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THEORY OF THE S-RELATIVITY

Solomon Shapiro

Contents

ABSTRACT

The relativistic concepts, which were previously presented in the research papers [2,3], are crystallized and amalgamated in this paper in the theory of s-relativity, comprising relativistic kinematics and dynamics. The mathematical formulation of the proposed theory of the s-relativity differs from that of the Lorentz transformation and the Einstein's special relativity.

The presented theory is a mathematical structure strictly based on the mathematically formulated axiomatic statements. The theory determines equations of spatial and timing relationship between uniformly moving inertial systems and objects, and formulates the dynamic laws of physics with consideration of the effects of extremely high speed of movement.

Keywords: Inertial systems; Directing equations; Direct transformation factor; Relativistic momentum; Relativistic Energy; Relativistic Force; Relativistic acceleration.

Three-dimensional reference frame **XYZ** is associated with the analyzed inertial systems. For simplification the systems analysis will be performed in two-dimensional **XY** coordinate plane, and **Z** coordinate can be optionally linked to the results. Analysis of the systems and objects movements will be oriented to the **x**-coordinate, but all operations and results are equally applicable to **Y** and **Z** coordinates by straight substitution of **x** designators by the **y** or **z** coordinates, respectively.

S-relativistic kinematics

1. S-transformation equations for the systems carrying propagation of light

Consider two inertial systems (Fig.1): system **K**, carrying space-time coordinates (**X, Y, Z, t**) originated at zero point **O**, and system **K'**, carrying coordinates (**X', Y', Z', t'**) originated at point **O'**. Both systems are arranged in such a way that abscissa **X'** of system **K'** coincides with abscissa **X** of system **K** (the **y** offset of **X'** from **X** on Fig.1 is shown for illustrative purpose). It's been conditionally assumed that system **K** is stationary and system **K'** is moving with velocity $\mathbf{v_x}$ along positive direction of axis **X**. The moment when the origin **O'** coincides with the origin **O** is considered as time zero point ($\mathbf{t_x=t'_x=0}$), and at this moment a light pulse will be generated in system **K'** from the origin **O'** along positive direction of axis **X'**. Upon elapsing of local time $\mathbf{t'_x}$ the light pulse will reach point **B'** in moving system **K'**, traveling distance **x'**, and the same pulse, viewed from stationary system **K**, will reach point **B**, coinciding with **B'**, traveling distance **x**. At the same period of time system **K'** will advance along axis **X** for distance $\mathbf{v_x t}$.

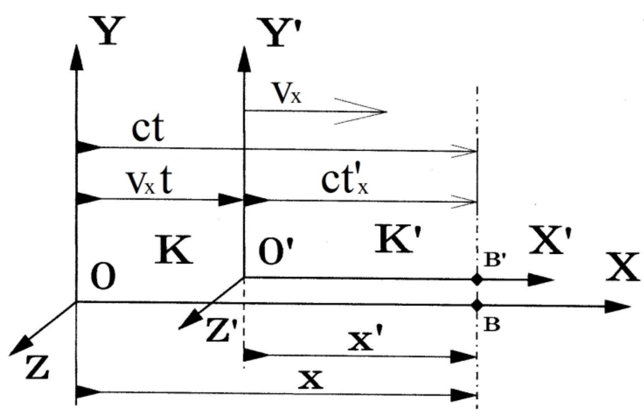

Fig.1

As it is follows from Fig. 1:

$$x'=x-v_x t_x \qquad (1\text{-}1)$$

The described system's kinematic state (1-1) is distinguished by the fact that the object propagating in system **K'** is a light pulse which is characterized with special unique property of its velocity. The transformation of spatial coordinates and the timing of the light pulse in the moving system K' will be defined in accordance with the formulated directing axiomatic statements:

1. Velocity of light measured in any inertial system is always of the same value, regardless of position in the space or relative movement of the origin of light.
2. Intervals of time in any inertial system can never be negative.

The axiom 1 can be expressed in form of the directing axiomatic equations (see Fig.1):

$$x'=ct'_x \qquad (1\text{-}2)$$

$$x=ct \qquad (1\text{-}3)$$

In equation (1-3) time **t** is not indexed to **x** coordinate because system **K** is assigned as the stationary system, and in stationary system time **t** is isotropic in all directions, therefore it is not to be referenced to any coordinate. Timing in the stationary reference frame is always $\mathbf{t_x=t_y=t_z=t}$.

Then equation (1-1) will be:

$$x'=x-v_x t \qquad (1\text{-}4)$$

In equation (1-4) substitute: **x'** from (1-2), **x** from (1-3) and **t** as $\mathbf{\frac{x}{c}}$ from (1-3):

$$ct'_x=ct-v_x\frac{x}{c} \qquad (1\text{-}5)$$

Determine t'_x from (1-5):

$$t'_x=t-\frac{v_x x}{c^2} \qquad (1\text{-}6)$$

Equations (1-4) and (1-6) constitute the x-oriented transformation equations for inertial systems carrying propagation of light, and this type of transformation will be further referred as sl-transformation. Equations (1-2) and (1-3) are the directing equations for the sl-transformation.

Ultimately the x-oriented sl-transformation equations and the directing equations are:

$$x'=x-v_x t \qquad (1\text{-}7)$$

$$t'_x=t-\frac{v_x x}{c^2}$$

$$x'=ct'_x$$

$$x=ct$$

Inverting the moving and stationary systems in the sl-transformation can be achieved by reversing direction of velocity **v** (**v**→**-v**) in the transformation equations (1-7):

$$x = x' + vt'_x \qquad (1\text{-}7a)$$
$$t = t'_x + \frac{vx'}{c^2}$$

Transformation equations (1-7) are functional forms of equations, representing the spatial and timing transformation procedure as a function of combination of the spatial an timing parameters. The transformation procedure can be expressed in form of direct transformation of spatial and timing coordinates.

The direct transformation can be determined from the functional sl-transformation equations.
From equation (1-3): substitute **t** as $\mathbf{t=\frac{x}{c}}$ in the **x'** equation of (1-7), and substitute **x** as **x=ct** into the **t'** equation of (1-7):

$$x' = x - v_x\frac{\mathbf{x}}{\mathbf{c}} = x\left(1 - \frac{v_x}{c}\right) \qquad (1\text{-}8)$$
$$t'_x = t - \frac{v_x}{c^2}ct = t\left(1 - \frac{v_x}{c}\right) \qquad (1\text{-}9)$$

Expressions (1-8) and (1-9) are the **x**-oriented sl-transformation equations in the direct coordinates transformation form.

For further discussions the analyzed velocities will be expressed in form of the normalized speed **N** which represents speed as fraction of velocity of light: $\mathbf{N=\frac{v}{c}}$. Then equations (1-8) and (1-9)will be:

$$x' = x(1 - N_x) \qquad (1\text{-}8a)$$
$$t'_x = t(1 - N_x) \qquad (1\text{-}9a)$$

In (1-8a) and (1-9a) the equations for **x'** and t'_x have identical transformation factors $\mathbf{(1\text{-}N_x)}$, so in the case of light propagation they can be combined into one unified expression, which can be referred as the direct transformation factor $\boldsymbol{\eta_x}$ related to axis **X:**

$$\eta_x = \frac{x'}{x} = \frac{t'_x}{t} = (1 - N_x) \qquad (1\text{-}10)$$

Equation (1-10) express linear and time dilation in x'-direction of moving system, as it appears to the stationary system. As it follows from (1-10) the range of variation of $\boldsymbol{\eta_x}$, depending on $\mathbf{N_x}$, is:

when $\quad -1 \leq N_x \leq 1 \qquad (1\text{-}11)$

then $\quad 2 \geq \eta_x \geq 0$

According to (1-11) the linear and time dilation in the sl-transformation can be either contracted or stretched, depending on relative direction of the moving system and the light pulse: coinciding or opposite. The linear and time dilation can never be negative or exceed twofold. This limitations are imposed by fundamental constancy of velocity of light.

For the **Y** and **Z** directions of movement the equations for the direct transformation factors will be similar:

$$\eta_y = \frac{y'}{y} = \frac{t'_y}{t} = \left(1 - N_y\right) \qquad (1\text{-}10a)$$
$$\eta_z = \frac{z'}{z} = \frac{t'_z}{t} = (1 - N_z) \qquad (1\text{-}10b)$$

2. Combining (adding) velocities

If a moving system is traveling at speed $\mathbf{v_x}$ in **X** direction relative to stationary reference frame, and an object is traveling in the moving system at speed $\mathbf{u_x}$ relative to **X'** direction**,** then the speed $\mathbf{w_x}$ of the traveling object relative to X direction of the stationary reference frame can be determined by differentiating of equations (1-7a):

$$dx=dx'+v_x dt'$$
$$dt=dt'+\frac{v_x}{c^2}dx'$$

The derivative $\frac{dx}{dt}$ will be:

$$\frac{dx}{dt}=\frac{\frac{dx'}{dt'}+v_x}{1+\frac{v_x\frac{dx'}{dt'}}{c^2}} \qquad (2\text{-}1)$$

Note that $\frac{\mathbf{dx}}{\mathbf{dt}}=\mathbf{w_x}$ and $\frac{\mathbf{dx'}}{\mathbf{dt'}}=\mathbf{u_x}$. Then expression for combining (adding) velocities relative to **X** direction, as it is judged from the stationary reference frame, will be:

$$w_x=\frac{u_x+v_x}{1+\frac{u_x v_x}{c^2}} \qquad (2\text{-}2)$$

Formula (2-2) for combined x-directed velocities can be written in normalized form as follows:

$$N_{wx}=\frac{N_{ux}+N_{vx}}{1+N_{ux}N_{vx}} \qquad (2\text{-}3)$$

3. Generalized s-transformation for arbitrary speeds

Previous discussion was related to the inertial systems, where the moving system carried propagation of light. The unique feature of such arrangement is that the speed of the object traveling in the moving system is fundamentally invariable and always equal **c**.

If the object in the moving system travels at arbitrary speed, then the directing equations have to contain this speed and take into consideration special property of velocity of light to comply with the primary axiom statements.

In this case consider that in the moving system a material object **j'** travels relative to axis **X'** at speed $\mathbf{u_x}$ (Fig.2)**.** Speed $\mathbf{u_x}$ relative to axis **X** of the stationary system will appear as combined speed $\mathbf{u_x}$ and $\mathbf{v_x}$. According to formula (2-2) such combined speed will be $\mathbf{w_x}=\frac{\mathbf{u_x+v_x}}{\mathbf{1+\frac{u_x v_x}{c^2}}}$. Thus the coordinates **x** and **x'** will be defined as:

$$x'=t'_x u_x \qquad (3\text{-}1)$$
$$x=t\frac{u_x+v_x}{1+\frac{u_x v_x}{c^2}} \qquad (3\text{-}2)$$

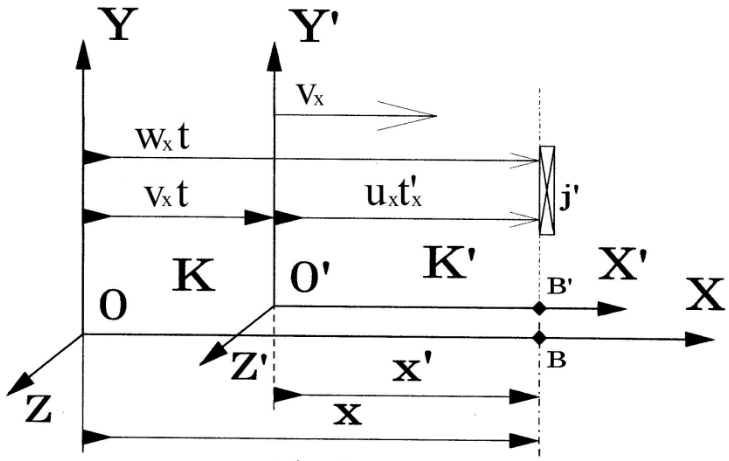

Fig.2

Equations (3-1) and (3-2) are the directing equations for the generalized case of the s-transformation.

Returning to Fig. 2, distance **x**' traveled by the object in system K' will be:

$$x' = x - v_x t \qquad (3\text{-}3)$$

Substitute **x** and **x'** from (3-1) and (3-2) into (3-3):

$$u_x t'_x = t\frac{u_x + v_x}{1 + \frac{u_x v_x}{c^2}} - v_x t \qquad (3\text{-}4)$$

Expanding and rearranging equation (3-4) gives direct timing transformation equation:

$$t'_x = t\frac{1 - \frac{v_x^2}{c^2}}{1 + \frac{u_x v_x}{c^2}} \qquad (3\text{-}5)$$

For determining the direct transformation equation for X' coordinate rewrite equation (3-3):

$$x' = x\left(1 - v_x \frac{t}{x}\right) \qquad (3\text{-}6)$$

Substitute **x** in brackets of equation (3-6) from (3-2):

$$x' = x\left(1 - v_x \frac{1 + \frac{u_x v_x}{c^2}}{u_x + v_x}\right) \qquad (3\text{-}6a)$$

After expanding and rearranging of (3-6a) it will be:

$$x' = x\frac{1 - \frac{v_x^2}{c^2}}{1 + \frac{v_x}{u_x}} \qquad (3\text{-}7)$$

Equations (3-7) and (3-5) constitute the s-transformation in direct transformation form, where the moving system, traveling at speed $\mathbf{v_x}$, carries an object traveling at speed $\mathbf{u_x}$:

$$x' = x\frac{1 - \frac{v_x^2}{c^2}}{1 + \frac{v_x}{u_x}}$$

$$t'_x = t\frac{1 - \frac{v_x^2}{c^2}}{1 + \frac{u_x v_x}{c^2}} \qquad (3\text{-}8)$$

$$x = t\frac{u_x + v_x}{1 + \frac{u_x v_x}{c^2}}$$ - directing equation

Equations (3-8) of the s-transformation in normalized form will be:

$$x' = x\frac{1-N_{vx}^2}{1+\frac{N_{vx}}{N_{ux}}} \qquad (3\text{-}8a)$$

$$t'_x = t\frac{1-N_{vx}^2}{1+N_{vx}N_{ux}}$$

$$x = t\frac{N_{ux}+N_{vx}}{1+N_{ux}N_{vx}} \text{ - directing equation}$$

The functional form of the timing equation can be defined by manipulation of the timing transformation equation (3-5):

$$t'_x = t\frac{1-\frac{v_x^2}{c^2}}{1+\frac{u_x v_x}{c^2}} = t - \left(t - t\frac{c^2-v_x^2}{c^2+u_x v_x}\right)$$

Expanding and rearranging of the above equation gives:

$$t'_x = t - \frac{c^2 t + u_x v_x t - c^2 t + v_x^2 t}{c^2+u_x v_x} = t - \frac{v_x}{c^2}\left(t\frac{u_x+v_x}{1+\frac{u_x v_x}{c^2}}\right)$$

The term in brackets in the above equation is the **x** directing equation (3-2). After substituting the directing equation in brackets as **x** the above expression for **t_x'** will be:

$$t'_x = t - \frac{x v_x}{c^2} \qquad (3\text{-}9)$$

Equations (3-3) and (3-9) constitute functional form of the s-transformation equations. These equations are identical to equations of the sl-transformation. The difference between the functional form of the sl-transformation and s-transformation is in structure of the directing equation.

The generalized s-transformation in functional form will be:

$$x' = x - v_x t$$

$$t'_x = t - \frac{x v_x}{c^2} \qquad (3\text{-}10)$$

$$x = t\frac{u_x+v_x}{1+\frac{u_x v_x}{c^2}} \quad \text{– directing equation}$$

The following is review of the s-transformation equations (3-8), exercised at particular values of **u_x** and **v_x**, characterizing distinctive ranges of applicable velocities:

a) v_x=c (moving system travels at speed of light)

$x'=0$
$t'_x=0$
$x=tc$

b) v_x=0 (moving system is motionless)

$x'=x$
$t'_x=t$
$x=tu_x$

c) v_x<<c; u_x<<c (moving system and the object traveling insight the moving system move very slow in comparison to speed of light)

$$x'=x\frac{u_x}{u_x+v_x}$$
$$t_x'=t$$
$$x=t(u_x+v_x)$$

d) u_x=c (moving system carries propagation of light pulse):

$$x'=x\left(1-\frac{v_x}{c}\right)$$
$$t_x'=t\left(1-\frac{v_x}{c}\right)$$
$$x=tc$$

e) u_x=0 (no **X'** directed moving activity in the moving system):

$$x'=0$$
$$t_x'=t\left(1-\frac{v_x^2}{c^2}\right)$$
$$x=tv_x$$

Here are evaluations of the above review:

a) If, hypothetically, the moving system moves at speed of light, then all coordinates and activities within the moving system have to appear to the stationary system as zeroes in order to maintain combined speeds of the moving system and all entities insight it as not exceeding velocity of light. The directing equation is defined by the velocity of light.

b) If the moving system is motionless relative to the stationary system, then the "moving" and stationary systems become the same, and the directing equation is defined by the velocity $\mathbf{u_x}$.

c) In this case the timing becomes $t_x'=t$, then the speed of the object within the moving system can be defined as: $u_x=\frac{x'}{t}$.

Substitute $\mathbf{u_x}$ in the equation for **x'**: $$x'=x\frac{\frac{x'}{t}}{\frac{x'}{t}+v_x} \qquad (a)$$

Rearrange (a): $$1=x\frac{1}{x'+v_x t}$$

or for **x'** it will be: $$x'=x-v_x t \qquad (b)$$

Equation (b) is the Galilean transformation equation, therefore at $\mathbf{v_x}$**<<c;** $\mathbf{u_x}$**<<c** the s-transformation approximates the Galilean transformation.

The directing equation is corresponding to the Galilean concept of transformation which does not impose any limitations on speeds.

d) When the travelling object in the moving system is a light pulse, then the s-transformation and the directing equation turn to the sl-transformation equations and directing equation (1-8), (1-9), (1-3). Therefore the sl-transformation is a particular limited version of the s-transformation when $\mathbf{u_x}$**=c**.

e) When no objects travel in the moving system then coordinate **x'** stays at **0** and the directing equation points to **x** position of the origin **O'**.
This case is special and needs to be evaluated specially.
The timing equation in this case represents timing relationship of two moving systems **K** and **K'**. Since neither of the systems carries any moving entities, then there is no basis for designation of the stationary and the moving system, which makes them equal. In such case there is no need for the common reference frame and the systems could be considered as equal moving bodies. Equation $\mathbf{x=tv_x}$ expresses current distance between two moving bodies at the same trajectory. The timing equation affirms that the internal timing rate within each moving body, as it appears to another body, depends on their relative velocities and is always looks contracted by factor $\left(\mathbf{1-\frac{v_x^2}{c^2}}\right)$**.** This timing state does not depend on the relative direction of the bodies' movement as long as they are on the same path. It means:

$$t'_x = t\left(1-\frac{v_x^2}{c^2}\right) \qquad (3\text{-}11)$$
$$t = t'_x\left(1-\frac{v_x^2}{c^2}\right)$$

Equations (3-11) express natural ultimate limitation of two bodies' mutual relative speed**:** velocity $\mathbf{v_x}$ cannot exceed speed of light **c**, otherwise the relative timing rate in each body becomes negative, which is contrary to the reality and to the stated initial axiom.

Relative contraction of the timing rate entails relative contraction of the length in this direction, since length is defined by the timing required by light to pass through this length.

There is a special point in usage of the s-transformation: $\mathbf{u_x=-v_x}$. At this point the directing equation becomes **x=0**, which means that the traveling object in the moving system "freezes" at position **x=0**, and denominator in the **x'** equation (3-8) becomes **0**. As a result the **x'** equation (3-8) becomes undetermined of the $\mathbf{\frac{0}{0}}$ type. In this case the functional type of the **x'** equation (3-10) should be used with assigning **x=0**. Then the result for the **x'** will be: $\mathbf{x'=-v_xt}$.

As it follows from the above evaluation, the s-transformation complies with whole range of velocities of the inertial systems without exceptions, including velocity of light. Therefore the s-transformation can be recognized as the universal transformation, applicable to all of physically possible velocities.

S-relativistic dynamics

4.<u>S-relativistic momentum</u>

If body **K'** with mass **m** is moving at uniform speed **v** relative to another body **K**, positioned on continuation of trajectory of the body **K'**, then the mechanical momentum **p'** of the body K' with relation to the body **K** will be defined as:

$$p'=mv \qquad (4\text{-}1)$$

Kinematic interrelation of the bodies is equal and their timing interrelation, according to equations (3-11), are:

$$t' = t\left(1 - \frac{v^2}{c^2}\right) \qquad (4\text{-}2)$$

$$t = t'\left(1 - \frac{v^2}{c^2}\right)$$

If **S** is the current instant distance between the moving bodies, then equation (4-1) can be expressed as:

$$p' = m\frac{dS}{dt'} \qquad (4\text{-}3)$$

From prospective of **K** body the mechanical momentum of **K'** body can be defined by substitution of **t'** in (4-3) from (4-2):

$$p = m\,\frac{dS}{d\left[t\left(1-\frac{v^2}{c^2}\right)\right]} \qquad (4\text{-}4)$$

Rearranging (4-4)::

$$p = m\frac{\frac{dS}{dt}}{1-\frac{v^2}{c^2}} \qquad (4\text{-}4a)$$

Note that $\frac{\mathbf{dS}}{\mathbf{dt}} = \mathbf{v}$, then:

$$p = mv\,\frac{1}{1-\frac{v^2}{c^2}} \qquad (4\text{-}5)$$

Equation (4-5) expresses momentum of the body of mass **m**, moving at speed **v** relative to another body.

Equation (4-5) can be expressed in normalized form by applying of normalized velocity $\mathbf{N}=\frac{\mathbf{v}}{\mathbf{c}}$, representing speed of the body as fraction of velocity of light:

$$p = mc\frac{N}{1-N^2} \qquad (4\text{-}5a)$$

5. S-relativistic energy

Kinetic energy of **K'** body of mass **m** moving at speed **v** relative to **K** body is defined as:

$$E_k = \int v\,dp \qquad (5\text{-}1)$$

Where **p** is momentum of **K'** body relative to **K** body, as expressed in (4-5).

Substitute **p** in (5-1) from (4-5):

$$E_k = \int v\,d\left(m\,\frac{v}{1-\frac{v^2}{c^2}}\right) \qquad (5\text{-}1a)$$

Integrating of (5-1a) by parts gives:

$$E_k = m\frac{v^2}{1-\frac{v^2}{c^2}} - m\int \frac{v}{1-\frac{v^2}{c^2}}\,dv \qquad (5\text{-}2)$$

Rearranging (5-2):

$$E_k = mc^2 \frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} - \frac{1}{2} mc^2 \int \frac{1}{1-\frac{v^2}{c^2}} d\left(\frac{v^2}{c^2}\right) \qquad (5\text{-}2a)$$

Integrating equation (5-2a):

$$E_k = mc^2 \frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} + \frac{1}{2} mc^2\, ln\left(1 - \frac{v^2}{c^2}\right) + E_0 \qquad (5\text{-}3)$$

$\mathbf{E_0}$ is an integration constant, and it can be defined from the condition: if **v=0**, then $\mathbf{E_k=0}$. It gives $\mathbf{E_0=0}$. Then:

$$E_k = mc^2 \frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} + \frac{1}{2} mc^2\, ln\left(1 - \frac{v^2}{c^2}\right) \qquad (5\text{-}3a)$$

Rearrange equation (5-3a):

$$E_k = mc^2\left[\frac{1}{1-\frac{v^2}{c^2}} - 1 + \frac{1}{2} ln\left(1 - \frac{v^2}{c^2}\right)\right] \qquad (5\text{-}3b)$$

Equation (5-3b) can be presented as follows:

$$E_k = mc^2\left[\frac{1}{1-\frac{v^2}{c^2}} + \frac{1}{2} ln\left(1 - \frac{v^2}{c^2}\right)\right] - mc^2 \qquad (5\text{-}4)$$

Equation (5-4) represents kinetic energy of the moving body.
The separate term $\mathbf{mc^2}$ in the right part of equation (5-4) does not depend on speed of the body, therefore this item represents the internal energy $\mathbf{E_r}$ of the body's mass at rest:

$$E_r = mc^2 \qquad (5\text{-}5)$$

It can be concluded that the full energy **E** of the moving body is sum of kinetic energy and the body's rest energy:

$$E = E_k + E_r = mc^2\left[\frac{1}{1-\frac{v^2}{c^2}} + \frac{1}{2} ln\left(1 - \frac{v^2}{c^2}\right)\right] \qquad (5\text{-}6)$$

Equations for the moving body kinetic energy (5-3a) and for full energy (5-6) of the moving body can be expressed in normalized form, where normalized speed is $\mathbf{N} = \frac{\mathbf{v}}{\mathbf{c}}$:

$$E_k = mc^2 \left[\frac{N^2}{1-N^2} + \frac{1}{2} ln(1 - N^2)\right] \qquad (5\text{-}7)$$

$$E = mc^2 \left[\frac{1}{1-N^2} + \frac{1}{2} ln(1 - N^2)\right] \qquad (5\text{-}8)$$

If body moves at speed considerably slower than velocity of light (**N<<1**), then item $\mathbf{N^2}$ can be disregarded in comparison to **1**, but this is not applicable to logarithm functions.

The function $\boldsymbol{ln}\mathbf{(1\text{-}N^2)}$ can be expressed by the Taylor series:

$$\mathbf{ln(1\text{-}N^2)} = -N^2 - \frac{N^4}{2} - \frac{N^6}{3} \ldots$$

If **N<<1**, then only the first term of the Taylor series should be taken and the others can be disregarded. Then at the very low speed of body the kinetic energy according to equation (5-7) will be:

$$\mathbf{E_{k(slow)}=mc^2}\left(N^2-\frac{1}{2}N^2\right)=\frac{1}{2}\mathbf{mc^2N^2}=\frac{1}{2}\mathbf{mv^2},$$

which is in compliance with conventional formula for kinetic energy.

6. S-relativistic force and acceleration

As it's been postulated, the theory of special relativity is considering the inertial systems which are moving at uniform speed without influence of any external force or gravity. In this connotation such physical categories as the force and the acceleration should not be in the scope of the theory of special relativity. However, the force is the primary source of movement of systems and objects and it causes acceleration of speed. Correspondingly, the relativistic aspect of the physical force and acceleration are considered to be included in special relativity and will be analyzed herein.

The force applied to the freely moving body causes acceleration of the body's speed in the direction of the applied force. This acceleration, characterizing changing of the body's speed relative to external reference frame, is the speed acceleration $\boldsymbol{a}_\mathbf{v}$:

$$a_\mathrm{v}=\frac{d\mathrm{v_t}}{dt} \qquad (6\text{-}1)$$

where $\mathbf{v_t}$ is instant speed of the body as function of time **t**.

There is another aspect of acceleration, given by the Newtonian second law, characterizing dynamic reaction of a free moving massive body in response to applied external force. This acceleration can be called as the force related acceleration $\boldsymbol{a}_\mathbf{f}$:

$$a_\mathrm{f}=\frac{\mathrm{F}}{\mathrm{m}} \qquad (6\text{-}2)$$

where **m** is the body's mass, and **F** is the external force applied to the body

The force related acceleration $\boldsymbol{a}_\mathbf{f}$ is absolute in sense that it does not depend on any external reference frames and can be totally defined within the moving body by measuring internally its mass and the applied force. If mass of the moving body and the applied force are constant, then the force related acceleration $\boldsymbol{a}_\mathbf{f}$ will be constant, regardless of speed of the body relative to any external frame of reference.

If external force **F** is applied to a freely moving body **K'** of mass **m**, then the body **K'** will be moving on its trajectory with constant acceleration $\boldsymbol{a}_\mathbf{v}$. From prospective of reference body **K**, positioned on the extension of trajectory of the body **K',** the accelerating body **K'** is propelled by the force $\mathbf{F_t}$, which is defined at each point of time by equation:

$$\mathrm{F_t}=\frac{d\mathrm{p_t}}{dt} \qquad (6\text{-}3)$$

where $\mathbf{p_t}$ is mechanical momentum of body K' as function of varying speed $\mathbf{v_t}$.

Applying formula (4-5) of section 4, the momentum $\mathrm{p_t}$ at each given instant of time will be:

$$\mathrm{p_t}=\mathrm{mv_t}\frac{1}{1-\frac{\mathrm{v_t^2}}{\mathrm{c^2}}} \qquad (6\text{-}4)$$

Make substitution of $\mathbf{p_t}$ from (6-4) in (6-3):

$$\mathrm{F_t} = \frac{d}{dt}\left(m\frac{\mathrm{v_t}}{1-\frac{\mathrm{v_t^2}}{\mathrm{c^2}}}\right) \qquad \text{(6-5)}$$

Derivative of (6-5) is:

$$\mathrm{F_t} = \mathrm{m}\frac{\left(1-\frac{\mathrm{v_t^2}}{\mathrm{c^2}}\right)\frac{d\mathrm{v_t}}{dt}+2\frac{\mathrm{v_t^2}}{\mathrm{c^2}}\frac{d\mathrm{v_t}}{dt}}{\left(1-\frac{\mathrm{v_t^2}}{\mathrm{c^2}}\right)^2} \qquad \text{(6-6)}$$

Rearrange (6-6):

$$\mathrm{F_t} = \mathrm{m}\frac{d\mathrm{v_t}}{dt}\frac{1+\frac{\mathrm{v_t^2}}{\mathrm{c^2}}}{\left(1-\frac{\mathrm{v_t^2}}{\mathrm{c^2}}\right)^2} \qquad \text{(6-6a)}$$

Substitution of $\frac{d\mathrm{v_t}}{dt}$ from (6-1) in (6-6a) produces equation for the force $\mathbf{F_t}$:

$$\mathrm{F_t} = \mathrm{m}a_{\mathrm{v}}\frac{1+\frac{\mathrm{v_t^2}}{\mathrm{c^2}}}{\left(1-\frac{\mathrm{v_t^2}}{\mathrm{c^2}}\right)^2} \qquad \text{(6-7)}$$

Equation (6-7) can be presented in normalized form, applying substitution $\frac{\mathbf{v_t}}{\mathbf{c}} = \mathbf{N_t}$**:**

$$\mathrm{F_t} = \mathrm{m}a_{\mathrm{v}}\frac{1+\mathrm{N_t^2}}{\left(1-\mathrm{N_t^2}\right)^2} \qquad \text{(6-7a)}$$

Equations (6-7) and (6-7a) express instant value of the external force $\mathbf{F_t}$ applied to body **K'** and causing constant acceleration $\boldsymbol{a}_{\mathbf{v}}$ of body K' relative to body **K**. As it follows from equation (6-7), in order to maintain constant acceleration $\boldsymbol{a}_{\mathbf{v}}$ the force $\mathbf{F_t}$ has to continuously increase, approaching infinity when the body's accelerated speed $\mathbf{v_t}$ approaches speed of light ($\mathbf{N_t \rightarrow 1}$).

If to assign the external force **F** as constant, then the speed acceleration $\boldsymbol{a}_{\mathbf{vt}}$ can be defined.
In equation (6-7a), assign the force **F** as constant and the acceleration $\boldsymbol{a}_{\mathbf{v}}$ as time dependent $\boldsymbol{a}_{\mathbf{vt}}$:

$$\frac{\mathrm{F}}{\mathrm{m}} = a_{\mathrm{vt}}\frac{1+\mathrm{N_t^2}}{\left(1-\mathrm{N_t^2}\right)^2} \qquad \text{(6-8)}$$

From equation (6-8) the time dependant speed acceleration $\boldsymbol{a}_{\mathbf{vt}}$ is defined as follows:

$$a_{\mathrm{vt}} = \frac{\mathrm{F}}{\mathrm{m}}\frac{\left(1-\mathrm{N_t^2}\right)^2}{1+\mathrm{N_t^2}} \qquad \text{(6-9)}$$

According to equation (6-9), when external constant force is applied to the free moving body, the speed of the body will be accelerated, but the acceleration $\boldsymbol{a}_{\mathbf{vt}}$ will not be constant. It will decrease approaching **0** ($\boldsymbol{a}_{\mathbf{vt}}\mathbf{\rightarrow 0}$) upon increasing of the body's normalized speed $\mathbf{N_t}$ approaching velocity of light ($\mathbf{N_t \rightarrow 1}$).

Correlation between the relative speed acceleration and the absolute force related acceleration is defined by substitution (6-2) in (6-9):

$$a_{\mathrm{vt}} = a_{\mathrm{f}} \frac{\left(1-\mathrm{N_t^2}\right)^2}{1+\mathrm{N_t^2}} \qquad (6\text{-}10)$$

7. Summation

The s-transformation equations and relativistic formulation by the s-relativity of the dynamic equations of physics are presented in summarizing table. The equations are presented with the speeds expressed in actual form and in normalized form as $\mathbf{N=\frac{v}{c}}$. The equations are oriented to **X** directions, but they are equally valid for **Y** and **Z** directions by substituting of the **x** designators by **y** or **z**.

Summarizing table of the s-relativity equations

Definition	Actual speeds	Normalized speeds
S-transformation equations in the functional form for arbitrary speeds	$x'=x-v_xt$ $t'=t-\frac{v_xx}{c^2}$ $x=t\frac{u_x+v_x}{1+\frac{u_xv_x}{c^2}}$ - directing equation	$x'=x-N_{vx}ct$ $t'=t-N_{vx}\frac{x}{c}$ $x=ct\frac{N_{vx}+N_{ux}}{1+N_{vx}N_{ux}}$ -directing equation
S-transformation equations in the direct transformation form for arbitrary speeds	$x'=x\frac{1-\frac{v_x^2}{c^2}}{1+\frac{v_x}{u_x}}$ $t'_x=t\frac{1-\frac{v_x^2}{c^2}}{1+\frac{u_xv_x}{c^2}}$ $x=t\frac{u_x+v_x}{1+\frac{u_xv_x}{c^2}}$ - directing equation	$x'=x\frac{1-N_{vx}^2}{1+\frac{N_{vx}}{N_{ux}}}$ $t'=t\frac{1-N_{vx}^2}{1+N_{vx}N_{ux}}$ $x=ct\frac{N_{ux}+N_{vx}}{1+N_{ux}N_{vx}}$ -directing equation
Combined speed of two inertial objects relative to common reference frame	$w_x=\frac{u_x+v_x}{1+\frac{u_xv_x}{c^2}}$	$N_{wx}=\frac{N_{vx}+N_{ux}}{1+N_{vx}N_{ux}}$
Internal timing of inertial objects moving at speed **v** relative to each other	$t'=t\left(1-\frac{v^2}{c^2}\right)$ $t=t'\left(1-\frac{v^2}{c^2}\right)$	$t'=t(1-N_v^2)$ $t=t'(1-N_v^2)$
Mechanical momentum	$p=mv\frac{1}{1-\frac{v^2}{c^2}}$	$p=mc\frac{N}{1-N^2}$
Full energy of moving body	$E=mc^2\left[\frac{1}{1-\frac{v^2}{c^2}}+\frac{1}{2}ln\left(1-\frac{v^2}{c^2}\right)\right]$	$E=mc^2\left[\frac{1}{1-N^2}+\frac{1}{2}ln(1-N^2)\right]$
Energy of the body at rest.	$E_r=mc^2$	$E_r=mc^2$
Kinetic energy of moving body	$E_k=mc^2\left[\frac{1}{1-\frac{v^2}{c^2}}+\frac{1}{2}ln\left(1-\frac{v^2}{c^2}\right)-1\right]$	$E_k=mc^2\left[\frac{1}{1-N^2}+\frac{1}{2}ln(1-N^2)-1\right]$
External force applied to moving body, maintaining constant acceleration	$F_t=ma_v\frac{1+\frac{v_t^2}{c^2}}{\left(1-\frac{v_t^2}{c^2}\right)^2}$	$F_t=ma_v\frac{1+N_t^2}{(1-N_t^2)^2}$
Acceleration of moving body under applied constant force	$a_{vt}=\frac{F}{m}\frac{\left(1-\frac{v_t^2}{c^2}\right)^2}{1+\frac{v_t^2}{c^2}}$	$a_{vt}=\frac{F}{m}\frac{(1-N_t^2)^2}{1+N_t^2}$

8. Afterthought

The theory of the s-relativity presented in this paper covers the same segment of physics as the Einstein's special theory of relativity, but produces equations which are different from the Einstein's theory. Therefore, the question arises: which of the theories is accurate. This question can be answered in two ways: analytical and experimental.

The Einstein's special relativity was analytically examined in [2] and shown that the Lorentz transformation equations, which are at the core of the Einstein's special relativity, are not accurate for the relativistic transformation procedures, particularly introduction of the **γ** factor. The **γ** factor is helpful as a mathematical tool of solving certain types of equations, but, as shown in [2], it does not naturally emerge over derivation of the transformation equations. In certain cases, as it shown in [2], the **γ** factor causes violation of constant velocity of light. These facts may indicate inaccuracy of the Einstein's relativity.

With regards to the s-relativity, the examination of the s-relativity's equations and their applications has not revealed so far any physical inaccuracies, however it still remains to be scrutinized. Analytical evaluation of kinematic and dynamic equations of the s-relativity at specific speed ranges, characterizing already known applications, showed full compliance of the s-relativity with conventional formulations of the laws of physics without exceptions..

Regarding experimental assessments of the s-relativity and the Einstein's relativity it should be noted that significant difference between them occurs at the velocity range close to velocity of light. For instance, at speed **v=0.5c** the kinetic energy defined in the s-relativity is **1.22** times higher than in the Einstein's relativity. At speed **v=0.9c** the kinetic energy in the s-relativity is **2.65** times higher. At **v=0.999c** the kinetic energy in the s-relativity is **23.22** times higher

Experimental determination of correlation between kinetic energy and speeds of particles in the range of velocity of light can determine the correct theory.

References:

1. Albert Einstein. "RELATIVETY. The Special and the General Theory" , Crown Publisher, Inc., One Park Avenue, New York, N.Y.10016, 1952
2. Solomon Shapiro. "Variations of the relativistic transformation - 2", The General Science Journal, www.gsjournal.net June 2014
3. Solomon Shapiro. "Relativistic mechanics in the s-relativity and comparison with the Einstein relativity" The General Science Journal, www.gsjournal.net July 2014

Appendix

Comparison of the s-transformation equations with the Lorentz transformation and the dynamic equations of the s-relativity with the Einstein's relativistic equations are presented in a summarizing tables. Speeds in the equations are expressed in normalized form $\mathbf{N=\frac{v}{c}}$. All equations are **x**-coordinate oriented, but equally applicable to the **Y** and **Z** coordinates. The time dependant variables are subscript marked as $(_t)$.

Comparison of the s-transformation equations and the Lorentz transformation

DESCRIPTION	S-RELATIVITY	LORENTZ TRANSFORMATION
Sl-transformation equations in functional form for the moving system carrying propagation of light	$x'=x-N_x ct$ $t'=t-N_x\frac{x}{c}$ $x'=ct'$; $x=ct$ - directing equations	$x'=\frac{1}{\sqrt{1-N_x^2}}(x-N_x ct)$ $t'=\frac{1}{\sqrt{1-N_x^2}}(t-N_x\frac{x}{c})$
Sl-transformation equations in the direct transformation form for the moving system carrying propagation of light	$x'=x(1-N_x)$ $t'= t(1-N_x)$ $x'=ct'$; $x=ct$ - directing equations	$x'=x\sqrt{\frac{1-N_x}{1+N_x}}$ $t'=t\sqrt{\frac{1-N_x}{1+N_x}}$
Ranges of coordinates variations, depending on mutual directions of the system and the light pulse	when $-1\leq N_x\leq 1$, then: $2x\geq x'\geq 0$ $2t\geq t'\geq 0$	when $-1\leq N_x\leq 1$, then: $\infty\geq x'\geq 0$ $\infty\geq t'\geq 0$
S-transformation equation in the functional form for arbitrary speeds	$x'=x-N_{ux}ct$ $t'=t-N_{ux}\frac{x}{c}$ $x=ct\frac{N_{vx}+N_{ux}}{1+N_{vx}N_{ux}}$ - directing equation	- - - - - - - - - - - - - - -
S-transformation equations in the direct transformation form for arbitrary speeds.	$x'=x\frac{1-N_{vx}^2}{1+\frac{N_{vx}}{N_{ux}}}$ $t'=t\frac{1-N_{vx}^2}{1+N_{vx}N_{ux}}$ $x=ct\frac{N_{vx}+N_{ux}}{1+N_{vx}N_{ux}}$ - directing equation	- - - - - - - - - - - - - - -
Combining speed of inertial objects relative to common reference frame.	$N_w=\frac{N_v+N_u}{1+N_vN_u}$	$N_w=\frac{N_v+N_u}{1+N_vN_u}$
Internal timing of two inertial objects moving relative to each other on the same trajectory	$t'=t(1-N^2)$ $t=t'(1-N^2)$	- - - - - - - - - - - - - - -

Comparison of the s-relativity equations and the Einstein's special relativity

DESCRIPTION	S-RELATIVITY	EINSTEIN'S RELATIVITY
Mechanical momentum.	$p=mc\frac{N}{1-N^2}$	$p=mc\frac{N}{\sqrt{1-N^2}}$
Full energy of moving body.	$E=mc^2\left[\frac{1}{1-N^2}+\frac{1}{2}ln(1-N^2)\right]$	$E=mc^2\ \frac{1}{\sqrt{1-N^2}}$
Energy of the body at rest.	$E_r=mc^2$	$E_r=mc^2$
Kinetic energy of moving body.	$E_k=mc^2\left[\frac{1}{1-N^2}+\frac{1}{2}ln(1-N^2)-1\right]$	$E_k=mc^2\left(\frac{1}{\sqrt{1-N^2}}-1\right)$
External force applied to moving body, maintaining constant acceleration	$F_t=ma_v\frac{1+N_t^2}{(1-N_t^2)^2}$	$F_t=ma_v\frac{1}{\left(\sqrt{1-N_t^2}\right)^3}$
Acceleration of moving body under applied constant force	$a_{vt}=\frac{F}{m}\ \frac{(1-N_t^2)^2}{1+N_t^2}$	$a_{vt}=\frac{F}{m}\ \left(\sqrt{1-N_t^2}\right)^3$