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RELATIVISTIC MECHANICS FOR INERTIAL STATES

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As it is currently understood, Einstein's Special Relativity Theory[1] (SRT) leads to mathematical and philosophical inconsistencies that do not correspond to the explained phenomenon when it is applied to inertial body and electromagnetic wave events. An example is that of relativistic aberration as given by SRT whereas the derived equations are devoid of any basic geometrical relationship[2] and are instead derived utilizing vector components of an electromagnetic wave-train. When the variables of these relativistic aberration equations are oriented within a well-understood geometrical relationship such that the relativistic aberration equations naturally result, resolving this geometrical relation utilizing the Law of Cosine's results in temporal equations that do not support SRT. Aberration is a known geometrical effect as a result of inertial motion, yet SRT cannot resolve the relativistic variables into any comprehensibly acceptable geometrical relationship without inducing inconsistencies within the theory.

Another example involves the predictions of the Lorentz Transformations (LT) as contained with SRT. The inconsistency arises when calculating the flight time ratio of an event, either inertial body or electromagnetic wave, in an inertial frame of reference as compared to the identical event occurring within a rest frame of reference. The principle of relativity, first postulate of SRT, would indicate that these ratios must equal each other as relativistic effects on the flight time of an inertial body event must also correspond to an identical effect on an electromagnetic wave event in order for the first postulate to be upheld and no motion be ascertainable by such a simple experiment. When comparing the calculations of the LT with regards to the flight time ratio of an inertial body and an electromagnetic wave along an identical one-way path length, the LT predict different results. The LT predict unequal flight time ratios for a one-way flight path, yet equal flight time ratios for a round-trip flight path. As a result of this inconsistent mathematical property of the LT, the first postulate of SRT can only be enforced for round-trip events and therefore breaks down for one-way events. We should not readily accept such restrictions within a theory.

Another example is that of the format of the LT. The LT are defined as equations that are applicable along the axes that separate the quadrants of a Cartesian coordinate system that are comprised of the three spatial dimensions (x, y, and z.) along with the temporal dimension (Δt). In order to apply the LT to an event that does not occur parallel or perpendicular to the axes, complex mathematical vector and matrix treatments must be utilized as the LT are devoid of any generalized trigonometric format. We would expect that such generalization of the LT is possible within the SRT framework.

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[1] Einstein, Albert. "On the Electrodynamics of Moving Bodies." *Annalen der Physik* (1905): Volume 17:891–921. Print.

[2] Russo, Daniele. "Stellar Aberration: the Contradiction between Einstein and Bradley." *Apeiron* (2007): Volume 14, Number 2:95–112. Print.

These examples along with many concerns regarding the correct derivation of the LT as applied to both inertial body and electromagnetic wave events raises concern that the LT may not be displayed in the most accurate format that supports the known and observed phenomenon. Therefore, we must find the trigonometric form for the LT that satisfies the geometric requirements of aberration and supports the first postulate for both one-way and round-trip events. What follows will encompass a revised relativistic theory of mechanics hereby defined as Inertial Relativity Theory[3].

I. KINEMATICS

§ 1. Definition of Velocity

We will confine our treatment to those inertial events that take place within a Cartesian coordinate system that is void of all external force fields of influence. Both postulates[4] of Dr. Albert Einstein's SRT are strictly enforced within this treatment, yet clarification of the second postulate[5] is taken into account. For any event that occurs in an inertial state, the equations that represent their basic mechanics of motion must encompass only those variables that define such inertial motion For basic mechanics, these variables are that of path distance (r), time interval (Δt), and inertial event velocity (u). The velocity (v) will always indicate that of the inertial frame of reference with respect to the rest frame of reference. There are only two required frames of reference from which measurements will be analyzed; the rest frame of reference designated as (k), and the inertial frame of reference designated as (k'). There are three required measurement points of view for an event; events that originate in a rest frame and are measured with respect to the rest frame, events that originate in an inertial frame and are measured with respect to the inertial frame, and events that originate in an inertial frame and are measured with respect to the rest frame.

Within this treatment, subscript notation is used to denote the frame of reference that the measurement is made relative to, where (Δt_k) indicates the time span "relative to" the rest frame k. For measurements of events that originate within a rest frame of reference, the variables that define this inertial motion with respect to this rest frame are given as the velocity (u), the path length (r), and the time interval (Δt) that this event transpired with respect to. For measurements of events that originate within an inertial frame of reference, the variables that define this inertial motion with respect to this inertial frame are given as the velocity ($u_{k'}$), the path length ($r_{k'}$), and the time interval (Δt_k) that this event transpired with respect to. For measurements of events that originate within an inertial frame of reference, the variables that define this inertial motion with respect to the rest frame are given as the velocity (u_k), the path length (r_k), and the time interval (Δt_k) that this event transpired with respect to. These equations are generalized as follows.

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3 Tieman, Robert. "Inertial Relativity: Understanding Physics in Motion." Amazon.com, 2014. Print.

4 Einstein, Albert. "Relativity: The Special and General Theory." London: Methuen & Co Ltd, 1916. Print.

5 Tolman, Richard. "The Second Postulate." *Physical Review* (1910), Volume 31, Series 1:26–40. Print.

$$\begin{cases} u = \dfrac{r}{\Delta t} \\ u_{k'} = \dfrac{r_{k'}}{\Delta t_k} \\ u_k = \dfrac{r_k}{\Delta t_k} \end{cases}$$

When analyzing inertial events within a Cartesian coordinate system comprised of three dimensions (x, y, z), these analyses involve at minimum two systems, or reference frames k and k', whose axes are parallel, i.e. x is parallel to x', and whose relative velocity vector with respect to each origin lies along the positive x-axis. During an analysis of any inertial body event, it is evident that any inertial event moving parallel to the velocity vector of the reference frame will in no way be deviated from such straight travel without external influence as is demanded by Newton's laws of motion.

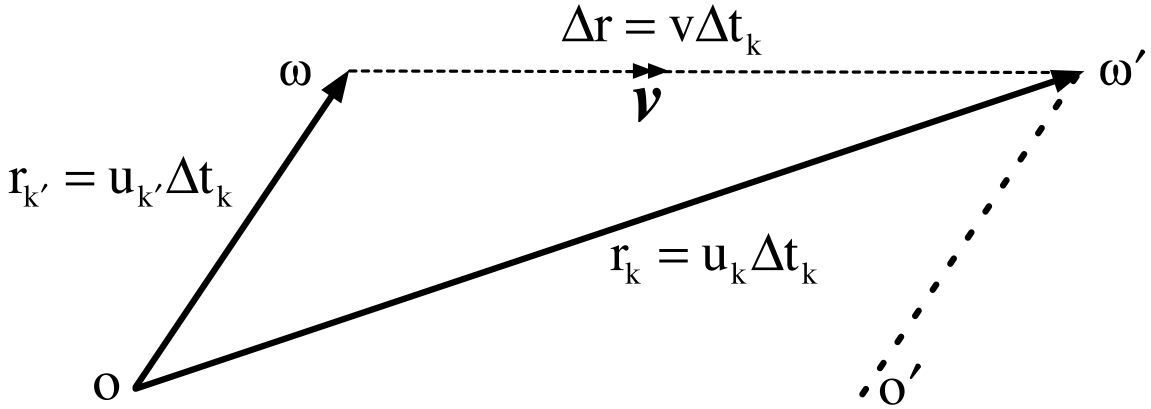

Figure 1

In accordance with the preceding figure, an event transpiring within a rest frame ($v = 0$) would traverse the path ($r_{k'}$) from point (o) to point (ω) as given by the known and accepted mechanics of motion. For an inertial frame at velocity (v), an identical event would traverse the path (r_k) from point (o) to point (ω'), while the inertial frame advances by the distance (Δr), as given by the known and accepted mechanics of motion. These are undisputable facts that must be supported by any theory of mechanics for this geometrical relationship is observationally and experimentally confirmed. Therefore, the geometrical relationship for analyzing an inertial event can be surmised by a simple obtuse triangle. For any inertial event that occurs within an inertial frame of reference, the following geometric relationship, hereby defined as the fundamental geometric of mechanics, must be adhered to in accordance with the known and observed mechanics of motion.

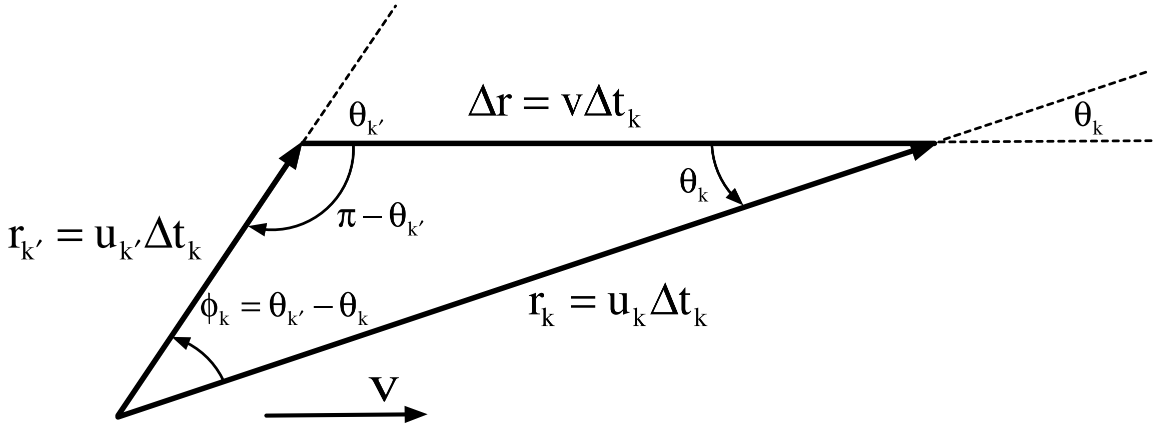

Figure 2

The preceding geometric relationship is the required geometric relationship regardless of how we define length, time or velocity in a relativistic sense. Utilizing the Law of Cosine's, we can make the following mathematical relationship.

$$u_k^2 \Delta t_k^2 = u_{k'}^2 \Delta t_k^2 + v^2 \Delta t_k^2 + 2u_{k'} v \Delta t_k^2 \cos\theta_{k'}$$

Allowing the event velocity to represent an electromagnetic event, we merely allow the substitution of (u = c), and then solve for the electromagnetic velocity with respect to the inertial frame while assuming compliance to the second postulate of SRT.

$$c_{k'} = c\left(\sqrt{1-\beta^2 \sin^2\theta_{k'}} - \beta\cos\theta_{k'}\right)$$

This is the velocity of an electromagnetic wave with respect to an inertial frame of reference, where $\beta = v/c$. We must take note that this is the calculated velocity in accordance with the geometrical-mechanical relationship, not the measured velocity as would be observed with test instruments.

§ 2. Michelson-Morley Temporal Identity

As reported by Albert A. Michelson and Edward W. Morley in 1887, their famous interferometry experiment[6] obtained a null result within the known limits of error and measurement capabilities. This experiment entailed measurements of two round-trip electromagnetic wave events ocurring orthogonal with respect to each other. It is well known and accepted that the transverse event can be resolved into a geometrical right triangle relationship among the variables and resolved for the resulting flight time given as follows.

$$\Delta t_\perp = \frac{2\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$$

The null result of this experiment yielded the fact that both events occurred within the same timeframe. Since the measurements were based upon round-trip events, the equation which manifests this property validated by experiment is given as follows for a

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[6] Michelson, Albert; Morley, Edward. "The Relative Motion of the Earth and the Luminiferous Ether." *American Journal of Science* (1887): Volume 22:120–129. Print.

unique angle (θ) with respect to the velocity vector of the inertial frame.

$$\frac{2\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{r_{k'}}{{}_{\theta}u_{k'}}+\frac{r_{k'}}{{}_{\theta+\pi}u_{k'}}$$

Therefore, a round-trip path transvere to the velocity vector of the inertial frame is equal to the sum of; the ratio of the path distance ($r_{k'}$) in the inertial frame to the event velocity (${}_{\theta}u_{k'}$) with respect to the inertial frame that is parallel to the velocity vector of the inertial frame, and the ratio of the path distance ($r_{k'}$) in the inertial frame to the event velocity (${}_{\theta+\pi}u_{k'}$) with respect to the inertial frame that is opposite to the velocity vector of the inertial frame. This equation is generalized for all inertial body and electromagnetic events as required by the first postulate. For linear electromagnetic events occurring in an inertial frame at any angle whose action is such that half its journey has a constant velocity vector in the opposite direction of the other half, then the sum of both halves times will be equal to twice the time of an identical half-event occurring transverse to the velocity vector of the inertial system.

We must note that this equation is dependent upon the assumption that transverse lengths remain invariant among all inertial frames due to compliance with the first postulate. If transverse lengths were allowed to vary, then the first postulate would be violated. This effect is hereby defined as the principle of invariant transverse length.

§ 3. Relativistic Length Contraction

Independently defined by George Francis Fitzgerald[7] in 1889 and Hendrik Antoon Lorentz[8] in 1892, inertial frames of reference undergo a length contraction along the axis that is parallel to the velocity vector of the inertial frame. This length contraction was postulated in order to explain the null result of the Michelson-Morley experiment and is known as the Lorentz-Fitzgerald contraction.

$$r_{k'}=r\sqrt{1-\frac{v^2}{c^2}}$$

This contraction, where the path distance relative to the inertial frame is representative of a physical length within the inertial frame, also occurs within the framework of SRT as this theory is based upon the LT. Although this contraction is defined along the velocity vector of the inertial frame typically assigned to the x axis, effects of this contraction occur at angles relative to this velocity vector where the effect must obviously be null orthogonal to the velocity vector. This implies that there must exist a trigonometric expression for this length contraction. We begin with the Michelson-Morley temporal identity for the path distance relative to the inertial frame traversed by an electromagnetic wave.

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7 Fitzgerald, George. "The Ether and the Earth's Atmosphere." *Science* (1889): Volume 13:328–390. Print.

8 Lorentz, Hendrik. "The Relative Motion of the Earth and the Ether." *Zittingsverlag Akad. V. Wet.* (1892): 1: 74–79. Print.

$$\frac{2\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{r_{k'}}{{}_{\theta}c_{k'}} + \frac{r_{k'}}{{}_{\theta+\pi}c_{k'}}$$

By finding a common denominator and recombining terms, it follows that

$$2\Delta t\gamma = r_{k'} \frac{{}_{\theta}c_{k'} + {}_{\theta+\pi}c_{k'}}{\left({}_{\theta}c_{k'}\right)\left({}_{\theta+\pi}c_{k'}\right)}$$

where

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

The electromagnetic velocity with respect to the inertial frame was previously defined as

$${}_{\theta}c_{k'} = c\left(\sqrt{1-\beta^2 \sin^2\theta_{k'}} - \beta\cos\theta_{k'}\right)$$

and the opposite vector (θ+π) relation yields

$${}_{\theta+\pi}c_{k'} = c\left(\sqrt{1-\beta^2 \sin^2\theta_{k'}} + \beta\cos\theta_{k'}\right)$$

Now we simply substitute these electromagnetic velocity equations into the preceding temporal identity and solve for the path distance ($r_{k'}$) relative to the inertial frame.

$$r_{k'} = \frac{r\sqrt{1-\beta^2}}{\sqrt{1-\beta^2 \sin^2\theta_{k'}}}$$

We have thus arrived at our trigonometric based Lorentz-Fitzgerald contraction for lengths within an inertial frame, where this effect is independent of any electromagnetic or inertial event. When resolved along the x, y and z-axes, the resulting relations are identical to those utilized by SRT.

$$x-\text{axis} \;\Rightarrow\; r_{k'} = r\sqrt{1-\beta^2}$$

$$y,z-\text{axis} \;\Rightarrow\; r_{k'} = r$$

§ 4. Relativistic Time Dilation

As found by the basic tenets of SRT, the forward progression of time was found to change as a result relative motion. With regards to SRT, time was mathematically treated as the measurements of known fundamental processes such as a mechanical clock or consistently repetitive events such as a light clock. Since any event travels different distances within different frames of reference just as demanded by classical mechanics, their resulting time of flight changes with respect to these different reference frames resulting in the clocks within these systems to alter their rates. Based upon the previous definition of velocity with respect to an inertial frame, the following is evident.

$$u_{k'} = \frac{r_{k'}}{\Delta t_k}$$

We will now apply this equation to an electromagnetic event (u = c) and solve for the temporal variable.

$$\Delta t_k = \frac{r_{k'}}{c_{k'}}$$

Both $(c_{k'})$ and $(r_{k'})$ were already derived previously and can now be substituted within the preceding equation yielding the following result.

$$\Delta t_k = \frac{\Delta t\sqrt{1-\beta^2}}{\sqrt{1-\beta^2\sin^2\theta_{k'}}\left(\sqrt{1-\beta^2\sin^2\theta_{k'}}-\beta\cos\theta_{k'}\right)}$$

We have thus arrived at our trigonometric based temporal dilation for events originating within an inertial frame, where the first postulate requires this relation to be applicable to both electromagnetic and inertial events. When resolved along the x, y and z-axes, the resulting relations are identical to those utilized by SRT.

$$x-\text{axis} \;\Rightarrow\; \Delta\vec{t}_k = \Delta t\sqrt{\frac{1+\beta}{1-\beta}}$$

$$y,z-\text{axis} \;\Rightarrow\; \Delta t_k = \frac{\Delta t}{\sqrt{1-\beta^2}}$$

§ 5. Inertial Transformations

Based upon the fundamental geometric of mechanics in conjunction with the Law of Cosine's, the relativistic path length is given as follows.

$$r_k^2 = r_{k'}^2 + v^2\Delta t_k^2 + 2r_{k'}v\Delta t_k\cos\theta_{k'}$$

Conversely, utilizing the velocity components in conjunction with the Law of Cosine's results in the following general equation for velocity.

$$u_k^2 = u_{k'}^2 + v^2 + 2u_{k'}v\cos\theta_{k'}$$

This is the equation for compound velocities within a relativistic framework. We must take note of this format as it greatly differs from that given by Einstein within SRT. For an event that occurs parallel to the velocity vector of the inertial frame, we have

$$u_k^2 = u_{k'}^2 + v^2 + 2u_{k'}v$$

or

$$u_k = u_{k'} + v$$

This format must be required for all theories of mechanics since no theory could adequately explain how the additive inertial frame velocity (v) could be altered as this velocity is directly measured by the rest frame and is uncontestable from a mechanics standpoint. Based upon the previously established definition of velocity with respect to an inertial frame, the following becomes evident.

$$u_{k'} = \frac{r_{k'}}{\Delta t_k}$$

Both $(r_{k'})$ and (Δt_k) were already derived previously and can now be substituted within this equation yielding

$$u_{k'} = u\left(\sqrt{1-\beta^2 \sin^2\theta_{k'}} - \beta\cos\theta_{k'}\right)$$

Based upon the fundamental geometric of mechanics in conjunction with the Law of Sine's, the relationship of angles is given as follows.

$$\sin\phi = \frac{v\Delta t_k \sin\theta_{k'}}{r_k} = \frac{v\Delta t_k \sin\theta_k}{r_{k'}}$$

related to an electromagnetic event yields

$$\sin\phi = \frac{v\Delta t_k \sin\theta_{k'}}{c\Delta t_k} = \beta\sin\theta_{k'}$$

This is none other than the relativistic version of James Bradley's stellar aberration. We can now summarize the inertial transformations as follows.

$$\begin{cases} \Delta t_k = \dfrac{\Delta t\sqrt{1-\beta^2}}{\sqrt{1-\beta^2\sin^2\theta_{k'}}\left(\sqrt{1-\beta^2\sin^2\theta_{k'}} - \beta\cos\theta_{k'}\right)} \\ r_{k'} = \dfrac{r\sqrt{1-\beta^2}}{\sqrt{1-\beta^2\sin^2\theta_{k'}}} \\ r_k^2 = r_{k'}^2 + v^2\Delta t_k^2 + 2r_{k'}v\Delta t_k\cos\theta_{k'} \\ u_{k'} = u\left(\sqrt{1-\beta^2\sin^2\theta_{k'}} - \beta\cos\theta_{k'}\right) \\ u_k^2 = u_{k'}^2 + v^2 + 2u_{k'}v\cos\theta_{k'} \\ \sin\phi_k = \dfrac{v\Delta t_k\sin\theta_{k'}}{r_k} = \dfrac{v\Delta t_k\sin\theta_k}{r_{k'}} \end{cases}$$

These are the transformations of Inertial Relativity defined for all angles that are compliant with the fundamental geometric of mechanics. Since all of these relations were derived based upon the fundamental geometric of mechanics, both the inertial transformations and the aberration formulae are based upon sound geometrical principles and are fully compliant with this geometrical relationship as expected. Since the inertial transformations only involve those variables that are directly relevant to their outcome, it is evident that the flight time ratio of an inertial body and an electromagnetic wave along an identical path length are found to be equal for both one-way and round-trip flight paths. This symmetry is best viewed by confirming that the ratio of flight path length remains invariant in all inertial frames therefore upholding the first postulate, where an electromagnetic time ($\Delta\tau$) and inertial body time (Δt) is utilized for an event occurring along the positive x axis.

$$\frac{u\Delta t}{c\Delta\tau} = \frac{u_{k'}\Delta t_k}{c\Delta\tau_k} = \frac{\left[u(1-\beta)\right]\cdot\left[\Delta t(1+\beta)\gamma\right]}{\left[c(1-\beta)\right]\cdot\left[\Delta\tau(1+\beta)\gamma\right]} = \frac{u\Delta t}{c\Delta\tau}$$

We must also note that the inertial transformations are no longer fully compliant with Einstein's invariant space-time interval ($x'^2 + y'^2 + z'^2 - c^2\Delta t'^2 = x^2 + y^2 + z^2 - c^2\Delta t^2$). When solved for electromagnetic events, the space-time intervals are invariant. When solved for inertial body events, the space-time intervals vary. The reason for this is found to be relatively simple. For an electromagnetic event compliant with the second postulate of SRT, all inertial frames may be considered as a rest frame relative to this electromagnetic event, or the measured velocity of light is a measured constant in all reference frames. For an inertial body event, only one reference frame may be considered as a rest frame, as the inertial body will have a differing non-zero velocity in all other reference frames. Since the invariant space-time intervals of SRT are mathematically based upon terms of electromagnetic length ($c\Delta t,\ c\Delta t'$), it should be no surprise that only electromagnetic events are found to be in compliance with this interval lest we unjustly force electromagnetic properties, namely the second postulate, upon inertial bodies.

What is important to note is that the inertial transformations were derived directly from a known and accepted geometrical relationship regarding inertial objects. The relativistic format and properties derived from this geometrical relationship are not choices but consequences that must be accepted regardless of current dogma.

II. ELECTRODYNAMICS

§ 6. Doppler's Principle

In 1842, Christian Andreas Doppler discovered that that the measured frequency of a propagating wave depended upon the relative speed of the source and the observer, which is referred to as the Doppler effect. We seek to find the relativistic expression for this effect for all angles relative to this motion. We begin by redefining the relativistic velocity equations for electromagnetic phenomenon, where the velocity of light (c) is related to its frequency (ν) and wavelength (λ).

$$\begin{cases} c = \nu\lambda \\ c_{k'} = \nu_{k'}\lambda_{k'} \\ c_k = \nu_k\lambda_k \end{cases}$$

For a uniform spherical electromagnetic wave expansion within a rest frame, the resulting radii (r) of this sphere are merely products of the expansion velocity (c) and the time of expansion (Δτ). Now the same spherical electromagnetic wave expansion occurs within an inertial frame where the resulting radii ($r_{k'}$) of this sphere are the products of the expansion velocity ($c_{k'}$) and the time of expansion ($\Delta\tau_k$). In order for the first postulate to be upheld, the transverse expansion radius within the inertial frame must be identical to the expansion radius that occurred within the rest frame in order to satisfy the principle of invariant transverse length. For a transverse electromagnetic event

$$r_{\perp} = c\Delta\tau = c_{k'}\Delta\tau_k$$

where solving for the time of expansion yields

$$\Delta\tau_k = \Delta\tau\gamma$$

Therefore, the first postulate requires an inertial expansion rate of ($\Delta\tau\gamma$). Utilizing these relations, we can now ascertain the spherical electromagnetic wave expansion within an inertial frame. For any electromagnetic wave whose expansion ($\lambda_{k'}$) occurs with respect to an inertial frame (k') in a fundamental unit of time ($\Delta\tau_k$), normally associated with the period of the electromagnetic wave, the resulting radius for this expansion can be expressed utilizing the electromagnetic expression for wavelength as

$$\lambda_{k'} = c_{k'} \Delta\tau_k$$

We have previously derived the velocity of light with respect to the inertial frame, and the preceding analysis has given us the expansion rate as well. With these values, we can simply this equation further.

$$\lambda_{k'} = \lambda\gamma\left(\sqrt{1-\beta^2 \sin^2\theta_{k'}} - \beta\cos\theta_{k'}\right)$$

This is the electromagnetic length within an inertial frame. Just as the Lorentz-Fitzgerald contraction defines physical lengths within an inertial frame, the preceding equation defines electromagnetic lengths within an inertial frame. This equation defines how electromagnetic lengths are contracted within an inertial frame, but we know that electromagnetic phenomenon also undergo aberration as a result of relative motion. Therefore, there must exist a combinatorial effect whereas the resulting relativistic Doppler effect must be comprised of both electromagnetic length contraction and electromagnetic aberration. Electromagnetic length contraction is an effect that occurs along the velocity vector of the inertial frame, whereas the effect of aberration occurs orthogonal to the velocity vector of the inertial frame. In order to ascertain this aberration, we return again to the fundamental geometric of mechanics as relates to electromagnetic length.

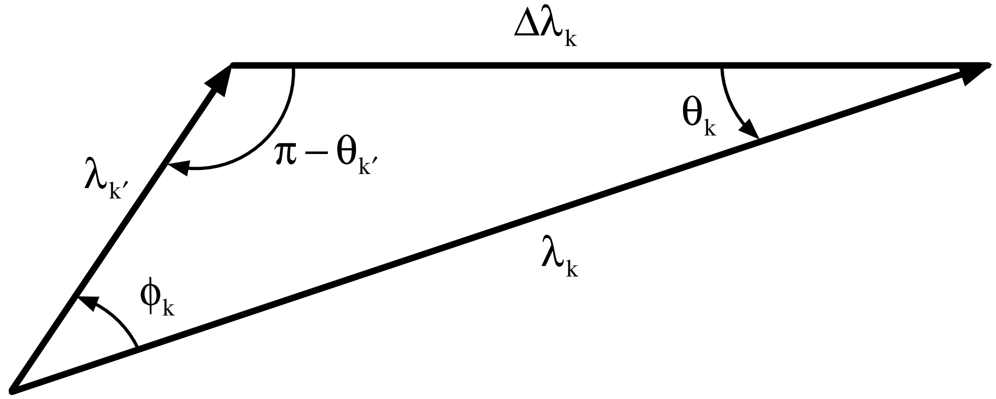

Figure 3

Utilizing the Law of Sine's, we solve for the change in wavelength ($\Delta\lambda_k$).

$$\Delta\lambda_k = \lambda_k \frac{\sin\phi_k}{\sin\theta_{k'}}$$

The relation of angles ($\sin\phi_k = \beta\sin\theta_{k'}$) was already established previously for this geometrical relation and can now be substituted for further simplification.

$$\Delta\lambda_k = \beta\lambda_k$$

This geometrical relationship merely shows the wavelength that is to undergo aberration in the inertial frame of reference, not the relationship due to spatial and aberrational distortion. Electromagnetic length contraction is an effect that occurs along the velocity vector of the inertial frame, whereas the effect of aberration occurs orthogonal to the velocity vector of the inertial frame. For simplicity, we merely overlay this orthogonal aberration requirement over the previous geometrical relation in order to show how the aberrational factor is correlated orthogonal to the wavelength path since we seek the resulting aberrational effect and not the traditional path length. What the two relationships have in common is the aberration angle, and that is what we use as the common factor between the two.

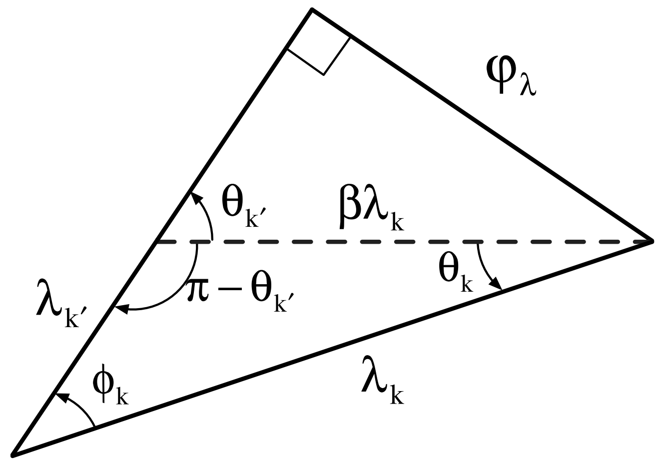

Figure 4

Utilizing the Law of Sine's, we solve for the orthogonal aberration factor (φ_λ).

$$\varphi_\lambda = \beta\lambda_k \sin\theta_{k'}$$

This geometrical relationship is simply an overlay illustrating the combinatorial effects of length contraction and aberration. The final geometrical relationship we seek is that which encompasses the combinatorial effects associated with electromagnetic emission within an inertial frame that also conserves the angle of aberration established within the fundamental geometric of mechanics.

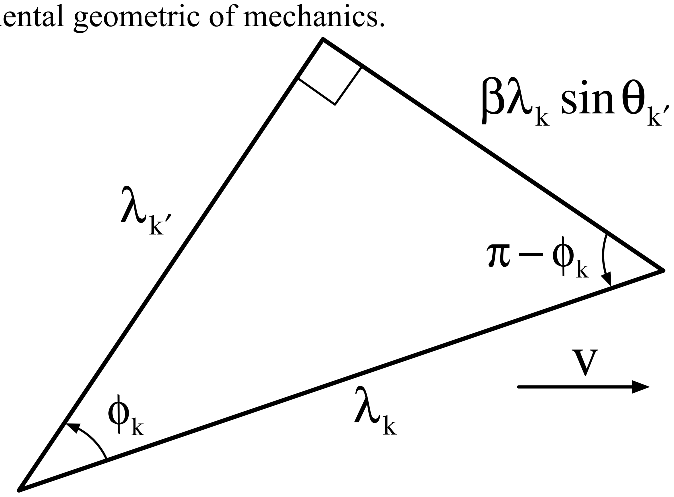

Figure 5

This is the electromagnetic geometric of aberration. Now we are capable of determining the second combinatorial effect utilizing this geometrical relation. Utilizing the Pythagorean theorem, we solve for the resulting wavelength (λ_k) with respect to the inertial frame otherwise known as the relativistic Doppler wavelength.

$$\lambda_k = \frac{\lambda_{k'}}{\sqrt{1-\beta^2 \sin^2\theta_{k'}}}$$

Now we substitute the previously derived electromagnetic length and continue to simplify.

$$\lambda_k = \lambda\gamma\left(1-\frac{\beta\cos\theta_{k'}}{\sqrt{1-\beta^2\sin^2\theta_{k'}}}\right)$$

Thus we have found the relativistic Doppler effect for all angles relative to the velocity vector of the inertial frame. Conversely, the remaining relativistic Doppler equations can be found utilizing the known electromagnetic relations ($c = \nu\lambda$) and ($\varepsilon = \rho c$), where the velocity of light (c) is related to the electromagnetic; frequency (ν), wavelength (λ), momentum (ρ), and energy (ε).

$$\begin{cases} \lambda_k = \lambda\gamma\left(1-\dfrac{\beta\cos\theta_{k'}}{\sqrt{1-\beta^2\sin^2\theta_{k'}}}\right) \\ \nu_k = \dfrac{\nu}{\gamma\left(1-\dfrac{\beta\cos\theta_{k'}}{\sqrt{1-\beta^2\sin^2\theta_{k'}}}\right)} \\ \rho_k = \dfrac{\rho}{\gamma\left(1-\dfrac{\beta\cos\theta_{k'}}{\sqrt{1-\beta^2\sin^2\theta_{k'}}}\right)} \\ \varepsilon_k = \rho_k c \end{cases}$$

We have thus arrived at the relativistic Doppler effect based upon trigonometric relations allowing application through all angles relative to the velocity vector of the inertial frame. When the electromagnetic energy is resolved along the x, y and z-axes, the resulting relations are identical to those utilized by SRT.

$$x-\text{axis} \;\Rightarrow\; \vec{\varepsilon}_k = \varepsilon\sqrt{\frac{1+\beta}{1-\beta}}$$

$$y,z-\text{axis} \;\Rightarrow\; \varepsilon_k = \frac{\varepsilon}{\gamma}$$

§ 7. Energy of Emission from an Inertial Frame

Suppose we have two fundamental particles that undergo annihilation with an angle of emission with respect to the velocity vector of the inertial system. For an event within the rest frame, conservation of momentum would require equal emission in opposite directions in order to conserve the total momentum of the two-particle system. We will analyze this event that occurs within an inertial frame, where the resulting emission is comprised of a single photon in each direction. The total energy of this photon pair is the sum of both photons.

$$\sum\varepsilon = \frac{{}_{\theta}\varepsilon}{2} + \frac{{}_{\theta+\pi}\varepsilon}{2}$$

Utilizing the relativistic Doppler equations for each direction (θ) and ($\theta+\pi$), the resulting total energy of this event can be calculated.

$$\sum \varepsilon = \varepsilon\gamma - \varepsilon\gamma\beta^2 \sin^2\theta_{k'}$$

Utilizing the fundamental trigonometric identity ($\sin^2\theta_{k'} = 1-\cos^2\theta_{k'}$), the resulting relationship can be defined.

$$\sum \varepsilon = \frac{\varepsilon}{\gamma} + \varepsilon\gamma\beta^2 \cos^2\theta_{k'}$$

This is the combined energy of the photon pair and is equal to the sum of all photon pairs created at an angle differing from that of the velocity vector of the inertial system. We can now summarize the electromagnetic expressions for the total energy before emission.

$$\begin{cases} \varepsilon_k = \varepsilon\gamma = h\nu\gamma \\ \rho_k = \rho\gamma = \dfrac{h\gamma}{\lambda} \end{cases}$$

For emission along the velocity vector of the inertial frame, the resulting total energy is given as ($\varepsilon\gamma$) which is identical to the total energy resident prior to annihilation. For emission transverse to the velocity vector of the inertial frame, the resulting total energy is given as (ε/γ). It immediately becomes evident that we must account for the residual energy not found within the created photon pair. This energy is the additive to the total energy as given previously.

$$\Delta\varepsilon = \varepsilon\gamma\beta^2 \cos^2\theta_{k'}$$

This energy is maximized along the velocity vector of the inertial frame; yet this energy is null transverse to this velocity vector. Whenever electromagnetic emission occurs at any angle other than the velocity vector of the inertial frame, then there must exist an additional emission along this velocity vector that is separate and distinct as a result of the conservation of energy. The energy not imparted to the primary photon emission must be resident along the DeBroglie kinetic vector as was present prior to emission. This residual energy is merely the difference of what is imparted to the photon and what was present prior to emission.

$$\psi = \varepsilon\gamma\beta^2 - \varepsilon\gamma\beta^2 \cos^2\theta_{k'}$$

or

$$\psi = \varepsilon\gamma\beta^2 \sin^2\theta_{k'}$$

In order to maintain compliance to the conservation of momentum, then equal emission must occur in opposite directions with respect to the velocity vector of the inertial frame such that the following equation represents the sum of this energy.

$$\psi = \frac{\varepsilon\gamma\beta^2 \sin^2\theta_{k'}\left(1-\beta\right)}{2} + \frac{\varepsilon\gamma\beta^2 \sin^2\theta_{k'}\left(1+\beta\right)}{2}$$

Therefore, we should expect to measure an emission along the velocity vector of the inertial system regardless of the photon-pair emission angle. Now we return to our description of the combined energy of the photon pair evaluated along the velocity vector of the inertial frame.

$$\sum \varepsilon_{\theta=0} = \ddot{\varepsilon} = \varepsilon\gamma = \frac{\varepsilon}{\gamma} + \varepsilon\gamma\beta^2$$

Now were merely multiply each side by the energy term (εγ) and perform the substitution (ε = ρc).

$$\ddot{\varepsilon}_k{}^2 = \varepsilon^2 + \left(\rho c\gamma\right)^2$$

We should immediately recognize this format as that of the energy-momentum relation contained within SRT. Now we can see that this SRT relation is merely a special case for electromagnetic emission along the velocity vector of the inertial frame.

§ 8. Relativistic Inertia

In 1924, Louis De Broglie[9] proposed that material objects in motion exhibit wave-like properties commonly referred to as matter waves. Clinton Davisson and Lester Germer experimentally[10] confirmed this property in 1927. For an inertial object, the DeBroglie wavelength is given as

$$\lambda_k = \frac{h}{P_k}$$

The DeBroglie wavelength (λ_k) is related to the ratio of Planck's constant of action (h) to the momentum of the inertial object (P_k). We must immediately recognize the fact that the momentum of this DeBroglie wave (ρ_k) must be one in the same as the momentum of the inertial object (P_k). Based upon the previously derived relation for electromagnetic emission within an inertial system, the total energy calculated along the velocity vector of the inertial frame represents the total energy present before emission occurs. Therefore, this is the potential electromagnetic energy present before emission occurs. Now we merely relate these expressions by virtue of their equal momentum.

$$P_k = \rho_k = \frac{h\gamma}{\lambda}$$

Since inertial momentum (P) is the product of the inertial mass (M) and the inertial velocity (v), and the velocity is directly measurable by the rest frame, then the only variable that can change in a relativistic sense would be that of the inertial mass. Therefore, the following becomes readily apparent.

$$P_k = M_k v = \frac{h\gamma}{\lambda}$$

Owing to the fact that the relativistic factor (γ) is responsible for the momentum/energy increase, the non-relativistic inertial momentum expressed as a DeBroglie wave ($P = Mv = \frac{h}{\lambda}$) allows further simplification.

$$M_k = M\gamma$$

[9] De Broglie, Louis. "The Wave Nature of the Electron." Nobel Lecture (1929).
[10] Davisson, Clinton. "The Diffraction of Electrons by a Crystal of Nickel." *Bell System Tech. J.* (1928): 7 (1): 90–105. Print.

This is none other than relativistic inertia, which is identical to the expression utilized within SRT. This fundamental relationship is possible due to the DeBroglie wavelength in conjunction with the conservation of momentum/energy. Contrary to postulation by SRT for the sole purpose of conserving relativistic momentum, relativistic inertia has actually been hereby derived from first principles. Let us now return to our derivation of the total energy encompassed by an inertial mass prior to emission.

$$\varepsilon\gamma = \frac{\varepsilon}{\gamma} + \varepsilon\gamma\beta^2$$

Owing to the equivalence of mass and energy as given within SRT, this is the energy that must be possessed by the inertial mass prior to emission. Therefore, this must be the potential electromagnetic energy of the inertial mass as given by the famous relation ($E = Mc^2$).

$$E_k = \frac{E}{\gamma} + E\gamma\beta^2$$

Where the SRT mass-energy relation can now be utilized to simplify this equation.

$$E_k = \frac{Mc^2}{\gamma} + Mv^2\gamma$$

We know that the (Mc^2/γ) is ascribed to the energy provided solely by the inertial mass, as this can be proven when evaluating electromagnetic emission transverse to the velocity vector of the inertial system. Therefore, the remaining term ($Mv^2\gamma$) must be ascribed to the DeBroglie wave, hereby defined as the DeBroglie kinetic (κ). In accordance with SRT, the relativistic kinetic energy is merely the difference between the relativistic energy and the rest energy of an inertial mass.

$$\Delta E_k = Mc^2(\gamma - 1)$$

We can now fully describe the inertial state of a relativistic mass with complete confidence.

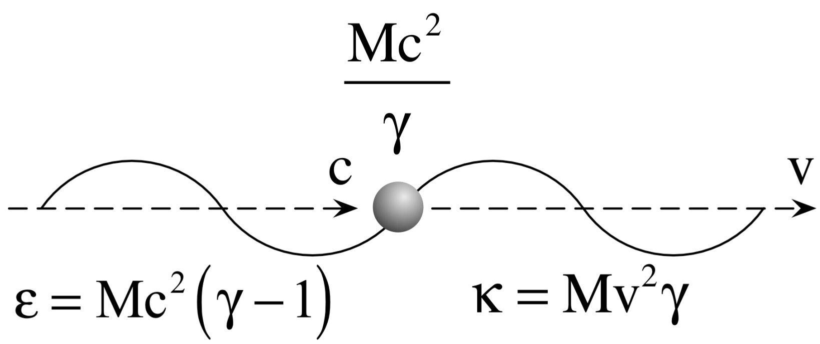

Figure 6

For an incident energy ($Mc^2(\gamma$-1)) that propels a rest mass (M) to a uniform velocity (v), the inertial state results in a harmonic balance between the inertial mass-energy (Mc^2/γ) and the DeBroglie kinetic ($Mv^2\gamma$) whose combinatorial effects comprise the entirety of the inertial effect.

III. GEOMETRICS

§ 9. Geometrical Properties of an Inertial Field

For a uniform spherical electromagnetic wave expansion within an inertial frame, the previously established electromagnetic length within an inertial frame defines the radii.

$$r_{k'} = r\gamma\left(\sqrt{1-\beta^2\sin^2\theta_{k'}} - \beta\cos\theta_{k'}\right)$$

and resolves into separate terms gives

$$r_{k'} = r\gamma\sqrt{1-\beta^2\sin^2\theta_{k'}} - r\gamma\beta\cos\theta_{k'}$$

This relation reflects the spherical expansion, where the ($r\gamma\beta\cos\theta_{k'}$) term indicates a spatial offset along the velocity vector of the inertial system that shifts towards the -x axis. This is the electromagnetic length, which is synonymous with the properties of the spatial medium and must not be confused with physical lengths already derived previously. This electromagnetic length will determine the apparent spatial properties of the medium that the inertial objects traverses, which we know is altered by motion as was found previously regarding a transverse electromagnetic event whose resulting relativistic distance was found to be ($r\gamma$) with respect to the rest frame. Since transverse lengths remain invariant for all frames of reference, this relativistic transverse path length of ($r\gamma$) can only be attributed to an apparent change in the spatial medium due to motion. We will now map this electromagnetic length in 5^o increments through 2π relative to a common origin as compared to an identical spherical expansion within a rest frame.

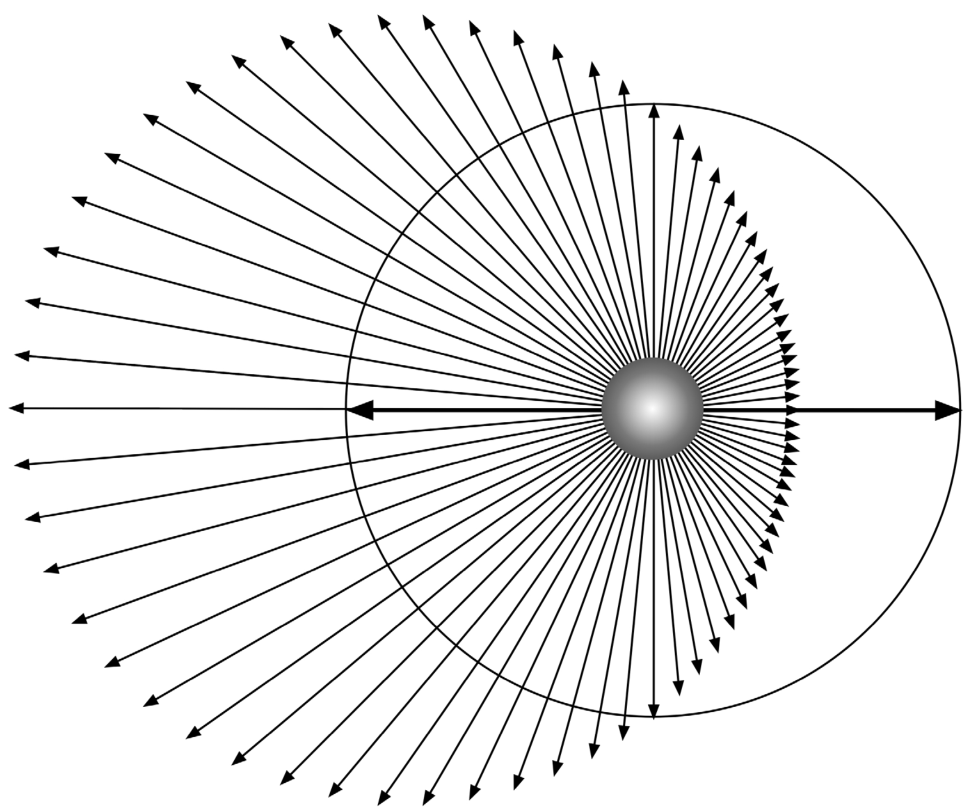

Figure 7

The result is an alternate spherical wave front whose apparent origin is offset from the actual origin by the previously established spatial offset ($-r\gamma\beta\cos\theta_{k'}$). The diameter of this sphere can be determined by summing the lengths for opposite directions with respect to both axes.

$$\varnothing_x = r_{\theta=0} + r_{\theta=\pi} = r\gamma(1-\beta) + r\gamma(1+\beta) = 2r\gamma$$

$$\varnothing_y = r_{\theta=\pi/2} + r_{\theta=3\pi/2} = r\gamma + r\gamma = 2r\gamma$$

It is evident that spherical electromagnetic expansion within an inertial system results in a field that dilates by ($r\gamma$) and is offset along the velocity vector of the inertial system by ($-r\gamma\beta\cos\theta_{k'}$). It is obvious that if the velocity approached (c), then the spherical offset would be maximized. This situation could occur with particle annihilation. Given this scenario, the following becomes readily apparent where the distances are scaled to match the resulting wavelength of the electromagnetic energy that would result from particle annihilation.

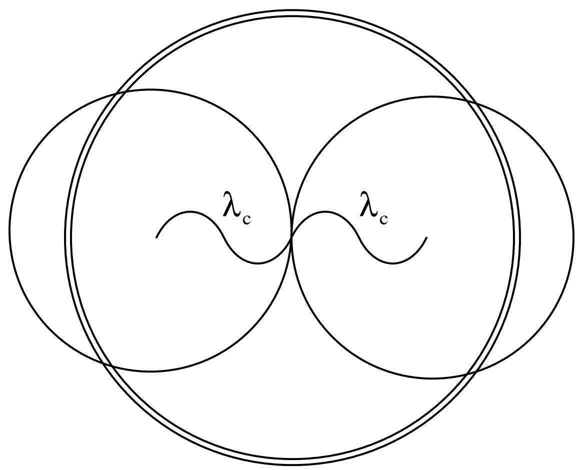

Figure 8

As illustrated above, two particles undergoing annihilation whose origins coincide as a natural result of this annihilation will actually have their corresponding fields of force completely offset such that these particles never truly occupy the same space. Particle annihilation is now seen to occur within a finite distance between the particles. Consider Coulomb's law, where the electric force (F) between two charges (Q, q) is directly proportional to the product of their charges and inversely proportional to the square of the distance (r) between their centers. Now we merely analyze this limit as the two charges approach one another.

$$\lim_{r\to 0}\left(K\frac{Qq}{r^2} \right) = \infty$$

Using this equation, one would calculate an infinite force associated with particle annihilation. Based upon spatial offset, we can readily see that particles that undergo annihilation maintain a positive distance with respect to their true origin and therefore do not experience infinite forces. Therefore, the classic infinity paradox of the field equations is hereby resolved.

§ 10. Vis Insita and the Relativistic Work Principle

With regards to the classical work function, there are currently two methods of derivation allowed.

$$\underline{dW = \mathbf{F}\ dr}$$

$$dW_i = M\mathbf{a}\ dr \qquad dW_\varepsilon = \left(\frac{d\mathbf{P}}{dt}\right) d\mathbf{r}$$

$$dW_i = M\left(\frac{d\mathbf{v}}{dt}\right) d\mathbf{r} \qquad dW_\varepsilon = \left(\frac{d\mathbf{r}}{dt}\right) d\mathbf{P}$$

$$dW_i = M\left(\frac{d\mathbf{r}}{dt}\right) d\mathbf{v} \qquad \boxed{dW_\varepsilon = \mathbf{v}\ d\mathbf{P}}$$

$$dW_i = M\mathbf{v}\ d\mathbf{v}$$

$$\boxed{dW_i = \mathbf{P}\ d\mathbf{v}}$$

Evaluating the integral for the classical mechanics case yields

$$\int \mathbf{P}\ d\mathbf{v} = \int \mathbf{v}\ d\mathbf{P} = \frac{1}{2}Mv^2$$

This is expected due to the classical definition of force with respect to the rate of change of momentum. Based upon the previously established equations of Inertial Relativity, evaluating the integral for the relativistic case yields

$$W_i = \int \mathbf{P}\ d\mathbf{v} = Mc^2\left(1 - \frac{1}{\gamma}\right)$$

$$W_\varepsilon = \int \mathbf{v}\ d\mathbf{P} = Mc^2\left(\gamma - 1\right)$$

It becomes evident that whereas the classical case yielded an identical result, the relativistic case reveals an inequality between these two expressions for work representing a unique relativistic property associated to each. It is evident that the electromagnetic work function (W_ε) is equal to relativistic kinetic energy as defined within SRT. This is the incident energy that induces motion within the inertial mass. The inertial work function (W_i) should be familiar as well.

$$W_i = Mc^2 - \frac{Mc^2}{\gamma}$$

This is none other than the reduction in the potential electromagnetic energy of the inertial mass as was found previously. This derivative is treated by SRT as a mathematical anomaly due to the incompatibility between classical mechanics and SRT. What we can clearly see is that it definitely has a relativistic function and cannot be ignored from a theoretical standpoint. During our previous evaluation of emission we have already established that the inertial energy is (Mc^2/γ). Therefore, all remaining energy must be resident within the DeBroglie kinetic. In order to validate this concept, we merely evaluate the known elements associated with an inertial mass.

$$E_k = \frac{Mc^2}{\gamma} + Mv^2\gamma$$

We also know that in accordance with SRT, the relativistic kinetic energy is additive to the rest energy in order to equal the relativistic energy of an inertial mass.

$$E_k = Mc^2 + Mc^2(\gamma - 1)$$

Therefore, we can relate these two equations and solve for the inertial work function (W_i).

$$Mc^2\left(1 - \frac{1}{\gamma}\right) = Mv^2\gamma - Mc^2(\gamma - 1)$$

Just as expected, the inertial work function (W_i) represents the mechanical energy input that, along with the electromagnetic energy input, comprises the DeBroglie kinetic.

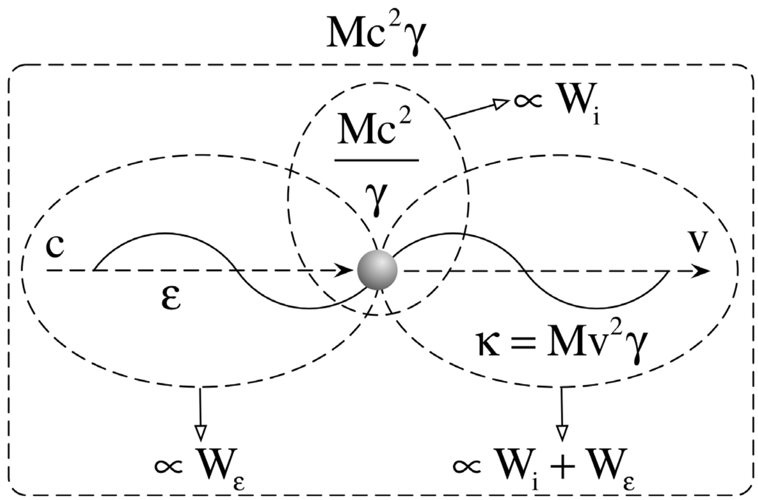

Figure 9

This relationship can be simplified as follows.

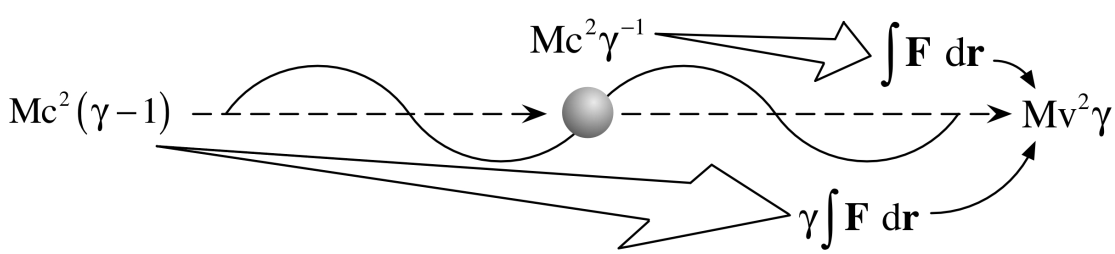

Figure 10

The implications of this relationship could not be any clearer. What we are viewing is an equal energy that only differs with respect to the relativistic transformation between the frames of reference. These energies are the same with respect to their frame of reference. Whenever a force (**F**) is imparted upon a mass (M) in order to achieve a uniform acceleration (**a**), the resulting work ($\int \mathbf{F}\, d\mathbf{r}$) done upon the mass is with respect to the inertial frame K’ that this mass is at rest with respect to. This is due to the fact that the mass (M/γ) with respect to the inertial frame is identical to the rest mass (M) with respect to the rest frame, where they only differ with respect to the relativistic transformation between the frames of reference. The simplest explanation is that in an inertial frame, the fundamental spatial separation increases by ($\mathbf{r}\gamma$) while the inertial mass reduces by (M/γ) maintaining a constant relationship between inertial mass and the medium that it resides within.

The fundamental relationship that is revealed is that for a given amount of work ($\int \mathbf{F}\, d\mathbf{r}$) done upon a mass with respect to the inertial frame, an equal amount of energy ($\gamma\int \mathbf{F}\, d\mathbf{r}$) is required with respect to the rest frame. Since this energy is resident as a vector electromagnetic wave, the resulting DeBroglie kinetic must become the sum of the input energy ($\gamma\int \mathbf{F}\, d\mathbf{r}$) and the corresponding work ($\int \mathbf{F}\, d\mathbf{r}$) done upon the mass. Whether we wish to state that work ($\int \mathbf{F}\, d\mathbf{r}$) or ($\gamma\int \mathbf{F}\, d\mathbf{r}$) was required to bring the mass into motion is merely a matter of perspective from a mathematical and theoretical standpoint.

The fact that the two relativistic work functions are equal with respect to their own frame of reference cannot be a coincidence. We know that ($\int \mathbf{F}\, d\mathbf{r}$) is a function of the mechanical force applied, whereas ($\gamma\int \mathbf{F}\, d\mathbf{r}$) is a function of the incident electromagnetic energy. We must immediately recognize that these two relativistic work functions represent the two known causes for changes in inertial states. The first cause for a change in inertial state is obviously mechanical in nature and is accompanied by a mechanical force causing such change as represented by incidental energy of ($\int \mathbf{F}\, d\mathbf{r}$). A mechanical change in an inertial state is known to be related to opposing fundamental fields associated with each mass, regardless of the illusion of macro physicality associated with each mass. The second cause for a change in an inertial state is known to be related to incident electromagnetic energy (photon) interacting with a mass as represented by incidental energy of (ΔMc^2).

In keeping with Newton’s methodology of the scientific method, the same natural effects must be assigned to the same causes. Therefore, the inertial effect represents a relationship in which the same effect will result regardless of the method of input of external energy. We now have a cohesive and symmetrical mathematical and theoretical description of how work is done upon a mass in relativistic terms.

§ 11. Wave-Particle Duality

Based upon the previous established geometric properties of an inertial mass, the following mathematical limits apply for each of these elements as their velocities approach (c).

$$\lim_{v\to c}\left(Mc^2\gamma\right)=\infty$$

$$\lim_{v\to c}\left(\frac{Mc^2}{\gamma}\right)=0$$

$$\lim_{v\to c}\left(Mv^2\gamma\right)=\infty$$

Therefore, as the energy of the DeBroglie kinetic increases, the energy of the inertial mass decreases in reciprocal fashion. The inertial property of the moving body is encompassed by the inertial energy. The electromagnetic property of the moving body is encompassed by the DeBroglie kinetic. These properties directly correlate to the classical wave-particle duality that is witnessed in nature. Based upon the previous analysis, we can confidently conclude the following. For the inertial component, we merely make a ratio of the inertial energy to the total energy as follows.

$$\Omega_i=\frac{Mc^2\gamma^{-1}}{Mc^2\gamma}=1-\frac{v^2}{c^2}$$

For the electromagnetic component, we merely make a ratio of the DeBroglie kinetic to the total energy as follows.

$$\Omega_\kappa=\frac{Mv^2\gamma}{Mc^2\gamma}=\frac{v^2}{c^2}$$

These relations, Ω_i and Ω_κ, satisfy the expected requirement that their sum must equal unity. In order to better visualize this mathematical summarization, we merely conduct the following analysis. For an object traveling at ½ the velocity of light, the following relationship ensues.

$$\Omega_i=\left(1-\frac{\left(c/2\right)^2}{c^2}\right)=\frac{3}{4}\propto 75\%$$

$$\Omega_\kappa=\frac{\left(c/2\right)^2}{c^2}=\frac{1}{4}\propto 25\%$$

For an object traveling at ½ the velocity of light, the object is comprised of 75% inertial (particle) and 25% electromagnetic (wave) properties. The closer an object's velocity approaches that of the velocity of light, the more electromagnetic and the less inertial it becomes. For an object traveling an indiscernible velocity below that of light, that object will look and behave almost identical to a pure electromagnetic wave, for that is essentially what this object has become due to such high velocity as given by Ω.