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The Doppler effect and the s-relativity

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ABSTRACT

The Doppler factor for the waves propagated in wave conducting medium is defined here for generalized arrangements. The s-relativistic Doppler factors for aligned and angled light propagation are derived. Relativistic and non-relativistic definitions of the derived Doppler factors are mutually transformable upon assigning specific values to the speed and angle parameters.

Keywords: Doppler effect, Doppler factor, S-relativistic Doppler factor, Wave conducting medium, Doppler neutral angles.

1. Definitions

The Doppler effect considered herein is presented as the kinematical phenomenon affecting the waves' length and caused by interaction of propagating waves with the moving waves emitter or receiver.

Let be a static wave conducting medium and two devices in it: one device is an emitter of waves, another device is a receiver of waves.

The wave emitter, when motionless relative to the wave conducting medium, generates waves of original wave length $\boldsymbol{\lambda_e}$, but when it moves relative to the wave conducting medium the length of the emitted waves becomes $\boldsymbol{\lambda_p}$.

The wave receiver, moving relative to the wave conducting medium, captures the oncoming waves of length $\boldsymbol{\lambda_p}$, measures them, and the result is registered as the wave length $\boldsymbol{\lambda_r}$.

The waves generated from the emitter propagate in the wave conducting medium at speed $\mathbf{u_w}$. The time interval of the waves repetition is a period, designated as $\mathbf{T_w}$. Characteristics of the waves propagated in the wave conducting medium are interrelated as:

$$\lambda_p=u_wT_w \qquad (1\text{-}1)$$

Movement of the wave emitting and receiving devices within the static wave conducting medium affects characteristics of the waves, particularly the wave length. Relationship of lengths of the originally generated, propagated and registered at the receiver waves can be expressed as:

For the moving wave emitter: $$\lambda_p=D_e\lambda_e \qquad (1\text{-}2)$$

For the moving wave receiver: $$\lambda_r=D_r\lambda_p \qquad (1\text{-}3)$$

Equations (1-2) and (1-3) express the Doppler effect for the wave emitter and the wave receiver moving relative to the wave conducting medium.

Factors **D** in equations (1-2) and (1-3) are designated as the Doppler factors that will be defined as:

For the emitter: $$D_e=\frac{\lambda_p}{\lambda_e} \qquad (1\text{-}4)$$

For the receiver: $$D_r=\frac{\lambda_r}{\lambda_p} \qquad (1\text{-}5)$$

Definition of the Doppler factor for the waves propagating between the wave receiver and wave emitter will be:

$$D_{er}=D_eD_r=\frac{\lambda_r}{\lambda_e} \qquad (1\text{-}6)$$

In equations (1-4), (1-5) the values of the waves length λ_p, λ_e and λ_r are positive, therefore value of the Doppler factors **D** are expected to be positive as well. The factor **D** appearing to be negative would signify existing of special conditions.

If the wave emitter generates electromagnetic waves (light), then, due to special fundamental property of light, the wave conducting medium becomes immaterial, and movement of the wave emitter cannot be referenced to the wave conducting medium. Therefore, the propagated light will be of the same wave length as the light originally generated: $\lambda_p = \lambda_e$; consequently the emitter's Doppler factor for light is $Dl_e = 1$. Movement of the wave receiver can be referenced to position of the light emitter and to path of the propagated light.
The Doppler factor **D*l*** for light will be defined as:

$$Dl_{er} = Dl_e \times Dl_r = \frac{\lambda_r}{\lambda_p} \qquad (1\text{-}7)$$

2. Doppler effect in the wave conducting medium

As it is stated above, movement and positioning of the wave emitter and receiver is affecting the Doppler effect, and in this section it will be analyzed at conditions, when $u_w \ll c$. In reality the waves are propagating in three-dimensional space, but the further analysis will be done within two-dimensional plane that can be extended to three-dimensional application when needed.

1) Consider an arrangement, where the wave emitting device is moving at speed v_e relative to static wave conducting medium.

The motionless emitter would generate waves of original length λ_e and period T_w. When the wave emitter moves, then, while the wave of original length λ_e is generated, the wave emitter will advance at speed v_e for distance $v_e T_w$ in the direction of the propagating wave. This distance will be subtracted from the original length of the wave, producing the wave of length λ_p, which will be propagating in the wave conducting medium in front of the wave emitter:

$$\lambda_p = \lambda_e - v_e T_w \qquad (2\text{-}1)$$

In equation (2-1) substitute T_w from (1-1), then after rearranging it will be:

$$\lambda_p = \lambda_e \left(1 - \frac{v_e}{u_w}\right) \qquad (2\text{-}2)$$

If the wave emitter generates circular waves, then at each radially selected direction at angle θ relative to path of the wave emitter the wave's length will be unique, because only projection of the emitter's speed v_e on the wave's selected direction will affect the Doppler's modification of the wave's length.

The length λ_p of the wave propagated in the wave conducting medium at angle θ relative to the path of the wave emitter's movement will be:

$$\lambda_p = \lambda_e \left(1 - \frac{v_e}{u_w} cos(\theta)\right) \qquad (2\text{-}2a)$$

Equation (2-2a) expresses the Doppler effect for the waves, propagated at angle **θ** relative to the path of the moving wave emitter. According to equation (1-4) the Doppler factor $\mathbf{D_{e\theta}}$ for the wave emitter is::

$$D_{e\theta} = \frac{\lambda_p}{\lambda_e} = 1 - \frac{v_e}{u_w} cos(\theta) \qquad (2\text{-}3)$$

As it follows from equation (2-3), the value if the Doppler factor $\mathbf{D_{e\theta}}$ depends on speed of the wave emitter and on the angle **θ**. The Doppler factor in front of the wave emitter at **θ=0** is $\mathbf{D_{e\theta}<1}$**,** and the Doppler factor behind the wave emitter at $\mathbf{\theta=180^o}$ is $\mathbf{D_{e\theta}>1}$. The angular boundary between these regions can be determined from equation (2-3) by assigning the Doppler factor $\mathbf{D_{e\theta}=1}$:

$$1 - \frac{v_e}{u_w} cos(\theta)=1$$

The solution for **θ** is: : $\theta = 90^o;\ \theta = 270^o$

So, the border between the expanding and contracting Doppler effect is always at right angles to the direction of the wave emitter's movement.

It can be a situation when $\mathbf{v_e>u_w}$. In this case the Doppler factor can become negative. Since by definition the Doppler factor cannot be negative, the absolute value should be taken in such case:

$$D_{e\theta} = \frac{\lambda_p}{\lambda_e} = \left|1 - \frac{v_e}{u_w} cos(\theta)\right| \qquad (2\text{-}3a)$$

It follows from equation (2-3a) that in the region in front of the wave emitter, when the Doppler factor appears negative the dependence of the Doppler factor on the speed of the wave emitter will be inversed: higher speed of the wave emitter produces higher Doppler factor.

There is an angular boundary line at some angle $\mathbf{\theta_b}$ relative to path of the wave emitter, separating regular and inverse (negative) regions of the Doppler factor. At these boundaries the Doppler factor will be $\mathbf{D_{e\theta}=0}$:

$$1 - \frac{v_e}{u_w} cos(\theta_b) = 0 \qquad (2\text{-}4)$$

Solution of equation (2-4) for $\mathbf{\theta_b}$ is:

$$\theta_b = \pm arccos \frac{u_w}{v_e} \qquad (2\text{-}5)$$

When $\mathbf{v_e=u_w}$, then the boundary angle $\boldsymbol{\theta_b = 0^o}$, which indicates that so-called "wave barrier" with the Doppler factor $\mathbf{D_{e\theta}=0}$ is formed in front of the wave emitter.

Since there are no reference frame associated with the moving wave emitter, the relativistic transformation procedures, particularly the dilation factor, cannot be applied.

2) Consider another arrangement, when waves of length $\mathbf{\lambda_p}$ propagate through the wave conducting medium at speed $\mathbf{u_w}$ toward the wave receiver, which moves relative to the wave conducting medium at speed $\mathbf{v_r}$ in the same direction as the oncoming waves.

While the wave of length $\mathbf{\lambda_p}$, enters the wave receiver's input, the receiver will be moving away from the wave at speed $\mathbf{v_r}$ to some length $\boldsymbol{l}$. The motions of the wave and the wave receiver will merge together at some time $\boldsymbol{t}$, which for the receiver will be $\boldsymbol{t} = \frac{\boldsymbol{l}}{\mathbf{v_r}}$, and for the wave it will be $\boldsymbol{t} = \frac{\boldsymbol{l}+\mathbf{\lambda_p}}{\mathbf{u_w}}$. Merging these two equations gives:

$$\frac{l}{v_r} = \frac{l+\lambda_p}{u_w} \qquad (2\text{-}6)$$

From (2-6) define ***l***:

$$l = \lambda_p \frac{v_r}{u_w - v_r} \qquad (2\text{-}7)$$

The wave length registered by the receiver will be:

$$\lambda_r = \lambda_p + l \qquad (2\text{-}8)$$

Substituting ***l*** from (2-7) in (2-8) produces the wave length registered by the receiver:

$$\lambda_r = \lambda_p + \lambda_p \frac{v_r}{u_w - v_r} = \lambda_p \frac{1}{1 - \frac{v_r}{u_w}} \qquad (2\text{-}9)$$

If the wave receiver is moving at angle $\boldsymbol{\alpha}$ relative to the path of oncoming waves, then only projection of the receiver's speed $\mathbf{v_r}$ on the wave's path will affect modification of the registered wave length. Then the registered wave length $\boldsymbol{\lambda_r}$, affected by the angular movement of the wave receiver, will be:

$$\lambda_r = \lambda_p \frac{1}{1 - \frac{v_r}{u_w} cos(\alpha)} \qquad (2\text{-}9a)$$

Equation (2-9a) expresses the Doppler effect caused by moving of the wave receiver relative to the wave conducting medium at angle $\boldsymbol{\alpha}$ relative to path of the oncoming waves. The Doppler factor $\mathbf{D_r}$ for the moving wave receiver will be:

$$D_r = \frac{\lambda_r}{\lambda_p} = \frac{1}{1 - \frac{v_r}{u_w} cos(\alpha)} \qquad (2\text{-}10)$$

It can be a situation when $\mathbf{v_r} > \mathbf{u_w}$. This condition can cause negative wave receiver's Doppler factor. It means that the wave receiver moves faster than waves, and the waves cannot reach it. In such case angle $\boldsymbol{\alpha_b}$ can be determined, defining angular region for the moving wave receiver, where the Doppler factor is positive $\mathbf{D_r > 0}$:

$$1 - \frac{v_r}{u_w} cos(\alpha_b) > 0 \qquad (2\text{-}11)$$

Solution of equation (2-11) for $\boldsymbol{\alpha_b}$, symmetrical to the path of the oncoming waves, is:

$$\alpha_b > \left| \pm \arccos \frac{u_w}{v_r} \right| \qquad (2\text{-}12)$$

Since there are no reference frame associated with the moving wave receiver, the relativistic transformation procedures, particularly the dilation factor, cannot be applied.

3) Consider that two aforementioned arrangements for the wave emitter and the receiver are combined.

The wave emitter moves at speed $\mathbf{v_e}$ relative to the wave conducting medium and generates waves of original wave length $\boldsymbol{\lambda_e}$, propagating circularly at speed $\mathbf{u_w}$ as the waves of length $\boldsymbol{\lambda_p}$.

The wave receiver is positioned at the selected radial direction of the propagated wave at angle $\boldsymbol{\theta}$ relative to path of the wave emitter, and moves with speed $\mathbf{v_r}$ at angle $\boldsymbol{\alpha}$ relative to the selected radial direction.

In this case the waves approaching the receiver will be defined by equation (2-3a). According to equation

(2-9a) the wave receiver will register incoming waves as:

$$\lambda_r = \lambda_e \left|1 - \frac{v_e}{u_w} cos(\theta)\right| \left(\frac{1}{1-\frac{v_r}{u_w}\cos(\alpha)}\right) \qquad (2\text{-}13)$$

The combined Doppler factor $\mathbf{D_{er}}$ for the moving emitter and receiver is:

$$D_{er} = \frac{\lambda_r}{\lambda_e} = \frac{\left|1-\frac{v_e}{u_w} cos(\theta)\right|}{1-\frac{v_r}{u_w}\cos(\alpha)} \qquad (2\text{-}14)$$

Negative value of $\mathbf{D_{er}}$ in equation (2-14) will mean that waves will not be able to reach the wave receiver since speed of the wave receiver is higher than speed of the waves.

3. S-relativistic linear Doppler effect

There are two issues related to the derived above Doppler factors:

1. The derived Doppler factors do not have provisions for containing speeds of the wave emitter and the receiver within limits of velocity of light.
2. If the emitter generates electromagnetic waves (light), then the wave conducting medium becomes immaterial, and movement of the devices cannot to be referenced to it.

In order to address these issues the Doppler factor will be derived based on the s-relativity.

Consider the arrangement, where a reference frame **E** containing static wave conducting medium is firmly attached to the wave emitter, and a reference frame **R** containing its static wave conducting medium is firmly attached to the wave receiver. The wave conducting media in the reference frames **E** and **R** have identical physical properties, but kinematically are independent. The reference frames are moving relative to each other at speed **v**, making the wave emitter and the receiver moving apart on common path. The wave emitter generates waves of length $\boldsymbol{\lambda_e}$ propagating in reference frame **E** toward the receiver at speed $\mathbf{u_w}$ relative to reference frame **E**.
By applying direct form of the s-transformation [3, section 3] the wave length $\boldsymbol{\lambda_e}$ can be transformed as moving segments of length from the reference frame **E** to the reference frame **R**, forming the wave length $\boldsymbol{\lambda_r}$ in the reference frame **R**:

$$\lambda_e = \lambda_r \frac{1-\frac{v^2}{c^2}}{1+\frac{v}{u_w}} \qquad (3\text{-}1)$$

From equation (3-1) the s-relativistic Doppler factor for the moving apart emitter and receiver will be defined as:

$$D_s = \frac{\lambda_r}{\lambda_e} = \frac{1+\frac{v}{u_w}}{1-\frac{v^2}{c^2}} \qquad (3\text{-}2)$$

If speed of the reference frames relative to each other is much lower than velocity of light (**v<<c**), then the relativistic equation (3-2) will approximate equation (2-3) for the Doppler factor at $\boldsymbol{\theta=180^o}$:

$$D_{es} = \frac{\lambda_r}{\lambda_e} = 1 + \frac{v}{u_w} \qquad (3\text{-}3)$$

If the wave emitter generates electromagnetic waves (light), then $\mathbf{u_w=c}$ and equation (3-2) for the Doppler factor for light will we:

$$\mathrm{D}l_s = \frac{\lambda_r}{\lambda_e} = \frac{1}{1-\frac{v}{c}} \qquad (3\text{-}4)$$

Relativistic equation (3-4) is identical to equation (2-10) for the wave receiver moving relative to the wave conducting media at angle **α=0°**. Therefore it can be concluded, that when the wave emitter and receiver are moving on common linear path the definition of the Doppler factor will be the same either with "classical" or with "relativistic" approach.

According to equation (3-4), variations of the light Doppler factor depending on relative speed of the emitter and receiver will be:

$$\text{when} \quad -1 < \frac{v}{c} < 1 \qquad (3\text{-}5)$$

$$\text{then} \quad \frac{1}{2} < \mathrm{D}_l < \infty$$

Conditional equations (3-5) assert that the Doppler factor for light, when the light emitter and receiver move on common path, cannot be less than $\frac{1}{2}$, but can be indefinitely great.

It is essential to emphasize, that for the relativistic transformation the wave conducting medium within each reference frame must be independent. If the reference frames share the same wave conducting medium, then speeds of each reference frame relative to common conducting medium must be specified. This action will brake symmetry of the reference frames relationship, which is violation of principal of the relativity, therefore this principle cannot be applied. This does not relate to the relativistic transformation of propagation of light where presence of the wave conducting medium is immaterial.

4. S-relativistic angular Doppler effect for light

Consider arrangement of two reference frames: reference frame **E** and reference frame **R** are moving apart along the common path at speed **v** relative to each other. A light emitting device is placed in the reference frame **E** and generates a light beam of wave length $\boldsymbol{\lambda_e}$ at angle $\boldsymbol{\alpha}$ relative to the direction of the reference frames movement. A light receiver is placed in the reference frame **R** and is positioned to capture light from the emitter and register the light's wave length as $\boldsymbol{\lambda_r}$.

According to the angular s-transformation [4] the segments of length $\boldsymbol{\Delta L_e}$ along the path of light in the reference frame **E** can be transformed to the reference frame **R** as $\boldsymbol{\Delta L_r}$:

$$\Delta L_e = \Delta L_r \left[\sqrt{1 - \frac{v^2}{c_2} sin^2(\alpha)} - \frac{v}{c} cos(\alpha) \right] \qquad (4\text{-}1)$$

The segments of lengths propagated in the reference frames correspond to the light wave length $\boldsymbol{\lambda}$ as $\boldsymbol{\Delta L_r \to \lambda_r}$: $\boldsymbol{\Delta L_e \to \lambda_e}$. Then the relativistic angular Doppler factor for light will be defined as:

$$\mathrm{D}_\alpha = \frac{\lambda_r}{\lambda_e} = \frac{1}{\sqrt{1 - \frac{v^2}{c_2} sin^2(\alpha)} - \frac{v}{c} cos(\alpha)} \qquad (4\text{-}2)$$

If the wave receiver is positioned on the emitter's path, then angle **$\alpha=0^o$**, and equation (4-2) will be:

$$D_{0^o} = \frac{\lambda_r}{\lambda_e} = \frac{1}{1-\frac{v}{c}} \qquad (4\text{-}3)$$

Equation (4-3) is identical to equation (3-4) expressing the receiver's Doppler effect for the case, when the light receiver and the light emitter are moving apart and aligned on the same path.

If the light receiver is positioned transverse to the emitter's path (**$\alpha=90^o$; 270^o**), then the Doppler factor according to equation (4-2) will be:

$$D_{90^o} = \frac{\lambda_r}{\lambda_e} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (4\text{-}4)$$

As it follows from (4-4), the Doppler factor **$D_{90}{}^o$** is always greater than **1** (**$D_{90}{}^o>1$**) and symmetrical around direction of movement of the light emitter.

At relatively slow motion of the light emitter and receiver, when **$v^2<<c^2$**, equation (4-2) will be transformed to equation:

$$D_{r\alpha} = \frac{\lambda_r}{\lambda_e} = \frac{1}{1-\frac{v}{c}cos(\alpha)} \qquad (4\text{-}5)$$

Equation (4-5) is identical to equation (2-10), and expresses the Doppler factor caused by moving of the wave receiver relative to the wave conducting medium at angle **α** relative to path of the oncoming waves.

According to equation (4-2) there are special neutral angles **α_o**, where the Doppler effect will not be present. These angles **α_o** separate regions of "blue shift" and "red shift" of the s-relativistic angular Doppler effect. In directions at these angles the Doppler factor will be **$D_\alpha=1$**:

$$D_\alpha = \frac{\lambda_r}{\lambda_e} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}sin^2(\alpha_o)}-\frac{v}{c}cos(\alpha_o)} = 1 \qquad (4\text{-}6)$$

Rearranging (4-6):

$$\sqrt{1-\frac{v^2}{c^2}sin^2(\alpha_o)} = 1+\frac{v}{c}cos(\alpha_o) \qquad (4\text{-}6a)$$

Square equation (4-6a):

$$1-\frac{v^2}{c^2}sin^2(\alpha_o) = 1+2\frac{v}{c}cos(\alpha_o)+\frac{v^2}{c^2}cos^2(\alpha_o) \qquad (4\text{-}7)$$

Rearrange equation (4-7):

$$0 = 2\frac{v}{c}cos(\alpha_o)+\frac{v^2}{c^2} \qquad (4\text{-}7a)$$

Solution for equation (4-7a) is:

$$\alpha_0 = \pm\left(arccos\frac{v}{2c}+180^o\right) \qquad (4\text{-}8)$$

Circular chart of distribution of the s-relativistic angular Doppler factor **D_α** is shown on **Fig.1**. The non-relativistic angular Doppler factor **$D_{r\alpha}$** is inserted in the same chart for comparison. The Doppler factors are calculated for the relative speed of the reference frames $\frac{v}{c}$ = **0.6**. Table 1 contains calculated values for the angular Doppler factors with **15^o** steps.
The neutral angles **α_o** for the selected relative speed of the reference frames will be:

$$\alpha_0 = \pm\left(arccos\frac{0.6}{2} + 180^{o}\right) = \pm 252.5^{o}$$

$$\alpha_{01} = 252.5^{o}; \qquad \alpha_{02} = -252.5^{o} + 360^{o} = 107.5^{o}$$

TABLE 1

α	D_α	$D_{r\alpha}$	α	D_α	$D_{r\alpha}$
0	2.5	2.5	180	0.625	0.625
15	2.449	2.378	195	0.638	0.633
30	2.302	2.082	210	0.679	0.658
45	2.078	1.737	225	0.752	0.702
60	1.804	1.429	240	0.866	0.769
75	1.516	1.184	255	1.031	0.866
90	1.25	1	270	1.25	1
105	1.031	0.866	285	1.516	1.184
120	0.866	0.769	300	1.804	1.429
135	0.752	0.702	315	2.078	1.737
150	0.679	0.658	330	2.302	2.082
165	.638	0.633	345	2.449	2.378

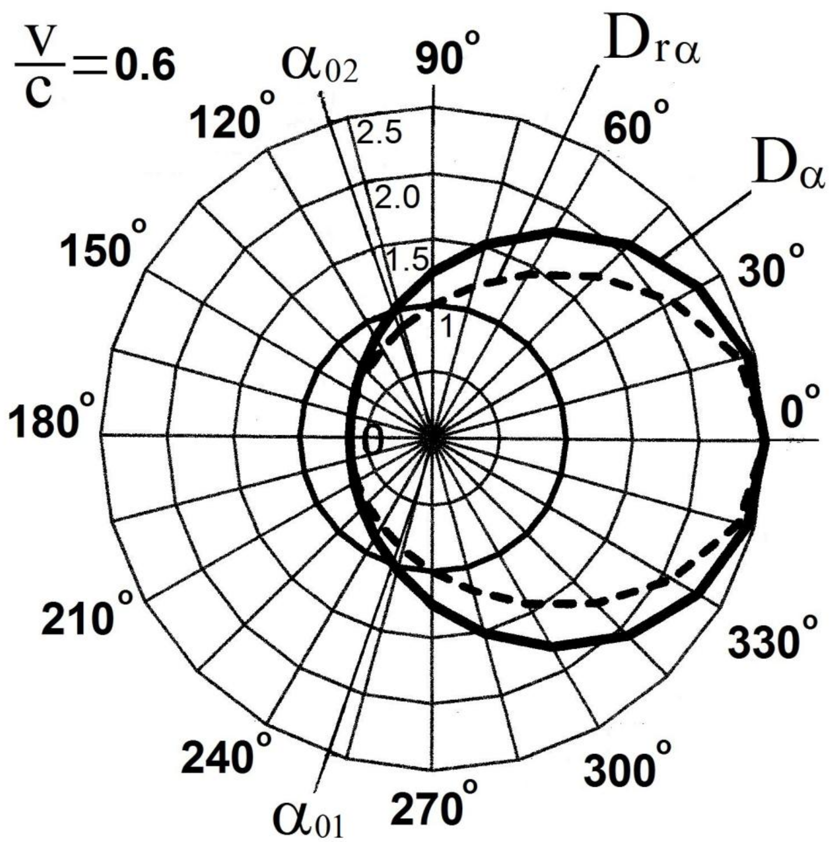

Fig.1

The angular distribution of the s-relativistic angular Doppler factor depicted on **Fig.1** shows circular pattern positioned symmetrically around linear path of the moving wave emitter. As it appears on the diagram of **Fig.1** the relativistic and the non-relativistic Doppler factors coincide at angles **0^o** and **180^o**, where the wave emitter and the receiver are moving on the same linear path. At angles **90^o** and **270^o** the non-relativistic Doppler factor **$D_{r\alpha}=1$**, indicating absence of the Doppler shift, but the relativistic Doppler factor **$D_\alpha=1.25$**, in conformity with equation (4-4). Neutral angles **α_0** mark boundaries between the "red shift" and "blue shift" for the relativistic Doppler effect. For non-relativistic Doppler effect these boundaries are always at **90^o** and **270^o**.

5. Summation

1. The Doppler effect is the kinematical phenomenon affecting the waves length, and is caused by interaction of the propagating waves with moving waves emitter or waves receiver.

2. The Doppler effect is characterized by the Doppler factor **D**, which is defined by relationship of:
   the wave length λ_e, originally generated by the wave emitter;
   the wave length λ_p, propagated in the wave conducting medium;
   the wave length λ_r, captured and measured by the wave receiver.

The Doppler factor **D** for the wave emitter and wave receiver is defined as:

For the wave emitter: $$D_e = \frac{\lambda_p}{\lambda_e}$$

For the wave receiver: $$D_r = \frac{\lambda_r}{\lambda_p}$$

For the wave emitter and the wave receiver combined: $$D_{er}=D_eD_r=\frac{\lambda_r}{\lambda_e}$$

3. The Doppler factor **D_{er}** for the waves propagated in the wave conducting medium at speed **u_w** is defined:
   by speed of the wave emitter **v_e** relative to the wave conducting medium,
   by speed of the wave receiver **v_r** relative to the wave conducting medium,
   by the devices' orientating angles:
   θ - angle at which the propagating waves are moving relative to the path of the wave emitter;
   α - angle at which the wave receiver moves relative to the path of the oncoming waves:

$$D_{er} = \frac{\lambda_r}{\lambda_e} = \frac{1-\frac{v_e}{u_w}\cos(\theta)}{1-\frac{v_r}{u_w}\cos(\alpha)}$$

4. The s-relativistic Doppler factor for the waves propagating in the wave conducting medium at speed **u_w**, when the wave emitter and receiver are aligned on common path and moving apart at speed **v** is:

$$D_s = \frac{\lambda_r}{\lambda_e} = \frac{1+\frac{v}{u_w}}{1-\frac{v^2}{c^2}}$$

For relatively slow motion of the wave emitter and receiver **$v \ll c$**, then the Doppler factor is:

$$D_{es} = \frac{\lambda_r}{\lambda_e} = 1 + \frac{v}{u_w}$$

For propagating of light the speed of waves is $\mathbf{u_w=c}$, then the Doppler factor is:

$$Dl_s = \frac{\lambda_r}{\lambda_e} = \frac{1}{1-\frac{v}{c}}$$

5. If the light emitter and light receiver are placed in different reference frames moving at relative speed **v**, and the light emitter generates a light beam at angle $\boldsymbol{\alpha}$ relative to the path of the reference frames movement, then the angular s-relativistic Doppler factor registered by the light receiver placed in the direction of angle $\boldsymbol{\alpha}$ will be:

$$Dl_\alpha = \frac{\lambda_r}{\lambda_e} = \frac{1}{\sqrt{1-\frac{v^2}{c_2}sin^2(\alpha)}-\frac{v}{c}cos(\alpha)}$$

At transverse position of the light receiver $\boldsymbol{\alpha=90^o}$ relative to the path of the light emitter movement the angular s-relativistic Doppler factor is:

$$Dl_{90^o} = \frac{\lambda_r}{\lambda_e} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

At position of the light receiver $\boldsymbol{\alpha=0^o}$ relative to the path of the light emitter movement, when the light receiver and emitter are aligned on the same path. the angular s-relativistic Doppler factor is:

$$Dl_{0^o} = \frac{\lambda_r}{\lambda_e} = \frac{1}{1-\frac{v}{c}}$$

6. The neutral angles $\boldsymbol{\alpha_0}$, separating regions of "red shift" and "blue shift" of the s-relativistic angular light Doppler effect are:

$$\alpha_0 = \pm\left(arccos\frac{v}{2c} + 180^o\right)$$

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