

Article: γ_{AB} is not appropriate as the distance contained in “time=distance/speed”

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Abstract

In §2 of the paper “On the Electrodynamics of Moving Bodies” [1], which is Albert Einstein’s first published work in 1905, he introduced the following equations:

$$t_B - t_A = \frac{\gamma_{AB}}{c - v} \quad \text{and} \quad t'_A - t_B = \frac{\gamma_{AB}}{c + v} .$$

These imply that an event in a moving system viewed from within that moving system differs from the same event when viewed from a stationary system. This perspective became the basis of the Special Theory of Relativity.

However, on thoroughly examining these equations, we find that the term γ_{AB} is not appropriate as the distance contained in the expression "time=distance/speed". In this manuscript, I expand on this issue from a mathematical point of view, employing practical examples containing numerical values.

1. Introduction

In §2 of his 1905 paper “On the Electrodynamics of Moving Bodies”[1], Albert Einstein introduced the following equations:

$$t_B - t_A = \frac{\gamma_{AB}}{c - v} \quad \text{and} \quad t'_A - t_B = \frac{\gamma_{AB}}{c + v}$$

In my article entitled “Unnecessary Equations in §2 of “On the Electrodynamics of Moving Bodies”” [2], I had proved that these equations are not necessary for unraveling the relativity of two systems, one moving and one stationary. However, for some people, it may not be satisfactory performance; because, I did not evaluate these equations themselves. Therefore, in this article, I let the paper [2] be aside, and then I evaluate this expression directly, by considering the term γ_{AB} . Through a consideration of γ_{AB} , I will demonstrate that it is not appropriate to use the term γ_{AB} as the distance contained in the following expression: time = distance / speed.

2. Background

In §1 of the paper “On the Electrodynamics of Moving Bodies” [1], which is Albert Einstein’s first published work in 1905, the following simple equation was shown:

$$t_B - t_A = t'_A - t_B \quad (1)$$

This equation presents a simple formalism to be used in confirming the synchronization of two clocks, placed at points A and B , using light to make a round trip between the two points. The variable t represents time, where t_A is the time at which the light departed from A , t_B is the time at which the light was reflected by a mirror placed at B , and t'_A is the time at which the light returned to A . The left-hand side of Eq.(1) represents the time required for the ray of light to “Go” (from A to B) and the right-hand side represents the time required for the ray of light to “Return” (from B to A). Based on the premise that the speed of light is constant, the right and left-hand sides are equal.

Subsequently, in § 2 of Einstein’s paper [1], the following equations were introduced:

$$t_B - t_A = \frac{\gamma_{AB}}{c - v} \quad \text{and} \quad t'_A - t_B = \frac{\gamma_{AB}}{c + v} \quad (2)$$

The form of both equations corresponds to an expression of time = distance / speed, and the premises set by Einstein were as follows:

1. Two co-ordinate systems: the moving system is described as a moving rigid rod, and the stationary system as the reference frame.
2. The rigid rod travels at a uniform velocity, undergoing a parallel translation with respect to the stationary system, along the positive direction of the X-axis.
3. The end-point of the rod: for the positive direction, the backward is referred to as A , and forward is referred to as B .
4. A light source is placed at A . A mirror is placed at B so as to reflect the light in the opposite direction.
5. A clock is placed at each of the points A and B .
6. The round trip of a ray of light between A and B is performed according to the formalism contained in Eq.(1).
7. v is the velocity of the moving rigid rod, c is the velocity of light.
8. t_A is the time point at which the light was emitted by the light source.

9. t_B is the time point at which the light was reflected by the mirror.
10. t'_A is the time point at which the light returned to A .
11. " γ_{AB} denotes the length of the moving rigid rod - measured in the stationary system".

When an observer in the stationary system observes this round trip of the light, Eq.(2) holds. The left-hand side of Eq.(2) corresponds to "Go" (the light leaving), and the right side corresponds to "Return" (the light returning). Unlike Eq.(1), the structure of the equation representing "Go" and "Return" is different in Eq.(2). Through the consideration about Eq.(2), Einstein implied that an event in a moving system, viewed within that moving system differs from the same event, viewed relatively from a stationary system. This perspective became the fundamental basis of "The Special Theory of Relativity".

[Note] In Section 1 of my paper [2] except the last seven lines, the same sentences until here exist.

At a glance, the above definition of γ_{AB} appears to be perfect. However, when this is considered in some detail, we find that this does not contain information about the time sequence of the system.

It is very important to consider the reasons for which time sequence information is required, when viewed from the stationary system; because, both points A and B are in motion with the progress of time. Therefore, unless we specify their respective time data, we cannot specify their respective positions; and if the positions of A and B are not known, we cannot determine the distance between them. Taken from this perspective, we see that, when measuring the light path between A and B from the stationary system, γ_{AB} is not an appropriate term as the distance contained in the expression: time = distance / speed.

3. Examinations and Discussion

Let us focus on the left side expression contained in Eq.(2),

$$t_B - t_A = \frac{\gamma_{AB}}{c - v} \quad (3)$$

In § 2 of Einstein's paper, before introducing Eq.(3), he emphasized the fact that the relationship between the velocity of light, its path, and the time interval taken to traverse this path can be described by the following expression: "*velocity = light path / time interval*" [1]. The form of this expression corresponds to "speed = distance/time". Thus, Einstein's term

“light path” corresponds to the distance contained in “speed = distance/time”. In other words, the distance between A at the time point at which the ray of light was emitted and B at the time point at which the ray reached B , is the “light path”. Besides, it corresponds to the term γ_{AB} given by Eq.(3).

After proposing Eq.(3) in § 2 of his paper [1], Einstein stated the following: “ γ_{AB} denotes the length of the moving rod – measured in the stationary system”. This sentence does not include any information as to the time sequence of the system. Therefore, I propose two possibilities for a more detailed definition of γ_{AB} , with the time sequence.

- The first possibility: γ_{AB} corresponds to the distance between A at the time point at which the ray was emitted and B at the time point at which the ray reached B .
- The second possibility: γ_{AB} is measured at “a definite time”, from the stationary system.

As we have previously determined the fact that the phrase “light path” corresponds to the distance the light travelled between points A and B , we must consider whether or not the second possibility may be allowed.

Before his definition of Eq.(3), Einstein stated:

“We now inquired as to the length of the moving rod, and imagine its length to be ascertained by the two operations”:

“(a) The observer moves together with the given measuring-rod and the rod to be measured, and measures the length of the rod directly by superposing the measuring-rod, in just the same way as if all three were at rest.”

“(b) By means of stationary clocks set up in the stationary system and synchronizing in accordance with § 1, the observer ascertains at what points of the stationary system the two ends of the rod to be measured are located at a definite time. The distance between these two points, measured by the measuring-rod already employed, which in this case is at rest, is also a length which may be designated “the length of the rod.” [1]

In the description of operation (b), the statement that the two ends of the rod are “located at a definite time” leads us to assume our second possible definition of γ_{AB} .

Now, let us examine a practical example containing numerical values in order to investigate Eq.(3) in more detail.

First of all, I assume that “ $c - v$ ” is the correct term as the speed contained in “speed =

distance/time", temporarily.

[Note] The reason why showing this assumption temporarily is: I already confirmed that there is no need to subtract v from c in my paper [2]; besides, I will show that " $c - v$ " is not appropriate as the velocity contained in "*velocity = light path / time interval*", in the next article of the near future.

In the preparation phase, I temporarily suspend the motion of the rigid rod. I then adjust point A of the rod so that it is at the origin of the stationary system. The distance between points A and B is set to the distance that light can travel in one second. Here, the distance between A and B corresponds to the length of the rigid rod, and the distance travelled by light in one second is hereafter denoted as l .

At $t = 00s$ (zero time), the rod starts to move at half the speed of light ($c \times 0.5$). The light source simultaneously emits a ray of light. The light advances according to "*the principle of the constancy of the velocity of light*", described in Einstein's work [1] as the following: "*Any ray of light moves in the "stationary" system of co-ordinates with the determined velocity c , whether the way be emitted by a stationary or by a moving body*". As a result, we obtain numerical values for the time interval and the length of the light path, both equal to 2.

If we assume the validity of the first possible definition of γ_{AB} , and, substitute the numerical values above into Eq.(3), we obtain the following:

$$2t - 0t = \frac{2l}{1c - 0.5c} \quad (4)$$

However, the result of calculation of Eq.(4) is $2t \neq 4t$; thus, Eq.(4) does not hold. This leads us to distrust our first definition of the term γ_{AB} (the first possibility, mentioned above).

Next, let us validate our second definition of γ_{AB} (the second possibility).

Assuming this definition to be true, let us measure the distance between points A and B from the stationary system at "*a definite time*". In contrast, the left-hand side of Eq.(3), which is used to describe the light path travelling between A and B , requires the determination of two different time point. From the point of view of time sequence, such a situation is not ideal, as it is difficult to know which time (t_A , t_B , or another time point) corresponds to the "*definite time*", to be used in measuring γ_{AB} . However, leaving this issue aside, we may substitute numerical values into Eq.(3).

Assuming that the rigid rod is located at a definite time, this is effectively the same as

considering the rod to be suspended temporarily in the stationary system at this definite time. Thus, the length of the rod can be measured using a measuring-rod, in the stationary system. We will know that this length is $l \times 1$. This value is same as the length viewed by an observer that is moving along with the moving rod. This implies that its value is same as the value that is found using operation (a) [1]. By substituting these values into Eq.(3), we obtain the following expression:

$$2t - 0t = \frac{1 l}{1c - 0.5c} \quad (5)$$

The result of calculation of Eq.(5) is that $2t = 2t$, which denotes that Eq.(5) holds and that the second possible definition of γ_{AB} is correct. However, after introducing operations (a) and (b), Einstein stated the following:

“The length to be discovered by the operation (b) we will call “the length of the (moving) rod in the stationary system.” This we shall determine on the basis of our two principles, and we shall find that it differs from l .” [1]

[Note] The two principles referred to in this sentence are the principle of relativity and the principle of the constancy of the velocity of light, defined in the beginning of § 2 of his paper [1]. The former was defined by Einstein to be the following:

4. Conclusions

Now, let us marshal the above.

1. Einstein defined “*the principle of the constancy of the velocity of light*”.
2. He introduced the operations (a) and (b), used to determine the length of a moving rod.
3. He prefigured that the length of the moving rod measured from a stationary system differs from the length of the same rod measured in the moving system.
4. Einstein’s expression Eq.(3) describing the rigid rod in a moving system, with reference to a stationary system, holds true, if γ_{AB} is equal to $l \times 1$.
5. Assuming the rigid rod to be located at “*a definite time*”, this is effectively equivalent to a rod that is suspended temporarily in the stationary system at this particular time.
6. Measuring the rod under the condition stated in the previous point gives a value of $\gamma_{AB} = l \times 1$.
7. $l \times 1$ is the same length value that may be obtained by an observer that is moving along with

the moving rod.

8. $l \times 1$ is also the same value that may be obtained using operation (a) .

In other words, when we assume that the value of γ_{AB} is variable, Eq.(3) is invalid. Assuming γ_{AB} to be constant, Eq.(3) is valid, contrary to the suggestion of Einstein that γ_{AB} is variable when measured from the stationary system. We can say that there is a contradiction here, and the term γ_{AB} is not appropriate as the distance contained in the expression "time=distance/speed".

The issue discussed in this work, which concerns the universal equation for time = distance/speed, is a purely mathematical problem at the elementary level. Therefore, a validation of my claims will be obtained through the field of mathematics.

Reference

[1] Perrett,W., Jeffery, G.B., On the Electrodynamics of Moving Bodies, English translation of "Zur Elektrodynamik bewegter Körper", German Das Relativitätsprinzip, [4th ed] Tuebner, 1922. Einstein, A. Zur Elektrodynamik bewegter Körper. Annalen der Physik. 17:891, (1905). This version of Einstein's paper appeared in the book "The Principle of Relativity", published in 1923 by Methuen and Company, Ltd. of London. All of these sources are now in the public domain.

[2] Masahiko Makanae, "Unnecessary Equations in §2 of "On the Electrodynamics of Moving Bodies"", publishing in the General Science Journal, March 11, 2015 .

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