes Actually Quark Stars?

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Abstract

A contradiction has been found between the gravitational singularity at the centre of a black hole predicted by general relativity and the exclusion principle. General relativity asserts that collapsing stars over a certain size mass have no stable orbits and will become a gravitational singularity forming a 'black hole', having finite mass within a point sized space within an event horizon. On the other hand, the Pauli-exclusion principle predicts that for large collapsing stars, a quark-gluon plasma will be created to form a quark star at the central core inside an event horizon. The exclusion principle will thus preclude the collapse of a star into a gravitational singularity at the centre of a black hole.

Investigations of the Schwarzschild and Kerr solutions inside an event horizon have been undertaken. Using an approach that neglects imaginary and space-like solutions it has been found that:

i) For the Kerr solution, an imaginary solution for the ring singularity occurs when $r = \pm\, i\, a$, and so it cannot exist physically. Only two time-like singularities are found to physically exist at the inner and outer event horizons.

ii) For the Schwarzschild solution, a revision of the equation occurs as the coordinate time term becomes space-like.

iii) For the Kerr and Schwarzschild models, a stellar object could then exist inside an event horizon with finite density at the centre and be stable at a fixed radius, precluding a gravitational singularity.

It has been shown that the gravitational energy of a collapsed star ($>3M_{\odot}$), near the core of a quark star, exceeds the energy to create a quark-gluon plasma and the de-confinement energy of quarks from the neutron. It has also been shown that a degenerate non-strange quark star with a maximum density of 1.1×10^{25} kg/m^3 could exist at the centre of an event horizon within the Schwarzschild radius, instead of a black hole. The quark star core would be an ultra-relativistic degenerate Fermi gas that is stable for masses ranging from $3M_{\odot}$ to $287M_{\odot}$.

Calculations have also shown for stellar and rotating black holes that the quark star radius exists at the centre and well within the radius of the Schwarzschild event horizon. The exclusion principle would preclude the formation of a gravitational singularity at the centre. The problem for empirical science is that the quark star with an event horizon will have no emission of radiation and appear to observers to be very similar to a black hole. Notwithstanding, the merger of two 'black holes' for GW150914, producing gravitational waves, offers empirical evidence in the remnant event horizon for the existence of quark stars.

1. Introduction

A black hole is a region of space-time exhibiting strong gravitational effects such that not even high energy particles nor electromagnetic radiation can escape from it. It has an event horizon from which the escape velocity is greater than the speed of light. The theory of general relativity predicts that a sufficiently compact mass can deform space-time to form a black hole. A black hole is effectively invisible and is relatively small in size but can only be observed by the effect it has on other stellar objects. According to contemporary relativistic theory, if the remnant star has a mass greater than about 3 solar masses, it continues collapsing to form a black hole with infinite density at the centre.

The Pauli exclusion principle is one of the most important principles in physics, mainly because the three types of particles from which ordinary matter is made electrons, protons, and neutrons are subject to it. The Pauli exclusion principle underpins many of the characteristic properties of matter, from the large-scale stability of matter to the existence of the periodic table of the elements. Furthermore, the exclusion principle has been applied to astrophysics to explain the stable existence of neutron stars and white dwarfs. In 1995 Elliott Lieb, in a spectacular confirmation of the exclusion principle, showed that it leads to the stability of neutron stars in intense magnetic fields [1].

A neutron star is the collapsed core of a large star, typically, neutron stars have a radius on the order of 10 kilometres and a mass between 1.4 and 3 solar masses. Neutron stars are some of the smallest and densest stars in the universe (excluding quark stars). They result from the supernova explosion of a massive star, combined with gravitational collapse, that compresses the core past the white dwarf star density to that of atomic nuclei. Most of the models for these objects imply that neutron stars are composed almost entirely of neutrons, as electrons have merged with protons to form neutrons. Neutron stars are supported against further collapse by neutron degeneracy pressure, a phenomenon described by the Pauli exclusion principle, just as white dwarfs are supported against collapse by electron degeneracy pressure.

2. The Pauli Exclusion Principle

The Pauli exclusion principle is the quantum mechanical principle that applies to fermions [2]. For fermions, particles with half-integer spin, the exclusion principle weak case states:

In a multi-electron atom there can never be more than one electron in the same quantum state.

A rigorous statement is that with respect to exchange of two identical fermions particles is the strong case [2]:

A system containing several electrons must be described by an anti-symmetric total eigenfunction.

In the case of electrons in atoms, the Pauli-exclusion principle can also be stated as follows:

It is impossible for two electrons of a poly-electron atom to have the same values of the four quantum numbers.

The four quantum numbers:

i) n, the principal quantum number,

ii) ℓ, the angular momentum quantum number,

iii) m_ℓ, the magnetic quantum number, and

iv) m_s, the spin quantum number.

One particularly important consequence of the principle is the elaborate electron shell structure of atoms and the way atoms share electrons, explaining the variety of chemical elements and their chemical combinations. Electrons, being fermions, cannot occupy the same quantum state as other electrons, so electrons have to stack within an atom, that is, have different spins while at the same electron orbital as described.

The exclusion principle is beguilingly simple to state, and many physicists have tried to skip relativity and find direct proofs that use ordinary quantum mechanics alone – albeit assuming spin, which is a genuinely relativistic concept. Pauli himself was puzzled by the principle, and in his Nobel lecture he noted [3]:

"*Already in my original paper I stressed the circumstance that I was unable to give a logical reason for the exclusion principle or to deduce it from more general assumptions. I had always the feeling and I still have it today, that this is a deficiency. …The impression that the shadow of some incompleteness fell here on the bright light of success of the new quantum mechanics seems to me unavoidable*."

Even Feynman, who usually outshone others with his uncanny intuition, felt frustrated by his inability to come up with a simple, straightforward justification of the exclusion principle [3]:

"*It appears to be one of the few places in physics where there is a rule which can be stated very simply, but for which no one has found a simple and easy explanation... This probably means that we do not have a complete understanding of the fundamental principle involved. For the moment, you will just have to take it as one of the rules of the world*."

Quarks are also fermions and have spin ½. There are six quarks: up, down, charm, strange, top, and bottom. All have a rest mass and a charge that may be ± 1/3 or ± 2/3. No exceptions to the exclusion principle have yet been found and so the exclusion principle can be expected to apply to quarks.

The Pauli exclusion principle was recently tested at CERN for electrons at LNGS [4]. To further enhance the sensitivity of the test on the exclusion principle, the experiment was upgraded to VIP 2, where silicon drift detectors (SDDs) replace CCDs as X-ray detectors. In 2018 the limit on violations of the exclusion principle at $\beta^2/2 < 10^{-29}$. It is concluded that it is effectively negligible that a violation of the exclusion principle can occur.

2.1 A Conjecture for Fermions and Bosons

The following conjecture is proposed:

i) Symmetric wave functions, bosons, behave predominantly as waves.

ii) Anti-symmetric wave functions, fermions, behave predominantly as particles.

It is acknowledged that fermions are usually associated with matter, whereas bosons are generally force carriers. Composite fermions, such as protons, and neutrons, and electrons are the key building blocks of ordinary matter. While, bosons exemplified by photons or gluons have zero rest mass and typically act as waves or force carriers of electromagnetism and the strong force respectively.

The dual nature of bosons and fermions is not contradicted and can be explained:

i) Waves can exhibit particle-like properties as solitons (a localised wave)

ii) Particles show wave-like properties as standing waves (particle resonance).

The equally important mass-energy relation ($E = mc^2$) and the complementary nature of wave-particle duality would both be satisfied if:

i) Fermions, as particles, have mass and spatial extent.

ii) Bosons, as waves, have energy and no necessary spatial extent.

For example, a photon, has zero rest mass, but energy, E = hf (where f is the frequency of the wave).
Similarly, an electron would have a rest mass of 9.1×10^{-31} kg and intrinsic spin.
The gluon, as a boson, also has zero mass.

The meson and W and Z particles are specified by their rest mass energies, but are considered to be 'virtual particles' and as force carriers are energy alone.

Moreover, fermions as particles would explain the universality and primacy of the exclusion principle.

The exclusion principle then could be re-phrased:

Two particles cannot occupy the same position in space at the same time.

The exclusion principle would therefore be a direct consequence of the logical principle of non-contradiction. This can be stated as it is impossible for the same thing to be and not be. For the proposition, X: $X \cap \neg X = \emptyset$.

Conversely, bosons as waves can superimpose in space and time forming a reinforced wave. The superposition principle states that, for all linear systems, the net response caused by two or more stimuli (waves) is the sum of the responses that would have been caused by each stimulus individually. The superposition of waves would explain why the exclusion principle does not apply to bosons.

Force carrying bosons must then be wave-like in nature. Particles have spatial extent and cannot occupy the same quantum state. Hence, the universal applicability of the exclusion principle can be explained.

3. General Relativity and Black Holes

In 1796, Laplace calculated, using Newtonian gravitation, the size of a stellar object so compact that the escape velocity would be greater than the speed of light. These were termed 'dark' or 'invisible stars'. The first relativistic calculation for this object was done by Schwarzschild in 1916. The Schwarzschild's radius, R_S, is the same as that for Laplace, where G is constant and M is the mass of the star, c the speed of light:

$$R_S = 2\,GM / c^2 \qquad (1)$$

General relativity is the geometric theory of gravitation published by Albert Einstein in 1915 and the description of gravitation in modern physics. General relativity generalizes special relativity and Newton's law of universal gravitation, providing a unified description of gravity as a geometric property of space and time, or space-time [5]. In particular, the curvature of space-time is directly related to the energy and momentum of whatever matter and radiation are present.

The Einstein field equations may be written in the form:

$$R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} / c^4 \qquad (2)$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, Λ is the cosmological constant, G is Newton's gravitational constant, c is the speed of light in vacuum, and $T_{\mu\nu}$ is the stress–energy tensor.

The Tolman–Oppenheimer–Volkoff limit is an upper bound to the mass of cold, non-rotating neutron stars. Taking account of the strong nuclear repulsion forces between neutrons, leads to estimates in the range from approximately 1.5 to 3.0 solar masses, that is, a neutron star can form for masses less than 3.0 solar masses. In contemporary physics it is accepted that a black hole may form for masses greater than three stellar masses, $3M_\odot$.

The Schwarzschild metric specifies the solution to the Einstein field equations that describes the gravitational field outside a spherical mass, on the assumption that the electric charge of the mass, angular momentum of the mass, and universal cosmological constant are all zero.

In Schwarzschild coordinates, for $r > r_s$, has the form:

$$ds^2 = c^2.d\tau^2 = -(1 - r_s/r).\ c^2.dt^2 + (1 - r_s/r)^{-1}dr^2 + r^2(\ d\theta^2 + sin^2\theta.d\varphi^2) \quad (3)$$

when $d\tau^2$ is positive, τ is the proper time, c is the speed of light, t is the time coordinate, r is the radial coordinate, θ is the co-latitude, φ is he longitude and r_s is the Schwarzschild radius of the massive body, a scale factor which is related to its mass M by $r_s = 2GM/c^2$, where G is the gravitational constant.

The Schwarzschild solution appears to have singularities at $r = 0$ and $r = r_s$ and some of the metric components become infinite at these radii. For the Schwarzschild radius, however, $r_s = 2GM/c^2$, lies interior to the radius of most bodies. For example, the radius of the Sun is 695,700 Km, while its hypothetical Schwarzschild radius, r_s, is only 3 km and the earth is 6,371 km while r_s is 8.8 mm. However, the same cannot be said for compact stellar objects with extremely high densities.

The coordinate singularity at $r = r_s$ divides the Schwarzschild coordinates into two regions. The exterior Schwarzschild solution with $r > r_s$ is the one that is related to the gravitational fields of stars and planets. The interior Schwarzschild solution with $0 \leq r < r_s$, which contains the singularity at $r = 0$, is completely separated from the outer by the singularity at $r = r_s$. The singularity at $r = r_s$ is called a *coordinate singularity,* because changing to a different coordinate system the metric becomes regular at $r = r_s$ and can extend the external region to values of r smaller than r_s.

The case, $r = 0$, is a true singularity and cannot be transformed by coordinates. The Schwarzschild solution is not valid for all r as one runs into a *gravitational singularity*, at the origin. At $r = 0$ the curvature becomes infinite, indicating the presence of a singularity. At this point the metric, and space-time itself, is no longer well-defined.

The Schwarzschild solution, taken to be valid for all $r > 0$, is called a Schwarzschild or 'stellar black hole'. For $r < r_s$ the Schwarzschild radial coordinate r becomes time-like and the time coordinate t becomes space-like. A curve at constant r is no longer a possible world-line of a particle or observer, not even if a force is exerted to try to keep it there; this occurs because space-time has been curved so much that the direction of cause and effect points into the singularity. The surface $r = r_s$ marks the event horizon of the black hole. It represents the point past which light can no longer escape the gravitational field. Any physical object whose radius r becomes less than or equal to the Schwarzschild radius will undergo gravitational collapse and become a black hole.

General relativity predicts that any object collapsing beyond the Schwarzschild radius would form a black hole, inside which a singularity would be formed (covered by an event horizon) where density becomes infinite at the centre of a black hole. The Penrose–Hawking singularity theorems define a singularity to have geodesics that cannot be extended in a smooth manner [6]. The termination of such a geodesic is considered to be the singularity.

A singularity in solutions of the Einstein field equations gives:

i) A space-like singularity where all in-falling matter is compressed to a point.

ii) A time-like singularity where light rays come from a region with infinite curvature.

Space-like singularities are a feature of non-rotating uncharged black-holes, while time-like singularities are those that occur in charged or rotating black hole exact solutions. Both of them have the property of geodesic incompleteness, in which either some light-path or some particle-path cannot be extended beyond a certain proper-time or affine-parameter .

A stellar black hole is a non-rotational and uncharged black hole formed by the gravitational collapse of a massive star. They have masses ranging from about three to several tens of solar masses. The process is observed as a hypernova explosion or as a gamma ray burst. It is believed that black holes formed in nature have spin, but no definite observation of the spin has been recorded. The spin of a stellar black hole is due to the conservation of angular momentum of the star that produced it.

The four types of black holes can be listed as follows:

i) Non-rotating ($J = 0$), Uncharged ($Q = 0$), Schwarzschild.

ii) Non-rotating ($J = 0$), Charged ($Q \neq 0$), Reissner–Nordström.

iii) Rotating ($J > 0$), Uncharged ($Q = 0$), Kerr.

iv) Rotating ($J > 0$), Charged ($Q \neq 0$), Kerr-Newman.

Rotating black holes are formed in the gravitational collapse of a massive spinning star or from the collapse of a collection of stars or gas with a total non-zero angular momentum. As most stars rotate it is expected that most black holes in nature are rotating black holes. In 2006, astronomers reported estimates of the spin rates of black holes in *The Astrophysical Journal*. For example, a black hole in the Milky Way, GRS 1915+105, may rotate 1,150 times per second [7].

The Kerr metric describes the geometry of space-time in the vicinity of a mass, M, rotating with angular momentum, J. The line element in Boyer–Lindquist coordinates is:

$$ds^2 = -(1 - rs.r/\Sigma)c^2.dt^2 + (\Sigma/\Delta)dr^2 + \Sigma\, d\theta^2 + (r^2 + a^2 + (r_s ra^2/\Sigma)\sin^2\theta)\sin^2\theta.d\phi^2 + (2r_s ra \sin^2\theta/\Sigma).c.\,dt.d\phi \quad (4)$$

$a = J / Mc$, and $J = I\omega$ where J is angular momentum, I is moment of inertia, ω angular velocity, $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = r^2 + a^2 - 2mr$

The Kerr inner event horizon, $r_E^- = m - \sqrt{(m^2 - a^2)}$, where $m = ½\, r_s = GM/ c^2$
The Kerr outer event horizon, $r_E + = m + \sqrt{(m^2 - a^2)}$.
A ring singularity exists at r = a.

The outer event horizon, r_E^+, of the rotating black hole is similar to the Schwarzschild radius for a stellar black hole. The inner event horizon, r_E^-, is termed the 'Cauchy horizon'. The ring singularity of the rotating black hole is similar to the gravitational singularity and contains all the mass of the black hole.

3.1 Investigation of the Schwarzschild Solution inside an Event Horizon

Inside the event horizon, $r < r_s$, the sign of dt^2 and dr^2 reverse such that Schwarzschild equation (3) becomes:

$$ds^2 = (1 - r_s/r).\, c^2.dt^2 - (1 - r_s/r)^{-1}dr^2 + r^2(\,d\theta^2 + \sin^2\theta.d\varphi^2) \quad (5)$$

As the coordinate time term, $c^2.dt^2$ has become positive it is a space-like world line. This would entail that the mass becomes imaginary and a bradyon (real particle with mass) travels faster than the speed of light and similarly for light that cannot travel in excess of c by definition. The space-like term is prohibited by relativity and is physically meaningless. The coordinate time term is therefore neglected and given that the dr^2 term is negative and a time-like world line, equation (5) becomes:

$$ds^2 = -(1 - r_s/r)^{-1}dr^2 + r^2(\,d\theta^2 + \sin^2\theta.d\varphi^2) \quad (6)$$

There are five cases as a particle trajectory is attracted by gravitational force to the centre of an event horizon:

i) As the centre is approached at $r \to 0$, $-(1 - r_s/r) \to \infty$ and $-(1 - r_s/r)^{-1} \to 0$ and $ds^2 \to 0$.

ii) For a particle moving with velocity, $v < c$, $ds^2 < 0$, and is time-like inside the event horizon.

iii) For light, $ds^2 < 0$, and light is time-like inside the event horizon as $(-(c\tau)^2 < 0)$

iv) As the Schwarzschild radius is approached at $r \to r_s$, $-(1 - r_s/r) \to 0$and $-(1 - r_s/r)^{-1} \to \infty$ and $ds^2 \to \infty$

v) In addition, r, may remain constant then dr^2, dθ and dφ are zero, so $ds^2 = 0$.

For i) At the gravitational singularity, r = 0, ds^2 vanishes to zero. Matter will be attracted to and can remain fixed at the gravitational singularity. It is not implied that an infinite density mass occurs at the centre.

For ii) The velocity of a real particle is v < c, and so is time-like inside the event horizon. The particles trajectory will be towards the centre.

For iii) The velocity of a photon is c, and this entails $-(c\tau)^2 < 0$ and so is time-like inside the event horizon. This phenomenon is also described in relativity as 'tipping light cones' and also explains why light acts as a particle and not a wave inside the event horizon. Note that the null light cone outside the event horizon (in space-time) has $ds^2 = 0$, causing confusion with a stationery particle in this case.

For iv) At the event horizon, $r = r_s$, a particle is prohibited from leaving the Schwarzschild radius, r_s, as the equation, ds^2, tends to infinity. Light or matter cannot exit the event horizon.

For v) a physical particle can remain fixed at a constant radius, r, from the gravitational singularity as ds^2 is zero. Hence, according to the Schwarzschild solution at the central core a stellar object can remain stable, contrary to usual interpretations.

Equation (6) is not a four dimensional space-time of general relativity, but a three-dimensional Newtonian physics of gravitation with special relativity. Hence, the behaviour of particles inside the event horizon is described by the Newtonian escape velocity of 'dark stars'. This would also explain why the gravitational radius of 'dark stars' is *identical* to the relativistic Schwarzschild radius.

It was noted that the Schwarzschild solution divides space-time into two regions. It is proposed that outside the event horizon relativistic space-time is obeyed while inside the event horizon Newtonian physics and special relativity are obeyed. That light and matter cannot escape from the event horizon would be a pre-condition of special relativity.

For Newtonian gravitational physics, the central cores of ordinary stellar objects such as the earth, sun and neutron stars are filled with compressed matter. In particular, the core of neutron stars is stable as neutron degeneracy pressure, by the exclusion principle, balances gravitational pressure. It is suggested that the gravitational singularity at the centre of a 'black hole' of general relativity would thereby, be precluded by physical particles.

Albert Einstein would appear to support the solution presented here as he considered that general relativity may be invalid for large density fields (inside an event horizon). To quote an excerpt from *Einstein on Peace* [8]:

"The present theory of relativity is based on a division of physical reality into a metric field (gravitation) on the one hand and into an electromagnetic field and matter on the other hand. In reality, space will probably be of uniform character and the present theory will be valid only as a limiting case. For large densities of field and matter, the field equations and even the field variables which enter into them will have no real physical significance. One may not therefore assume the validity of the equations for very high density, and one may not conclude that the beginning of the expansion must mean a 'singularity' in the mathematical sense. All we have to realise is that the equations may not be continued over such regions".

3.2 Investigation of the Kerr Solution inside the Outer Event Horizon

The Kerr solution is claimed to have three singularities as r approaches zero, $r \to 0$:

$$ds^2 = -(1 - rs.r/\Sigma)c^2.dt^2 + (\Sigma/\Delta)dr^2 + \Sigma\, d\theta^2 + (r^2+a^2+ (r_s ra^2/\Sigma) \sin^2\theta) \sin^2\theta.d\phi^2 + (2r_s ra \sin^2\theta/\Sigma).c.\, dt.d\phi \qquad (7)$$

$a = J / Mc$, $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = r^2 + a^2 - 2mr$

a) Time-like singularities occur when $\Delta = 0$, $r^2 + a^2 - 2mr = 0$

Singularities occur at $r = m \pm \sqrt{(m^2 - a^2)}$ (real positive solutions are termed' inner' and 'outer' event horizons)

Also, $dr^2 < 0$ (space-like): $\Delta < 0$ when $r_E^- < r < r_E^+$

$dr^2 > 0$ (time-like): $\Delta > 0$ when $r < r_E^-$ and when $r > r_E^+$

b) A space-like singularity could occur theoretically when $\Sigma = 0$, $r^2 + a^2\cos^2\theta = 0$

A singularity occurs when $\theta = 0$ at $r^2 = -a^2$, that is, $r = \pm\, ia$ (imaginary solutions are neglected)

A singularity cannot occur when r = a as there is no real positive solution at the 'ring singularity'.

Also, the time coordinate will be negative, $dt^2 < 0$, and time-like as $\Sigma > 0$ for all $r > 0$.

c) When $r = 0$, $ds^2 = -c^2.dt^2 + a^2\cos^2\theta.\, d\theta^2 + a^2.\, \sin^2\theta.d\phi^2$

We let $\theta = 0$ and $ds^2 = - c^2.dt^2$, is time-like and constant and so a stable mass can exist at the centre of a Kerr black hole.

There are seven cases for the Kerr black hole as a particle trajectory is attracted by gravitational force to the centre, $r \to 0$:

i) The Kerr solution line element, ds^2, for dr^2 is space-like inside the inner and outer event horizons and dr^2 can be neglected.

ii) The line element, ds^2, for dr^2 is time-like inside the inner event horizons and outside the outer event horizon.

iii) The 'ring singularity' at r = a does not exist as it is imaginary. Thus, when $r = a$, $\Sigma = 2r^2 > 0$.

iv) The Kerr solution, ds^2, for dt^2 is time-like for all $r > 0$.

v) The Kerr solution, $ds^2 < 0$ and is time-like when $r = 0$ at the centre.

vi) For $r = r_E^+$ a time-like singularity occurs when $dr^2 = ds^2 = \infty$.

vii) For $r = r_E^-$ a time-like singularity occurs when $dr^2 = ds^2 = \infty$.

When bradyons (real particles) and light become space-like for the Kerr solution inside the inner and outer event horizons, the Kerr component of dr^2 is imaginary and neglected. The outer event horizon would act the same as the Schwarzschild event horizon and trap matter and light. The inner event horizon is a secondary event horizon that also traps matter and light. However, the ring singularity is an imaginary solution and therefore cannot physically exist. The Kerr time coordinate, dt^2, remains time-like for all radii, r. Matter will be attracted to the centre at r = 0 where it would become stable.

The simplified Kerr solutions inside the outer event horizon, r_E^+, become:

i) $dr^2 < 0$ (space-like) (inside the outer event horizon) when $\Delta < 0$ and $r_E^- < r < r_E^+$

$$ds^2 = -(1 - rs.r/\Sigma)c^2.dt^2 + \Sigma\, d\theta^2 + (r^2 + a^2 + (r_s ra^2/\Sigma)\sin^2\theta)\sin^2\theta.d\phi^2 + (2r_s ra \sin^2\theta/\Sigma).c.\,dt.d\phi \quad (8)$$

ii) $dt^2 < 0$ (time-like) when $\Sigma > 0$ (and $rs,r/\Sigma < 1$) and $r < r_E^-$ (inside the inner event horizon), that is the full Kerr solution:

$$ds^2 = -(1 - rs.r/\Sigma)c^2.dt^2 + (\Sigma/\Delta)dr^2 + \Sigma\, d\theta^2 + (r^2 + a^2 + (r_s ra^2/\Sigma)\sin^2\theta)\sin^2\theta.d\phi^2 + (2r_s ra \sin^2\theta/\Sigma).c.\,dt.d\phi \quad (9)$$

iii) When r = 0, a stable stellar mass can exist at the centre as ds^2 is negative, time-like and constant:

$$ds^2 = -c^2.dt^2 + \cos^2\theta.dr^2 + a^2\cos^2\theta.\,d\theta^2 + a^2.\,\sin^2\theta.d\phi^2 \quad (10)$$

Equation (10) is flat Minkowski space-time that also suggests rotation, θ, at the centre.

We let $\theta = 0$ and $ds^2 = -c^2.dt^2 < 0$ and constant. (11)

(d) For a slowly rotating stellar core at r = 0 such that v << c and a = 0, the Kerr solution becomes:

$$ds^2 = (\Sigma/\Delta)dr^2 + \Sigma\, d\theta^2 + r^2.\sin^2\theta.d\phi^2 \quad (12)$$

$$ds^2 = 1/(1 - r_s/r)dr^2 + r^2(d\theta^2 + \sin^2\theta.d\phi^2) \quad (13)$$

$a = 0$, $\Sigma = r^2$, $\Delta = r^2 - 2mr$

when $\Delta = r^2 - 2mr = 0$, a space-like singularity occurs:

$r^2 = 2mr \therefore r = 2m = r_s$, this is the Schwarzschild radius and equation (13) is the Schwarzschild solution.

When r = 0 a space-like singularity could occur, however, the time-like solution has become space-like when $r < r_s$ and is therefore neglected as for the Schwarzschild solution.

The angular momentum, J, of a compact rotating spherical stellar core at the centre is given by:

$J = I\omega = (2MR^2\omega) / 5$

Where R is the radius, ω is the angular speed at the event horizon, M is the total mass of the compact star.

4. Quark Stars as Black Holes

Observations and theoretical predictions about quark stars have been expanding in recent times. These stars are usually considered to be more compact versions of neutron stars but with a smaller radius, lower luminosity, higher density, and masses between 1 and $3M_\odot$. For instance, it has been suggested that the collapsed core of supernova SN1987A may be a quark star with a mass ≈ 1 - $2M_\odot$[9].

However, in 2009 Kovacs suggested that stellar mass black holes, with masses in the range of $3.8M_\odot$ and $6M_\odot$, could be quark stars in the colour-flavour-locked (CFL) phase [10]. Rapidly rotating CFL quark stars can achieve higher masses than neutron stars, thus making them stellar mass black hole candidates. Furthermore, the radiation properties of accretion disks around black holes and CFL quark stars are very similar. Moreover, quark stars have a low luminosity and a completely absorbing surface such that in-falling matter on the surface of the quark star would be converted into quark matter, again emulating black holes.

Colour–flavour locking (CFL) is a phenomenon that is expected to occur in ultra-high-density strange matter, a form of quark matter. Strange matter consists of up, down and strange quarks and Witten has suggested that this might be the ground state of hadronic matter at all pressures [11]. The quarks form Cooper pairs, whose colour properties are correlated with their flavour properties in a one-to-one correspondence between three colour pairs and three flavour pairs. The CFL forms a Fermi liquid that is a superconductor [12].

It has also been suggested by Thakur in 2007 that a black hole does not collapse into a gravitational singularity, instead matter transmutes into a central core of a quark star [13]. It has been demonstrated that a gravitationally collapsing star acts as an ultra-high energy particle accelerator that continually accelerates particles comprising the matter into a central region. If the in-falling matter has a sufficiently strong gravitational field a quark-gluon plasma can be created. Experiment has shown that matter needs to be heated to a temperature in excess of 2 x 10^{12} °K , which amounts to an energy greater than 175 MeV per particle.

Thakur states that slightly higher temperatures and energies (for a neutron) are required to create the quark-gluon plasma:

"*When the energy, E, of the particles in the black hole is ≥ 10^2 GeV, or equivalently the temperature, T, of the matter in the black holes is ≥ 10^{15} °K, the entire matter in the black hole will be converted into quark-gluon plasma permeated by leptons. Since quarks and leptons are spin 1/2 particles, they are governed by Pauli's exclusion principle."*

It is clear that the gravitational energy of large stars (mass > $3M_\odot$) are capable of generating the necessary temperature and energy for the creation of a quark-gluon plasma. Calculations will also show that the energy for de-confinement of quarks from the neutron is exceeded. Consequently, one of the two possibilities will occur; either Pauli's exclusion principle would be violated and the black hole would collapse to a singularity, or the collapse of the black hole to a singularity would be inhibited by Pauli's exclusion principle, and a quark star would formed at the centre.

It has been acknowledged that a 'black hole' will form for any compact object with a radius smaller than the Schwarzschild radius. In this case the quark star core would fill up the gravitational singularity with degenerate quarks according to the exclusion principle. An event horizon at the Schwarzschild radius for a non-rotating quark star and inner and outer event horizons for a rotating quark star would also occur. Hence, there would no light or radiation emission from the quark star. The quark star would appear to the outside observer to be a 'black hole'.

Another type of non-strange quark matter is termed 'udQM'. This is termed 'non-strange' matter but is actually a meson composed of up and down quarks that has an energy per baryon of 930MeV [14]. The udQM generally has lower bulk energy per baryon than normal nuclei and strange matter and is a predicted to be a new form of stable matter just beyond the periodic table.

In a quark-gluon plasma created by a stellar mass greater than $3M_\odot$ free quarks would move into extremely close proximity by a combination of two powerful forces, that of, the gravitational force and electrostatic attraction between oppositely charged up and down quarks. When quark matter is compressed together at high energy the strong force between quarks drops to zero, allowing asymptotic freedom [15]. The electric force is attractive between up and down quarks and is very strong at close range (6.9 x10^7 N). Two down quarks (-1/3e) will be attracted to one up quark (+2/3e), cancelling electrically, and creating an overall electrically neutral body. The gluons will have no effect as the strong force is zero at close range. The energy per quark of the 'maximally packed' non-strange quark star is only 204.8 MeV and so this lower energy solution is preferred over udQM with an energy per baryon of 930MeV and strange quark matter.

It is posited that the quark star can be fully degenerate in the core and be 'maximally packed' that is, spherical quarks will be packed together like balls in a box and impinge physically. The quark star that exists inside the event horizon would be the densest fermion matter in the universe at 1.110 x 10^{25} kg/m^3. Only 538 millilitres of a non-strange quark star would be equivalent to the mass of the earth, 5.97 x10^{24} kg! The density of the quark star is comparable to the density of a quark.

Quark stars are supported against further gravitational collapse by quark degeneracy pressure, described by the Pauli exclusion principle. It is clear that the non-strange quark star with ultra-high density will be stable. However, even if non-strange matter core were unstable (and exploded) it could not escape the Schwarzschild event horizon, as the escape velocity exceeds the speed of light and would reform as a stable core.

The quark star is found to have a high Fermi energy of 10.2 x10^{-3} J or 63.8 x 10^{15} eV. The momentum, p = 34.0 x10^{-12} kg.m.s^{-1} and mc = 2.1 x 10^{-21} kg.m.s^{-1} and so p >> mc, the velocity of the quark fermions, v ≈ c, that is, the ultra-relativistic case. The quark star is therefore found to be a degenerate ultra-relativistic Fermi gas. The quark degeneracy pressure has a maximum of 302.8 x 10^{42} Pa, that will balance the gravitational pressure by forming partially degenerate quark matter cores. The electric pressure between quarks is also of the order of the quark degeneracy pressure, at 59.6 x10^{42} Pa. The electric pressure will improve the stability of the stellar core by resisting both the gravitational and quark degeneracy pressures. The end result is that the quark star will be stable.

On the stellar evolution of the quark star, it is noted that quarks are only 1.266% of the total mass energy of a neutron. Hence, to form a quark star of mass $3M_\odot$ a star of size 237 $M_\odot$ would need to collapse, however, this would exceed the theoretical maximum of stars in today's universe. Instead, it is more plausible that a neutron star (of mass ≈ $2M_\odot$) expands in mass by the accretion of matter or by merger with another neutron star to obtain a mass of $3M_\odot$ [16]. The neutron star at $3M_\odot$ will then transmute under gravitational pressure into a quark star.

As evidence, GW170817 was the recent observed collision and merger of two neutron stars. It is inferred that, instead of a small black hole, a quark star of mass $2.7M_\odot$ was generated [17]. The composition is probably that of a strange or 'udQM' quark matter. This compact stellar object with a higher mass than the maximum for neutron stars at $2.3M_\odot$ with low x-ray emission and a lower mass than the minimum for black holes at $3M_\odot$, strongly suggests that GW170817 is a quark star. This would also support the presented view of quark star evolution.

The process of growth of the quark star by accretion and or by stellar, neutron or quark star merger could continue indefinitely, however, in sparse stellar regions of the universe would be limited to approximately less than 100 $M_\odot$. This is the typical mass range for stellar mass and rotating 'black holes'.

The characteristics of the degenerate non-strange quark star are summarized as follows:

i) The quark star consists of a mixture of free up and down quarks in the ratio 2d: 1u and gluons.

ii) Quarks (up and down) are considered to be identical and spherical and have a radius of 4.3×10^{-19} m.

iii) Quark stars become fully degenerate and 'maximally packed' due to electrostatic attraction between up and down quarks. Asymptotic freedom of quarks is allowed as the strong force is zero at close range. They have the highest density for fermions, that of, 1.1×10^{25} kg/m^3.

iv) The quark stellar core is an ultra-relativistic degenerate Fermi gas composed of up and down quarks.

v) The partial (or full) degeneracy pressure of the quark star and electric pressure of the attractive quarks balances the gravitational pressure. The non-strange quark star has been found to be stable for star masses from $3M_\odot$ to $287M_\odot$.

vi) Quark stars evolve by the accretion of matter onto a neutron star and can continue to grow in mass by the accretion of matter and or by quark star merger.

4.1 Calculations for the Quark Star as Black Hole

The stellar material due to a supernova explosion or similar event would be mainly protons, electrons and neutrons. However, under intense pressure of gravitational collapse protons and electrons would degenerate to form neutrons. Neutrons would then again degenerate under greater pressure to form quarks. Neutrons are composed of one up quark and two down quarks. This gives a mean mass energy for the non-strange quark matter of 3.9666 MeV/ c^2.

(a) Density of quark star (up and down quarks), $\rho = M/V = 7.061 \times 10^{-31} / 6.366 \times 10^{-55} = 1.110 \times 10^{25}$ kg/m^3

(b) The gravitational energy of a free quark and neutron in the quark stellar core ($r \approx 50.0$ m) for $M = 3M_\odot$:

i) Mass up quark $m = 4.1 \times 10^{-30}$ kg and ii) $m = 1.675 \times 10^{-27}$ kg for a neutron

i) $PE = GmM / r = 32.8 \times 10^{-12}$ J = 204.8 MeV (per quark)

$E = ½ KT$, and $T = 2E/k = 4.7 \times 10^{12}$ °K

ii) $PE = GMm/r = 1.3 \times 10^{-8}$ J = 84.0 GeV (per neutron)

$T = 2E/k = 1.9 \times 10^{15}$ °K

An energy greater than 1 GeV is required to pull a quark out of the neutron as given by the MIT bag model of quark confinement [18]. The energy of 1 GV is far exceeded by gravitational energy of 84 GeV acting upon the neutron in the quark star core. A quark can therefore be liberated from a neutron. The quark-gluon plasma requires an energy of greater than 175 MeV per particle and temperature higher than 2×10^{12} °K. Similarly, the energy for a quark gas is achieved by free quarks.

(c) Calculations for the quark star stability and degenerate Fermi gas parameters:

For $M = 3 M_\odot$, Fermi energy, $E_F = (h/2\pi)^2/2m.(3\pi^2N/V)^{2/3} = 10.2 \times 10^{-3}$ J = 63.8×10^{15} eV

(for mean quark mass = 7.061×10^{-30} kg)

$p \approx E_F / c = 34.0 \times 10^{-12}$ and $mc = 2.1 \times 10^{-21}$ and so $p >> mc$ for ultra-relativistic quarks.

Ultra-relativistic quark degeneracy pressure, $P_{Q(max)} = k_2'(\rho/\mu_e)^{4/3} = 302.8 \times 10^{42}$ Pa.

($k_2' = 1.24 \times 10^{11}$, $\rho_{max} = 1.1 \times 10^{25}$, $\mu_e = 1$)

Gravitational pressure, $P_{G(max)} = - (4\pi/3)^{1/3}G/5.\ M^{2/3}.\rho^{4/3} = 17.3 \times 10^{42}$ Pa.

It is noted that the quark degeneracy pressure, P_Q, is slightly greater than the gravitational pressure, P_G. The gravitational and quark degeneracy pressures will balance, however, by less quarks becoming fully degenerate, thus reducing the quark pressure, such that $P_G \approx P_Q$. That is, a partially degenerate quark star will have a fully degenerate quark central core and udQM or strange quark matter of lower density surrounding the core. The density of fully degenerate quark matter will be a maximum when ρ_{max} = 1.1 x 10^{25} kg.m^{-3} and the corresponding radius of the quark stars will be a minimum. The radius of the quark star will thus be slightly larger for partially degenerate quark cores than those calculated in the following.

In addition, the electric force between 1 up quark and 2 down quarks, F_E is: F_E = 9.0 x10^9. 2/3x1.6x10^{-19}x 2/3x1.6x10^{-19}/ (2x 4.3 x10^{-19})^2 = 138.4 x10^7 N

Electric pressure, $P_E = F_E$ / A = 59.6 x 10^{42} Pa.

$\therefore$ The electric pressure, P_E, will add (or subtract) to quark degeneracy pressure, P_Q, to maintain stability in the core.

So for M = 3 $M_\odot$, $P_Q > P_G$ and $P_Q \approx P_G + P_E$. This means that the quark star will be internally stable.

The maximum gravitational and quark degeneracy pressures are achieved when M = 287$M_\odot$ then $P_{G(max)}$ = 362.4 x 10^{42} Pa and for $P_{Q(max)} + P_E = P_{G(max)}$, the quark star will just be internally stable.

However, when M > 287$M_\odot$ and $P_G > (P_Q + P_E)$ the quark star becomes internally unstable and will collapse gravitationally.

(d) For a stellar black hole a quark star radius and event horizon of three stellar masses, 3$M_\odot$, is calculated:

i) The radius of the event horizon is given by the Schwarzschild radius, R_S
$R_S = 2GM/c^2$ = 2 x 6.67 x 10^{-11} x 3 x 1.989 x 10^{30}/ (2.998 x 10^8)^2 = 8856.2 m = 8.856 Km

ii) The upper limit of radius of a quark [19] = 0.43 x 10^{-16} cm = 4.3 x 10^{-19} m
Volume of quark (maximally packed sphere inside cube) = $(2R)^3$= 6.36055 x $10^{-55}$$m^3$
Mass energy of mean up and down quarks = 3.9666 MeV/ c^2
Mean mass neutron quarks = E/ c^2 = 3.9666 x10^6 x 1.6 x 10^{-19} / (2.998 x 10^8)^2 = 7.061 x 10^{-30} kg
Total Mass of quark star = 3 x 1.989 x 10^{30} kg
Total number of quarks = 8.4505 x 10^{59}
Total Volume of Quark star = total number x volume of quark = 537496.42 m^3
Volume of a spherical quark star = 4/3π.R^3
Radius (minimum) of quark star (mean quarks) = (Volume x 3/4π)$^{1/3}$ = 50.44 m.

iii) Solving for the mean density and radius given by the balancing of pressures inside the quark star:

$P_Q = P_G + P_E$: 1.24 x$10^{11}\rho^{4/3}$ = 7.1 x$10^9\rho^{4/3}$ + 59.6 x10^{42}

ρ_{mean} = 3.393 x 10^{24} kg.m^{-3}, R_{mean} = 75.02 metres

iv) For a stellar black hole of three stellar masses, 3$M_\odot$, the quark star will be a minimum radius of 50.44 metres and maximum radius of 75.02 meters exist inside the event horizon of radius 8.856 Kilometres.

(e) For a stellar black hole, a quark star radius and event horizon of thirty stellar masses, 30$M_\odot$ is calculated:

i) The radius of the event horizon is given by the Schwarzschild radius, R_S

$R_S = 2GM/c^2$ = 2 x 6.67 x 10^{-11} x 30 x 1.989 x 10^{30}/ (2.998 x 10^8)^2 = 88562 m = 88.56 Km

ii) The upper limit of radius of a quark = 0.43 x 10^{-16} cm = 4.3 x 10^{-19} m
Volume of quark (maximally packed sphere inside cube) = $(2R)^3$= 6.36055 x $10^{-55}$$m^3$
Mean mass energy of up and down quarks = 3.9666 MeV/ c^2
Mass mean of neutron quarks = E/ c^2 = 3.9666 x10^6 x 1.6 x 10^{-19} / (2.998 x 10^8)^2 = 7.061 x 10^{-30} kg
Total Mass of quark star = 30 x 1.989 x 10^{30} kg
Total number of quarks = 8.4505 x 10^{60}
Total Volume of Quark star = total number quarks x volume of quark = 5374982.78 m^3
Volume of a spherical quark star = 4/3π.R^3
Radius of quark star (minimum) = (Volume x 3/4π)$^{1/3}$ = 108.67 m

iii) Solving for ρ from $P_Q = P_G + P_E$: 1.24 x$10^{11}\rho^{4/3}$ = 3.310 x$10^{10}\rho^{4/3}$ + 59.6 x10^{42}

ρ_{mean} = 4.0976 x 10^{24} kg.m^{-3}, R_{mean} = 151.77 metres

iv) For a stellar black hole of thirty stellar masses, $30M_{\odot}$, the quark star will be a minimum radius of 108.67 meters and mean radius of 151.77 metres and exist inside the event horizon of radius 88.56 Kilometres.

(f)For a rotating black hole, GRS 1915+105, $14.0 \pm 4.0\ M_{\odot}$ [20], a quark star radius and event horizon can be calculated:

i) The Kerr outer event horizon, $r_E^+ = m + \sqrt{(m^2 - a^2)} = 35421.95 = 35.421$ Km

The radius of the outer event horizon is given, $r_E^+ = 35421.95 = 35.421$ Km

ii) The Kerr inner event horizon, $r_E^- = m - \sqrt{(m^2 - a^2)} = 5907.16$ metres

$m = ½ R_s = GM / c^2 = 20664.5235$ metres
$R_S = 41.329$ Km
Spin factor, a = 0.7 [20]
$\therefore a = 0.7\ m = 14465.16$ metres
$\therefore\ r_E^- = 5907.16$ metres

iii) The ring singularity is at, R = a = 0.06849m = 14465.16 metres.

iv) The upper limit of radius of an up quark = 0.43×10^{-16} cm = 4.3×10^{-19} m
Volume of quark (maximally packed sphere inside cube) = $(2R)^3 = 6.36055 \times 10^{-55} m^3$
Mean mass energy of up and down quarks = 3.9666 MeV/ c^2
Mass mean of neutron quarks = $E/ c^2 = 3.9666 \times 10^6 \times 1.6 \times 10^{-19} / (2.998 \times 10^8)^2 = 7.061 \times 10^{-30}$ kg
Total Mass of quark star = $14 \times 1.989 \times 10^{30} = 2.7846 \times 10^{31}$ kg
Total number of quarks = 3.94363×10^{60}
Total Volume of quark star = total number quarks x volume of quark = 2508368.153 m^3
Volume of a spherical quark star = $4/3\pi.R^3$
Radius of quark star (minimum) = $(\text{Volume} \times 3/4\pi)^{1/3} = 84.29$ m

v) Solving for ρ from $P_Q = P_G + P_E$: $1.24 \times 10^{11}\rho^{4/3} = 1.99 \times 10^{10}\rho^{4/3} + 59.6 \times 10^{42}$

$\rho_{mean} = 3.7017 \times 10^{24}\ kg.m^{-3}$, $R_{mean} = 121.77$ metres

vi) For a rotating black hole, GRS 1915+105, $14.0\ M_{\odot}$ the quark star radius will be a minimum of 84.29 metres and mean radius 121.77 metres and exists inside the radius of the outer event horizon of the rotating 'black hole' GRS 1915+105 with radius 35.421 Km. The inner event horizon is at 5907.16 metres and theoretical ring singularity at 14.465 Km is outside the quark star radius. However, the Schwarzschild solution applies in this case ($v/c << 1$) and the Schwarzschild radius is at 41.329 Km and the core is inside at 84.29 Metres in radius.

(g)For a rotating black hole, GW150914, $62M_{\odot}$ [21], a quark star radius and event horizon can be calculated:

i) The Kerr outer event horizon, $r_E^+ = m + \sqrt{(m^2 - a^2)} = 157896 = 157.896$ Km

The radius of the outer event horizon is given, $r_E^+ = 157896 = 157.896$ Km.

ii) The Kerr inner event horizon, $r_E^- = m - \sqrt{(m^2 - a^2)} = 24304.6$ metres

iii) $m = ½ R_s = GM / c^2 = 91100.2$ metres
$R_S = 182200.45$ metres
spin factor, a = 0.68, a* = 0.68x91100.2 = 61948.1 metres
The ring singularity is at, R = a = 61948.1 metres.

iv) The upper limit of radius of an up quark = 0.43×10^{-16} cm = 4.3×10^{-19} m
Volume of quark (maximally packed sphere inside cube) = $(2R)^3 = 6.36055 \times 10^{-55} m^3$
Mean mass energy of up and down quarks = 3.9666 MeV/ c^2
Mass mean of neutron quarks = $E/ c^2 = 3.9666 \times 10^6 \times 1.6 \times 10^{-19} / (2.998 \times 10^8)^2 = 7.061 \times 10^{-30}$ kg
Total Mass of quark star = $62 \times 1.989 \times 10^{30} = 1.2332 \times 10^{32}$ kg
Total number of quarks = 1.7464×10^{61}
Total Volume of quark star = total number quarks x volume of quark = 11108487.54 m^3
Volume of a spherical quark star = $4/3\pi.R^3$
Radius of quark star (minimum) = $(\text{Volume} \times 3/4\pi)^{1/3} = 138.42$ metres.

v) Solving for ρ from $P_Q = P_G + P_E$: $1.24 \times 10^{11}\rho^{4/3} = 5.37 \times 10^{10}\rho^{4/3} + 59.6 \times 10^{42}$

$\rho_{mean} = 4.969 \times 10^{24}\ kg.m^{-3}$, $R_{mean} = 181.28$ metres

vi) a= J / Mc = 0.68 and $\omega = 2\pi \times 250$ = rad.s^{-1}

J_K = aMc = 2.5026 x 10^{40} (for a Kerr black hole)

J_Q = I. ω= (2/5. M. r_{mean}^2). ω = 2.5347 x 10^{39} (for a rotating compact spherical quark star)

Ratio = J_Q / J_K = 0.1013, that is, 10.13 %

$\therefore$ ω_K (mass at ring singularity) = J / I = 2.5253 x 10^{40}/ 62 x 2 x 10^{30}x 65601.6^2 = 0.047

$\therefore$ f = 7.5 x $10^{-3} \approx 0$ Hz!

vii) For GW150914 the quark star minimum radius will be 138.42 metres and mean radius 181.28 metres and exist inside the Schwarzschild radius of 182.2 Km. The outer event horizon of the rotating 'black hole' has radius 157.90 Km. The inner event horizon at 24.304 Km is not contained within quark star radius and theoretical ring singularity at 61948 metres is also outside the quark star radius. However, GW150914 is a slowly rotating quark star (v = 7.04 x10^5 and v/c << 1 and a = 0) so the Schwarzschild solution applies with a quark stellar mass at the centre.

5. Discussion

When equations for physical theories predict that some quantity becomes infinite or increases without limit, this is generally a sign that there is a missing piece in the theory. For instance, in the history of physics as in the ultraviolet catastrophe, re-normalization, and instability of a hydrogen atom predicted by the Larmor formula major changes in science have been affected. Quantum physics originated as a response to the ultraviolet catastrophe. Also, renormalization in quantum electrodynamics was first developed to make sense of infinite integrals in perturbation theory.

It is proposed that the exclusion principle acts as a 'renormalisation' of the space-time singularity in black holes predicted by general relativity. In particular, the infinite density of the gravitational singularity predicted at the centre of stellar black holes is problematic. The infinite density of the gravitational singularity challenges the notion of mass having spatial extent and raises questions about the physical existence of black holes. How can a point in space possibly have a strong gravitational influence?

The weak and the strong cosmic censorship hypotheses by Penrose are mathematical conjectures about the limitations on singularities arising in general relativity. Singularities that arise in the solutions of Einstein's equations are typically hidden within event horizons, and therefore cannot be seen from the rest of space-time. Singularities that are not hidden are called 'naked singularities'. The weak cosmic censorship hypothesis was conceived by Roger Penrose in 1969 and posits that no naked singularities, other than the Big Bang singularity, exist in the universe [22]. However, as the interior of a black hole is **in** the universe, it would follow that the naked gravitational singularity would not exist either.

On September 14, 2015, the LIGO Scientific Collaboration announced they had made the first observation of gravitational waves from GW150914 [23][24]. The gravitational waves converted about $3M_{\odot}$ into gravitational wave energy and originated from a pair of merging 'black holes' with masses $36M_{\odot}$ and $29M_{\odot}$ with a final mass of $62M_{\odot}$. By extrapolation, the merging of two 'black holes' at some time two singularities would be contained within an event horizon. This would be physically impossible and violate cosmic censorship and a solution to the quandary is required.

It is proposed that the 'black hole' merger of GW150914 creating gravitational waves was in fact the merger of binary quark stars. To begin with, general relativity applies outside the event horizon of quark stars and because the masses are identical for binary 'black holes' and quark stars, the amount of gravitational wave energy liberated would be identical. If the object were a quark star it would have an event horizon equal to the Schwarzschild radius. According to LIGO scientific data, the observed size of the event horizon for GW150914 is 183 Km (equatorial radius) and the remnant mass is $62M_{\odot} \pm 4$, [21][25][26]. The quark star Schwarzschild radius of 182.20 Km (Δ = 0.8 Km) is much closer to the observed size of GW150914 than the Kerr black hole outer event horizon of 157.90 Km (Δ = 25.1 Km). This result offers empirical support that GW150914 is a quark star with an event horizon.

Scientific data for the remnant merger of GW150914 gives a 'black hole' of $62M_{\odot}$, ringdown frequency of 250 Hz and a spin parameter 'a' = 0.68. Hence, the Kerr angular momentum, J = aMc, and quark star angular momentum, $J_O = (2M\omega\, R_{Omean}^2)/5$, can be calculated. For a rotating quark star, this would entail a mean radius of 181.28 metres and a frequency of 250 Hertz giving an internal angular momentum that is 10.13 % of that from the Kerr model. It should be noted that the quark star does not have a spin parameter (a = 0, v/c << 1, f = 250 Hz) and so an angular momentum would be anticipated to be lower than the Kerr model. However, these two figures are not really comparable. It should also be noted that the Kerr black hole parameters (with all mass inside the ring singularity) give a non-sensical rotational frequency of zero Hertz, that is, stationary.

The plausibility of gravitational singularities for black holes can be questioned on five significant points:

i) In the cosmos fermions, as particles, cannot disappear into a point of zero size at the gravitational singularity, as this would violate the exclusion principle.

ii) The gravitational centre for known physical stellar objects in the universe has been enveloped by matter. Similarly, matter in the form of a quark star, would also envelope a 'black hole' singularity.

iii) A black hole has physical mass and this implies that there are particles, residing within it, unlike a gravitational singularity that has no spatial extent but mass.

iv) The cosmic censorship hypothesis (weak) prohibiting naked singularities, would have to apply inside an event horizon of a black hole because for relativity the space-time inside is the same as the outside.

v) Black holes have been observed to merge creating gravitational waves and a paradox of nature with two internal singularities occurs. There would be no paradox if two quarks stars merged.

There are no quandaries, however, about the mass and extent of the black hole singularity when fermions are regarded as particles. A quark star, containing fermions, will have a rest mass and finite spatial extent with a density that is extremely high, but not infinite. There will be no gravitational singularity in this case. It will obey the exclusion principle and with a fixed mass have a corresponding strong external gravitational influence.

The quark star with an event horizon would be stable as the exclusion principle ensures that the quark degeneracy pressure balances the gravitational force. In addition, the strong electrostatic attraction between up and down quarks will improve the stability of the stellar core by resisting both the gravitational and quark degeneracy pressures.

Supermassive black holes, with masses of the order of hundreds of thousands to billions of solar masses, have gravitational pressures that exceed that the quark degenerate and electrostatic pressures of the non-strange quark star by some orders of magnitude. Perhaps Planck stars form at the central core for supermassive black holes again precluding a gravitational singularity or gravitational pressure equations or quark degeneracy equations do not hold? [27]

6. Conclusion

The plausibility of the relativistic model of a black hole has been questioned. An investigation of the Schwarzschild solution for a particle trajectory inside an event horizon has found a revision of theory as the coordinate time term, becomes space-like it is physically meaningless and is neglected. The revised solution inside an event horizon gives no general relativistic space-time, but Newtonian gravity and special relativity. The event horizon acts as a boundary with an escape velocity greater than the speed of light and matter and light are trapped inside while particles are gravitationally attracted to the centre. Consequently, a stellar object can exist at the centre with finite density and be stable at a fixed radius, precluding a gravitational singularity.

An analysis of the Kerr solution as the particle trajectory tends to zero has found only two time-like singularities at the inner and outer event horizons. The ring singularity at $r = a$, was found to be an imaginary solution and as such is a theoretical device. It has therefore been neglected. All matter in the Kerr black hole would be attracted inwards towards the centre where it could find a time-like and stable solution when $r = 0$. For a slowly rotating quark star ($v \ll c$ and $a = 0$) it has been found that the Kerr solution is identical to the Schwarzschild solution. A slowly rotating quark star will have a single event horizon at the Schwarzschild radius. Similarly, for the Schwarzschild and Kerr black holes a stellar object can exist at the centre at a fixed radius.

The Pauli exclusion principle applies universally, present both in the formation of the atomic periodic table and to astrophysics for the stability of neutron stars, white dwarves and quark stars. The formation of a stable quark star within the event horizon at the central core would be made possible by the exclusion principle. This would preclude a gravitational singularity.

A quark-gluon plasma can be generated by the gravitational energy of particles of matter for a star greater than three stellar masses. Furthermore, at the quark star core radius the bag model energy for de-confinement of quarks (1 GeV) from the neutron is exceeded and free quarks can occur. Quarks stars probably evolve from the accretion of matter upon neutron stars and can continue to grow by accretion of matter and or by quark star merger. The quark star is posited to be a degenerate and maximally packed combination of non-strange quark matter (up and down quarks), whereby free quarks have asymptotic freedom due to the strong force dropping to zero at close range.

The degenerate non-strange quark star can achieve a maximum density of fermions at 1.1×10^{25} kg/m^3. The quark star is found to have a high Fermi energy of 10.2×10^{-3} J or 63.8×10^{15} eV. The momentum, $p \gg mc$, and so the velocity of the fermions, v ≈ c, that is, the ultra-relativistic degenerate gas case for quarks. The quark partial degeneracy pressure balances the gravitational pressure in mass ranges from $3M_{\odot}$ to $287M_{\odot}$. In addition the strong attractive electric pressure of quarks improves the internal stability for the quark star. The stellar cores of supermassive black holes, however, cannot be explained by this approach.

Calculations have shown that for stellar and rotating black holes the quark star radius exists well within the radius of the Schwarzschild event horizon. For example, a stellar black hole of three stellar masses, $3M_{\odot}$, the quark star will be a minimum radius of 50.44 metres and mean radius of 75.02 metres and exist inside the event horizon of radius 8.856 Kilometres. For a stellar black hole of thirty stellar masses, $30M_{\odot}$, the non-strange quark star will have a minimum radius of 108.67 metres and mean radius of 151.77 metres and exist inside the event horizon of radius 88.56 Kilometres.

For a rotating 'black hole', GRS 1915+105, 14.0 $M_{\odot}$ the quark star radius will be a minimum of 84.29 metres and mean radius 121.77 metres and would exist inside the radius of the Kerr outer event horizon of 35.4 Kilometres. The inner event horizon is at 5907.16 metres. The fictitious ring singularity was found at a radius of 14465.16 metres. However, v/c = 0.002 and so v/c << 1 and the Schwarzschild solution applies, with a radius of 41.33 Km and mass at the centre.

It has been proposed that the gravitational wave detection of 2015 for GW150914 of a binary 'black hole' merger was, in fact, an equivalent binary quark star merger. It has been shown that the quark star Schwarzschild radius of 182.20 Km is much closer to the observed event horizon size of 183 Km than the Kerr black hole outer event horizon of 157.90 Km. This result would be the empirical support that GW150914 is actually a quark star. It has been calculated that the remnant quark star would have an angular momentum that is 10.13 % of that of the Kerr model, but this would be anticipated as the quark star does not have a spin parameter and the figures are not comparable. While the Kerr black hole having mass in the ring singularity gives a non-sensical rotational and ringdown frequency of zero.

Quark stars that reside behind an event horizon would appear to be the same as a 'black hole' and there would no light or radiation emission. A new empirical problem arises for stellar black holes as the two entities would appear to be identical. Albeit, the event horizon sizes vary slightly for rotating black holes and quark stars.

History shows that Einstein was uninterested in analysing the nature of the space-time singularities that appeared in solutions to his gravitational field equations for general relativity [28]. The existence of such monstrosities reinforced his conviction that general relativity was an incomplete theory which would be superseded by a singularity-free unified field theory.

In the eighteenth and nineteenth centuries 'black holes' were originally termed 'dark stars'. Later due to their relatively small size and unknown composition, in the twentieth century, they were known as 'compact stellar objects'. These objects would appear to be identified as 'quark stars'.

To quote American Astronomer Andrea Ghez:

"*We have this interesting problem with black holes. What is a black hole?*
It is a region of space where you have mass that's confined to zero volume, which means that the density is infinitely large, which means we have no means of describing, really, what a black hole is!"

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