the general science journal

# Why speed cannot be bounded

**Radwan M. Kassir**

radwan.elkassir@dar.com

**Abstract**

In this study, it was hypothesized that speed cannot be indefinite, and must be bounded by a universal limit. This hypothesis was rigorously analyzed using thought experiment and mathematical deduction. A time and space transformation was deemed necessary for the hypothesis to seem viable. However, the transformation turned out to be inconsistent when obtained using different derivation approaches. Furthermore, the obtained transformation equations resulted in critical contradiction. Hence, the universal speed limit hypothesis was deemed false and rejected.

*Key words: Universal speed limit; time transformation; space transformation*

## Introduction

Speed, in terms of space and time (i.e., the rate of change of space coordinates of an entity), is observed in nature basically through the movement of material bodies and waves. According to Newton's first law of motion, a material body in a state of uniform motion, or at rest, will change its state of motion only if it was acted upon by an external force. And, as per Newton's second law of motion, a material body of mass $m$, acted upon by a net constant force $F$, will accelerate with a constant acceleration $a$, according to the relation $F = m \times a$. If the force was applied indefinitely, will the body gain indefinite speed? As per the Special Relativity (SR), the answer is no; the body inertial mass will be increasing as its speed is increasing, and its acceleration will therefore keep decreasing to the point where there will be no further acceleration (i.e., the speed will reach a constant limit), as it approaches the speed of light, when its mass approaches an infinite value – since $a = F / m$. I believe there are two critical problems with this SR prediction.

First of all, the mass increase with velocity follows from the SR assumption that the speed of light is always constant irrespective of the light source or the observer speed, which implies that the speed of light cannot be exceeded. Hence, the mass increase with speed (result) follows from the assumption that the speed of light is the universal speed limit (cause). Hence, using the mass increase with speed (result) to prove the assumption that light speed is the universal speed limit (cause) is an invalid argument—in other words, the result of an assumption cannot be used as a fact to verify the assumption.

Secondly, the law of conservation of energy will be violated as the body approaches the speed of light and stop accelerating. The law of conservation of energy applied to an accelerating mass states that the work done on a material body is equal to its increase in kinetic energy. For an increasing mass with speed in line with SR, this law can be expressed as $F\Delta d = \Delta mc^2$, or $Fd_2 - Fd_1 = [m_{v_2} - m_{v_1}]c^2$, for any space interval along the material body path. But, when the body approaches the speed of light, there will be no further increase in its mass, or $m_{v_2} = m_{v_1}$ for any two points along its constant speed path, which results in the contradiction $F\Delta d = 0$, violating the law of conservation of energy.

As for the wave motion, by which energy is transferred through space via a medium, the speed of a wave depends on parameters such as the physical characteristics of the propagating medium. For instance, mechanical wave, such as sound waves, happen to take definite speed values in different media, be it liquids, solids, or gases, at different physical conditions of the media; so as electromagnetic waves, such as light waves, happen to take a definite value in vacuum, as well as lower values in different transparent media, such as water and glass. It is accepted in mainstream physics, in accordance with SR, that the speed of light in vacuum is supposed to be the universal speed limit. However, it is based on an assumption suggested by some empirical data, such as the result of the well-known Michelson-Morley experiment, with no solid proof or evidence. In fact, superluminal speeds have been reported in several studies. In addition, the possibility that there could exist some unknown media through which light can propagate faster than its speed in vacuum shouldn't be excluded, neither the existence of particles moving at superluminal speeds.

In this paper, it will be indisputably demonstrated, through a thought experiment for testing the hypothesis of the existence of a bounding speed, that speed cannot be restricted to any definite value, utilizing a straight forward analytical approach based on mathematical deductions. Such hypothesis will be shown to lead to absurdity and contradiction.

## Universal Speed Limit Assumption Consequences

### *Time Transformation*

Let's start by hypothesizing that there exists a universal upper speed limit, and let's represent the magnitude of this limit by $c$. Consider two reference frames, a "stationary" shore deck and a "traveling" ship moving seaward at uniform velocity $v$, with the coordinate systems $K(x, y, z)$ and $K'(x', y', z')$ attached to the deck and ship, respectively, such that the $x$- and $x'$-axes are overlapping and extending in the direction of motion.

Suppose there is a material body traveling seaward (in the ship motion direction), relative to an observer in the ship, at the assumed maximum possible speed $c$. Assuming the body was at the superimposed origins of the systems $K$ and $K'$ at time $t = 0$, it will be at a distance of $x' = ct$ from the origin of $K'$, with respect to a ship observer $O'$ at any time $t$. Using the intuitive velocity addition formula, one would conclude the velocity of the body with respect to a stationary observer $O$ in the deck frame should be $c + v$—higher than $c$—and the respective distance traveled by the body should be $x = (c + v)t$. However, as $c$ is supposed to be the maximum speed the body could reach, the speed of the body should remain at its maximum speed $c$ relative to $O$; and the respective traveled distance would be $x = ct$. But we have $x' = ct$, which would result in the wrong conclusion $x = x'$, unless the time elapsed relative to $O'$ was different than the respective time elapsed relative to $O$. Let $t'$ be the time elapsed for $O'$ and $t$ the corresponding time elapsed for $O$. Hence, the traveled distance by the body at time $t'$ relative to $O'$ is $x' = ct'$; and $x = ct$ relative to $O$ at the corresponding time $t$. Therefore, there must be a time transformation between the two relatively moving frames, in order for the universal speed limit assumption to have the prospect of being viable. Next, we will determine the mathematical form of this transformation.

Applying the Galilean transformation on the $x$- and $x'$-coordinates of the material body under consideration, we would get the expressions

$$x' = x - vt \tag{1}$$

$$x = x' + vt' \tag{2}$$

relative to $K$ and $K'$, respectively. But, these expressions will lead to $t = t'$, by substituting Eq.(2) into Eq.(1). Therefore, the above equations do not satisfy the universal speed limit requirement established earlier; that is $t \neq t'$. Hence, let's assume the appropriate transformation will take the following linear form:

$$x' = \gamma(x - vt), \tag{3}$$

relative to $K$, where $\gamma$ is an unknown factor to be determined. Equation (3) represents the $x$-coordinate transformation from the coordinate system $K(x, y, z)$ to $K'(x', y', z')$, where the reference frame of $K'$, the ship, is moving at the uniform speed $v$ relative to that of $K$, the shore deck. But, conversely, $K$ is moving at the speed $-v$ relative to $K'$. Therefore, the coordinate transformation from $K'$ to $K$ should take the same form as the former transformation (from $K$ to $K'$) with the speed being $-v$. Hence

$$x = \gamma(x' + vt'), \tag{4}$$

Using the fact that for $x' = ct'$, we have $x = ct$, under the universal speed limit assumption, as determined earlier, Eqs. (3) and (4) yield

$$ct' = \gamma(ct - vt)$$

$$ct = \gamma(ct' + vt')$$

or

$$t' = \gamma t(1 - v / c) \tag{5}$$

$$t = \gamma t'(1 + v / c) \tag{6}$$

Substituting Eq. (6) into Eq. (5) returns

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{7}$$

It follows that the universal speed limit hypothesis results in the transformation equations (3)–(6), where $\gamma$ is given by Eq.(7).

***Another Time Transformation***

Now, starting from scratch again, another time transformation will de deduced from the speed limit hypothesis, using a different approach.

Suppose now the material body is traveling transversally (perpendicular to the ship motion direction), relative to an observer in the ship, at the assumed maximum possible speed $c$ (Fig. 1). Assuming the body was at the superimposed origins of the systems $K$ and $K'$ at time $t = 0$, it will be at a distance of $y' = ct$ from the origin of $K'$, with respect to a ship observer $O'$, at any time $t$. Using the intuitive velocity addition formula, one would conclude the velocity of the body with respect to a stationary observer $O$ in the deck frame should be $\vec{c} + \vec{v}$ —higher than $c$ —and the respective distance traveled by the body should be $d = (c^2 + v^2)^{1/2} t$. However, as $c$ is supposed to be the maximum speed the body could reach, the speed of the body should remain at its maximum speed $c$ relative to $O$; and the respective traveled distance would be $d = ct$. But we have $y' = ct$, which would result in the wrong conclusion $d = y'$, unless the time elapsed relative to $O'$ was different than the respective time elapsed relative to $O$. Let $t'$ be the time elapsed for $O'$ and $t$ the corresponding time elapsed for $O$. Hence, the traveled distance by the body at time $t'$ relative to $O'$ is $y' = ct'$; and $d = ct$ relative to $O$.

The Galilean transformation applied on the $x$- and $y$- coordinate of the considered material body gives us $x = vt$ (since $x' = 0$) and $y = y'$. Using the Pythagorean Theorem, we can write, from the perspective of $O$, $d^2 = v^2t^2 + y^2$, or $c^2t^2 = v^2t^2 + c^2t'^2$, leading to

$$t = \frac{t'}{\sqrt{1 - v^2/c^2}}, \tag{8}$$

or

$$t = \gamma t'. \tag{9}$$

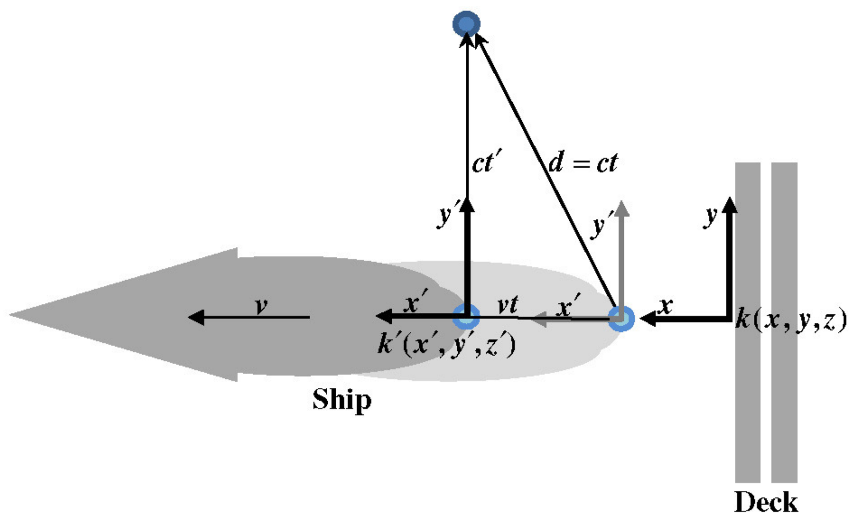


**Fig. 1.** Material body traveling transversally at the assumed speed limit $c$

***Inconsistency***

It follows that the universal speed limit assumption leads to two inconsistent time transformation equations (6) and (9), namely

$$t = \gamma t'(1 + v / c) \tag{10}$$

and

$$t = \gamma t', \tag{11}$$

resulting in $v = 0$.

In other words, the time it takes the material body to travel a certain distance relative to a ship observer is independent on whether the body is traveling in the longitudinal or transverse direction; a ship observer will estimate the same time $t'$ in both directions. But, under the universal speed limit assumption, a deck observer will get different times $(t = \gamma t'(1 + v / c)$ or $t = \gamma t')$ corresponding to the same time $t'$ estimated by a ship observer, depending on the body's direction of motion, which is obviously absurd.

Consequently, the universal speed limit hypothesis is deemed to be false.

Equation (10) can be artificially manipulated to reconcile it with Eq. (11). In fact, rewriting Eq.(10) in the form

$$t = \gamma(t' + vt'/c),$$

and substituting $x' = ct'$ $(t' = x'/c)$ in the second term of the equation, we get

$$t = \gamma(t' + vx'/c^2), \tag{12}$$

valid only for $x' = ct'$. Setting $x' = 0$ (requiring $t' = 0$) in Eq. (12), returns $t = \gamma t'$, which appears to be in agreement with Eq.(9), but in reality it is nothing but $t = 0$.

***Contradiction***

Furthermore, Eq.(5) can be written as

$$t' = \gamma(t - vx/c^2), \tag{13}$$

with $x = ct$.

Substituting Eq.(13) into Eq.(12) yields

$$t = \gamma\left(\gamma\left(t - \frac{vx}{c^2}\right) + \frac{vx'}{c^2}\right),$$

which can be simplified to

$$t\left(\gamma^2 - 1\right) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma x'}{x}\right). \tag{14}$$

Since, as shown earlier, Eqs.(12) and (13) are valid only for $x' = ct'$ and $x = ct$, then Eq.(14) can be written as

$$t\left(\gamma^2 - 1\right) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma t'}{t}\right). \tag{15}$$

Now, according to Eq.(13), for $t' = 0$, $t = vx/c^2$. Consequently, for $t \neq 0$, Eq. (15) would reduce to

$$t\left(\gamma^2 - 1\right) = t\gamma^2 \ , \tag{16}$$

yielding the contradiction,

$$\gamma^2 - 1 = \gamma^2 \ , \ \text{or} \quad 0 = 1 \ ,$$

reconfirming the invalidity of the universal speed limit hypothesis.

## Conclusion

The universal speed limit hypothesis involves space and time transformation. However, the resultant different time transformation equations are revealed to be inconsistent. In addition, they lead to fundamental mathematical contradiction. Hence, the universal speed limit hypothesis is deemed to be false and invalid.