

# RELATIVISTIC MECHANICS IN THE S-RELATIVITY AND COMPARISON WITH THE EINSTEIN RELATIVITY

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## CONTENTS

1. Relativistic momentum	page
A) Derivation of the s-relativistic momentum formula .....	2
B) Comparing of the relativistic momentums in the s-relativity and the Einstein relativity .....	3
2. Relativistic energy	
A) Derivation of the s-relativistic energy formulas .....	4
B) Comparing of the relativistic energies in the s-relativity and the Einstein relativity .....	6
3. Relativistic force and acceleration	
A) Derivation of the s-relativistic force and acceleration formulas .....	8
B) Comparing of the relativistic forces in the s-relativity and the Einstein relativity .....	10
C) Comparing of the relativistic accelerations in the s-relativity and the Einstein relativity .....	11
4. Summation .....	13

# RELATIVISTIC MECHANICS IN THE S-RELATIVITY AND COMPARISON WITH THE EINSTEIN RELATIVITY

## ABSTRACT

Derivation of relativistic equations for the major physical mechanical characteristics, such as mechanical momentum, kinetic and full energy, force and acceleration, based on the relativistic sg-transformation are presented in current research. The derived formulas are being evaluated and compared with similar Einstein relativity formulas by analytical and graphical means. The comparisons show closeness of values at relatively low speeds of movement (up to 20% -30% of speed of light) and considerable differences at speeds close to speed of light.

**Keywords:** sg-transformation, s-relativity, Einstein relativity, relativistic momentum, relativistic energy, relativistic force, relativistic acceleration.

The special theory of relativity is based on utilizing of the transformation equations, describing spatial and timing relationship of two inertial systems, moving relative to each other. Current research is based on utilization of the sg-transformation [2] for deriving of major physical mechanical relativistic equations: the mechanical momentum, the kinetic and full energy, the force and acceleration. Since the s-transformation equations differ from the Lorentz transformation equations, which are at the core of the Einstein relativity, it should be expected that mathematical descriptions of the relativistic mechanical characteristics, based on different transformation equations, will be different.

## 1. Relativistic momentum

### A) Derivation of the s-relativistic momentum formula

If body **K'** with mass **m** is moving at uniform speed **v** relative to another body **K**, positioned on continuation of the trajectory of the body **K'**, then the mechanical momentum **p'** of the body **K'** with relation to the body **K** will be defined as:

$$p'=mv \quad (1-1)$$

Since only two bodies are involved in the process, there is no grounds for defining of which body is moving and which is stationary. Their kinematical interrelation is equal and their timing interrelations, according to the conclusion of the sg-transformation [2], are:

$$t'=t\left(1 - \frac{v^2}{c^2}\right) \quad (1-2)$$
$$t=t'\left(1 - \frac{v^2}{c^2}\right)$$

If **S** is the current instant distance between the bodies, then equation (1-1) can be expressed as:

$$p'=m\frac{dS}{dt'} \quad (1-3)$$

From prospective of **K** body the mechanical momentum of **K'** body can be defined by substitution **t'** in (1-3) from (1-2):

$$p=m\frac{dS}{d\left[t\left(1-\frac{v^2}{c^2}\right)\right]} \quad (1-4)$$

Rearranging (1-4)::

$$p = m \frac{\frac{ds}{dt}}{1 - \frac{v^2}{c^2}} \quad (1-4a)$$

Note that  $\frac{ds}{dt} = v$ , then:

$$p = mv \frac{1}{1 - \frac{v^2}{c^2}} \quad (1-5)$$

Equation (1-5) can be expressed in normalized form by applying of the normalized velocity  $N = \frac{v}{c}$ , representing speed of the body as fraction of the velocity of light:

$$p = mc \frac{N}{1 - N^2} \quad (1-5a)$$

The s-relativistic momentum can be expressed by using the well known relativistic factor  $\gamma = \frac{1}{\sqrt{1 - N^2}}$ :

$$p = mcN\gamma^2 \quad (1-6)$$

### B) Comparing of the relativistic momentums in the s-relativity and the Einstein relativity

In order to make comparison of the relativistic momentums of the s-relativity and the Einstein relativity the momentum for the s-relativity will be marked as  $\mathbf{p}_s$ , and the momentum for the Einstein relativity will be marked as  $\mathbf{p}_e$ . Then it will be:

the momentum definition in the s-relativity is:

$$p_s = mc \frac{N}{1 - N^2}$$

the momentum definition in the Einstein relativity is:

$$p_e = mc \frac{N}{\sqrt{1 - N^2}}$$

The difference  $\mathbf{D}_p$  between momentums of the s-relativity and the Einstein relativity can be evaluated in percentage by the difference between  $\mathbf{p}_s$  and  $\mathbf{p}_e$  relative to  $\mathbf{p}_e$ :

$$D_p = \frac{p_s - p_e}{p_e} = \frac{p_s}{p_e} - 1 = 100 \left( \frac{1}{\sqrt{1 - N^2}} - 1 \right) \%$$

Table 1 presents results of calculations of  $\mathbf{p}_s$  and  $\mathbf{p}_e$ , and their relative difference  $\mathbf{D}_p$ , expressed in percentage, as function of normalized speed N. The factor  $\mathbf{mc}$  is assigned as  $\mathbf{mc} = 1$ .

TABLE 1

N	$P_s$	$P_e$	$D_p\%$
0	0	0	
0.1	0.101	0.101	0
0.2	0.208	0.204	1.96
0.3	0.33	0.314	5.1
0.4	0.476	0.436	9.17
0.5	0.667	0.577	15.6
0.6	0.938	0.75	25.07
0.7	1.373	0.98	40.1
0.8	2.222	1.333	66.69
0.9	4.737	2.065	129.39
0.95	9.744	3.042	220.32
0.99	49.749	7.018	608.88
0.999	499.75	22.344	2136.62
0.9999	4999.75	70.705	6971.28

Data in Table 1 shows that the mechanical momentum of the moving body defined in the s-relativity and Einstein relativity have similar trend of non-linear increasing upon linear increasing of speed of the body, but the s-relativity presents higher rate of the increasing, especially emphasized when speed of the body approaches speed of light.

Fig.1 presents plots of  $p_s$  and  $p_e$  in the N range 0 - 0.9, according to Table 1.

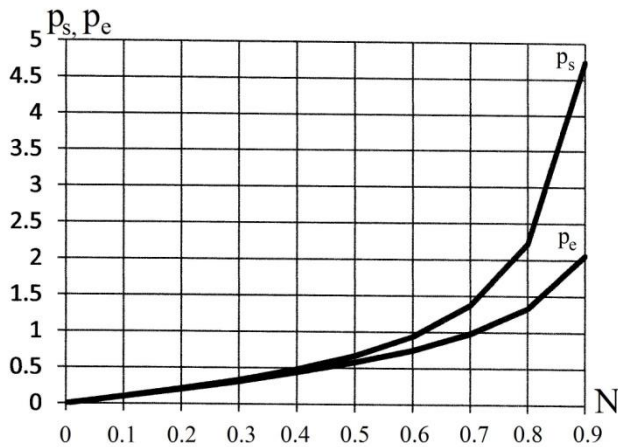


Fig.1

## 2. Relativistic energy

### A) Derivation of the s-relativistic energy formulas

The kinetic energy of the  $\mathbf{K}'$  body moving at speed  $\mathbf{v}$  relative to the  $\mathbf{K}$  body is defined as:

$$E_k = \int v dp \quad (2-1)$$

Where  $\mathbf{p}$  is the momentum of K' body relative to K body.

Substitute  $\mathbf{p}$  in (2-1) from (1-5):

$$E_k = \int v d \left( m \frac{v}{1 - \frac{v^2}{c^2}} \right) \quad (2-1a)$$

Integrating of (2-1a) by parts gives:

$$E_k = m \frac{v^2}{1 - \frac{v^2}{c^2}} - m \int \frac{v}{1 - \frac{v^2}{c^2}} dv \quad (2-2)$$

Rearranging (2-2):

$$E_k = mc^2 \frac{\frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} - \frac{1}{2} mc^2 \int \frac{1}{1 - \frac{v^2}{c^2}} d \left( \frac{v^2}{c^2} \right) \quad (2-2a)$$

Integrating equation (2-2a):

$$E_k = mc^2 \frac{\frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} + \frac{1}{2} mc^2 \ln \left( 1 - \frac{v^2}{c^2} \right) + E_0 \quad (2-3)$$

$E_0$  is an integration constant, and it can be defined from the condition: if  $\mathbf{v}=\mathbf{0}$ , then  $\mathbf{E}_k=\mathbf{0}$ . It gives  $\mathbf{E}_0=\mathbf{0}$ .

Then:

$$E_k = mc^2 \frac{\frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} + \frac{1}{2} mc^2 \ln \left( 1 - \frac{v^2}{c^2} \right) \quad (2-3a)$$

Rearrange equation (2-3a):

$$E_k = mc^2 \left[ \frac{1}{1 - \frac{v^2}{c^2}} - 1 + \frac{1}{2} \ln \left( 1 - \frac{v^2}{c^2} \right) \right] \quad (2-3b)$$

Equation (2-3b) can be presented as follows:

$$E_k = mc^2 \left[ \frac{1}{1 - \frac{v^2}{c^2}} + \frac{1}{2} \ln \left( 1 - \frac{v^2}{c^2} \right) \right] - mc^2 \quad (2-4)$$

Equivalent equations (2-3a) and (2-4) represent kinetic energy of the moving body.

The term  $mc^2$  in the right part of equation (2-4) does not depend on speed of the body, therefore it represents the internal energy  $\mathbf{E}_r$  of the body's mass at rest:

$$E_r = mc^2 \quad (2-5)$$

It can be concluded that the full energy  $\mathbf{E}$  of the moving body is the sum of the kinetic energy and the body's rest energy:

$$E = E_k + E_r = mc^2 \left[ \frac{1}{1 - \frac{v^2}{c^2}} + \frac{1}{2} \ln \left( 1 - \frac{v^2}{c^2} \right) \right] \quad (2-6)$$

Equations for the moving body kinetic energy (2-3a) and for the full energy (2-6) of the moving body can be expressed in normalized form, where normalized speed is  $\mathbf{N} = \frac{v}{c}$ :

$$E_k = mc^2 \left[ \frac{N^2}{1-N^2} + \frac{1}{2} \ln(1 - N^2) \right] \quad (2-4a)$$

$$E = mc^2 \left[ \frac{1}{1-N^2} + \frac{1}{2} \ln(1 - N^2) \right] \quad (2-6a)$$

If body moves at speed considerably slower than velocity of light ( $\mathbf{N} \ll 1$ ), then item  $\mathbf{N}^2$  can be disregarded in comparison to  $\mathbf{1}$ , but this is not applicable to the logarithm functions.

The function  $\ln(1-N^2)$  can be expressed by the Taylor series as follows:

$$\ln(1-N^2) = -N^2 - \frac{N^4}{2} - \frac{N^6}{3} \dots$$

When  $N \ll 1$ , then only the first term of the Taylor series should be taken and the others can be disregarded. Then for the very low speed of body, relative to speed of light, the kinetic energy according to equation (2-4a) will be:

$$E_{k(\text{slow})} = mc^2 \left( N^2 - \frac{1}{2} N^2 \right) = \frac{1}{2} mc^2 N^2 = \frac{1}{2} mv^2,$$

which is in compliance with the classical formula for kinetic energy.

The s-relativity energy equations (2-4a) and (2-6a) can be expressed in terms of the Lorentz relativistic

factor  $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ :

$$E_k = mc^2 \left( \gamma^2 - 1 + \frac{1}{2} \ln \frac{1}{\gamma^2} \right)$$

$$E = mc^2 \left( \gamma^2 + \frac{1}{2} \ln \frac{1}{\gamma^2} \right)$$

or:

$$E_k = mc^2 (\gamma^2 - \ln \gamma) - mc^2$$

$$E = mc^2 (\gamma^2 - \ln \gamma) \quad (2-7)$$

$$\text{When } \gamma=1, \text{ then: } E_k=0; E=mc^2$$

## B) Comparing of the relativistic energies in the s-relativity and the Einstein relativity

In order to make comparison of the relativistic energies, the energy for the s-relativity will be marked as  $E_s$ , and the energy for the Einstein relativity will be marked as  $E_e$ . Then it will be:

$$\text{the full energy in the s-relativity is :} \quad E_s = mc^2 \left[ \frac{1}{1-N^2} + \frac{1}{2} \ln(1 - N^2) \right]$$

$$\text{the full energy in the Einstein relativity is:} \quad E_e = mc^2 \frac{1}{\sqrt{1-N^2}}$$

The difference  $D_E$  between the energy equations in the s-relativity and the Einstein relativity can be evaluated in percentage by the difference between  $E_s$  and  $E_e$  relative to  $E_e$ :

$$D_E = \frac{E_s - E_e}{E_e} = \frac{E_s}{E_e} - 1 = 100 \left[ \frac{1}{\sqrt{1-N^2}} + \frac{1}{2} \sqrt{1-N^2} \ln(1 - N^2) - 1 \right] \%$$

TABLE 2 presents results of calculations of  $E_s$  and  $E_e$ , and their relative difference  $D_E$  expressed in percentage. The factor  $mc^2$  is assigned as  $mc^2=1$ .

TABLE 2

N	E <sub>s</sub>	E <sub>e</sub>	D <sub>E</sub> %
0	1	1	0
0.1	1.005	1.005	0
0.2	1.021	1.021	0
0.3	1.052	1.048	0.4
0.4	1.103	1.091	1.1
0.5	1.189	1.155	2.9
0.6	1.339	1.25	7.1
0.7	1.624	1.4	16
0.8	2.267	1.667	36
0.9	4.433	2.294	93.2
0.95	9.092	3.203	183.9
0.99	48.293	7.089	581.2
0.999	497.143	22.366	2122.8
0.9999	4995.991	70.712	6965.3

Fig.2 presents plots of E<sub>s</sub> and E<sub>e</sub> in the N range 0 - 0.9, according to Table 2.

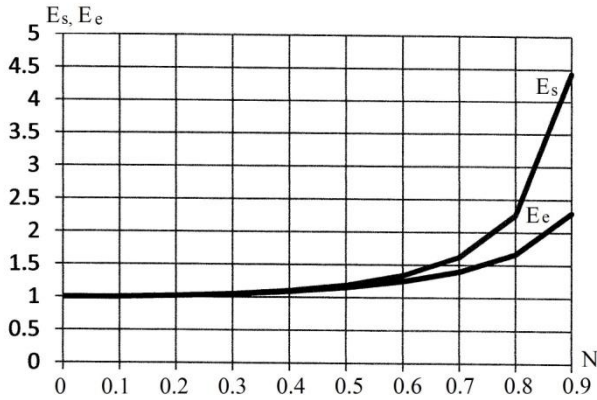


Fig.2

It would be informative to compare the relativistic energies from the s-relativity and the Einstein relativity as function of the relativistic factor  $\gamma$ :  $\mathbf{E=f(\gamma)}$ .

Table 3 presents values of the relativistic energy calculated according to the s-relativity and to the Einstein relativity as function of the  $\gamma$ -factor. The  $\gamma$ -factor is presented in the range 1-3 with steps 0.2. The normalized speed N corresponding to the  $\gamma$ -factor value is presented for reference as well.

TABLE 3

$\gamma$	$E_e$	$E_s$	$N$
1	1	1	0
1.2	1.2	1.258	0.553
1.4	1.4	1.624	0.7
1.6	1.6	2.09	0.781
1.8	1.8	2.652	0.831
2	2	3.307	0.866
2.2	2.2	4.052	0.891
2.4	2.4	4.885	0.909
2.6	2.6	5.804	0.923
2.8	2.8	6.81	0.934
3	3	7.901	0.943

Fig.3 presents plots of  $E_s$  and  $E_e$  in the  $\gamma$  range 1 - 3, according to Table 3.

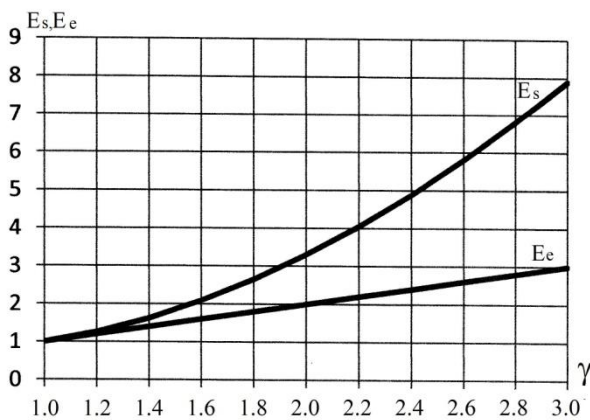


Fig.3

### 3. Relativistic force and acceleration

#### A) Derivation of the s-relativistic force and acceleration formulas

As it's been postulated, the theory of special relativity is considering the inertial systems which are moving at uniform speed without influence of any external force or gravity. In this connotation such physical categories as the force and the acceleration should not be in the scope of the theory of special relativity. However, the force is the primary source of movement of systems and objects and it causes acceleration of speed. Correspondingly, the relativistic aspect of the physical force and acceleration has been considered and analyzed in number of researches. So, the relativistic analysis of the physical force and the acceleration cannot be omitted in the current research.

The force applied to the freely moving body causes acceleration of the body's speed in the direction of the applied force. This acceleration, characterizing changing of the body's speed relative to external reference frame or body, is the speed acceleration  $a_v$  :



$$a_v = \frac{dv_t}{dt} \quad (3-1)$$

where  $v_t$  is varying instant speed of the body as function of time  $t$ .

There is another aspect of acceleration, given by the Newton's second law, characterizing dynamic proportionality between the applying force and the mass of the body. This acceleration can be called as the force related acceleration  $a_f$  :

$$a_f = \frac{F}{m} \quad (3-2)$$

where  $m$  is the body's mass, and  $F$  is the external force applied to the body

The force related acceleration  $a_f$  is absolute in sense that it does not depend on any external reference frames and can be totally defined within the moving body by measuring internally its mass and the applied force. If mass of the moving body and the applied force are constant, then the force related acceleration  $a_f$  will be constant, regardless of speed of the body relative to any external frame or body of reference.

If an external force  $F$  is applied to a freely moving body  $K'$  of mass  $m$ , then the body  $K'$  will be moving on its trajectory with acceleration  $a_v$ . From prospective of reference body  $K$ , positioned on the extension of trajectory of the body  $K'$ , the accelerating body  $K'$  is propelled by the force  $F_t$ , which is defined at each point of time by equation:

$$F_t = \frac{dp_t}{dt} \quad (3-3)$$

where  $p_t$  is varying mechanical momentum of body  $K'$  as function of varying speed  $v_t$ .

Applying formula (1-5) of section 1, the momentum  $p_t$  at each given instant of time will be:

$$p_t = mv_t \frac{1}{1 - \frac{v_t^2}{c^2}} \quad (3-4)$$

Make substitution of  $p_t$  from (3-4) in (3-3):

$$F_t = \frac{d}{dt} \left( m \frac{v_t}{1 - \frac{v_t^2}{c^2}} \right) \quad (3-5)$$

Derivative of (3-5) is:

$$F_t = m \frac{\left(1 - \frac{v_t^2}{c^2}\right) \frac{dv_t}{dt} + 2 \frac{v_t^2}{c^2} \frac{dv_t}{dt}}{\left(1 - \frac{v_t^2}{c^2}\right)^2} \quad (3-6)$$

Substitution of  $\frac{dv_t}{dt}$  from (3-1) in (3-6) produces equation for the force  $F_t$ :

$$F_t = ma_v \frac{1 + \frac{v_t^2}{c^2}}{\left(1 - \frac{v_t^2}{c^2}\right)^2} \quad (3-7)$$

Equation (3-7) can be presented in normalized form, applying substitution  $\frac{v_t}{c} = N_t$ , where  $N_t$  is the instant

normalized speed which grows in time due to acceleration of body's speed:

$$F_t = ma_v \frac{1+N_t^2}{(1-N_t^2)^2} \quad (3-7a)$$

Equations (3-7) and (3-7a) express instant value of the external force  $F_t$  applied to body  $K'$  and causing constant acceleration  $a_v$  of body  $K'$ , as it is judged from the reference body  $K$  at each instant of time  $t$ . As it follows from equation (3-7), in order to maintain constant acceleration  $a_v$  the force  $F_t$  has to continuously increase, approaching infinity when the body's accelerated speed  $v_t$  approaches speed of light ( $N_t \rightarrow 1$ ).

If to assign the external force  $F$  as constant, then the varying speed acceleration  $a_{vt}$  can be defined. In equation (3-7a), assign the force  $F$  as constant and the acceleration  $a_v$  as time dependent  $a_{vt}$ :

$$\frac{F}{m} = a_{vt} \frac{1+N_t^2}{(1-N_t^2)^2} \quad (3-8)$$

From equation (3-8) the time dependant speed acceleration  $a_{vt}$  is defined as follows:

$$a_{vt} = \frac{F}{m} \frac{(1-N_t^2)^2}{1+N_t^2} \quad (3-9)$$

According to equation (3-9) the speed acceleration  $a_{vt}$  decreases upon increasing of the body normalized speed  $N_t$ , approaching 0 when the body's speed approaches speed of light ( $N_t \rightarrow 1$ ).

Relativistic force of the s-relativity can be expressed in terms of the  $\gamma$ -factor. In this case the Lorentz  $\gamma$ -factor will not be a constant, but a function of changing speed  $v_t$ :  $\gamma_t = \frac{1}{\sqrt{1-N_t^2}}$

Modifying (3-7a):

$$F_t = ma_v \frac{1+1-(1-N_t^2)}{(1-N_t^2)^2} = ma_v \frac{2-\frac{1}{\gamma_t^2}}{\frac{1}{\gamma_t^4}} \quad (3-10)$$

Rearranging (3-10) gives:

$$F_t = ma_v \gamma_t^2 (2\gamma_t^2 - 1) \quad (3-10a)$$

### B) Comparing of the relativistic forces in the s-relativity and the Einstein relativity

In order to make comparison of the relativistic forces, the force for the s-relativity will be marked as  $F_{ts}$ , and the force for the Einstein relativity will be marked as  $F_{te}$ . Then it will be:

the relativistic force in the s-relativity is:  $F_{ts} = ma_v \frac{1+N_t^2}{(1-N_t^2)^2}$

the relativistic force in the Einstein relativity is:  $F_{te} = ma_v \frac{1}{(\sqrt{1-N_t^2})^3}$

The difference  $D_F$  between the force equations in the s-relativity and the Einstein relativity can be evaluated in percentage by the difference between  $F_{ts}$  and  $F_{te}$  relative to  $F_{te}$ :

$$D_F = \frac{F_s - F_e}{F_e} = \frac{F_s}{F_e} - 1 = 100 \left( \frac{1 + N_t^2}{1 - N_t^2} \sqrt{1 - N_t^2} - 1 \right) \%$$

TABLE 4 presents results of calculations of  $F_{ts}$  and  $F_{te}$ , and their relative difference  $D_F$  expressed in percentage. The factor  $ma$  is assigned as  $ma=1$ .

TABLE 4

N	$F_{ts}$	$F_{te}$	$D_F\%$
0	1	1	0
0.1	1.031	1.015	1.58
0.2	1.128	1.063	6.11
0.3	1.316	1.152	14.24
0.4	1.644	1.299	26.56
0.5	2.222	1.54	44.29
0.6	3.32	1.953	69.99
0.7	5.729	2.746	108.63
0.8	12.654	4.63	173.3
0.9	50.139	12.075	315.23
0.95	200.131	32.847	509.28
0.99	5000.126	356.222	1303.65
0.999	500000.1	11188.73	4368.78
0.9999	50000000	353579.9	14041.08

Fig.4 presents plots of  $F_{ts}$  and  $F_{te}$  in the  $N_t$  range 0 - 0.9, according to Table 4.

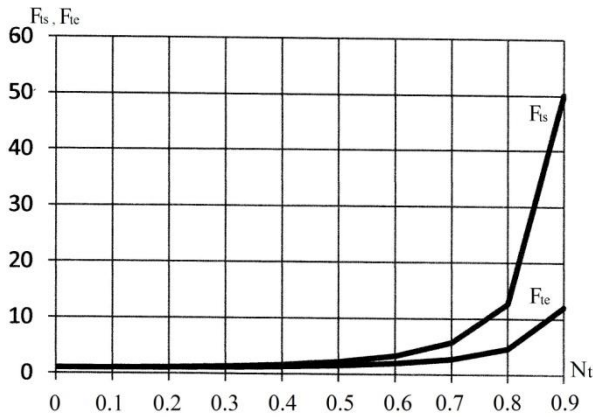


Fig.4

### C) Comparing of the relativistic accelerations in the s-relativity and the Einstein relativity

In order to make comparison of the relativistic speed accelerations, the instant acceleration for the s-relativity will be marked as  $a_{vts}$  and the acceleration for the Einstein relativity will be marked as  $a_{vte}$ . Then it will be:

the relativistic acceleration in the s-relativity is:

$$a_{vts} = \frac{F}{m} \frac{(1-N_t^2)^2}{1+N_t^2}$$

the relativistic acceleration in the Einstein relativity is:

$$a_{vte} = \frac{F}{m} \left( \sqrt{1-N_t^2} \right)^3$$

The difference  $D_a$  between the speed accelerations in the s-relativity and the Einstein relativity can be evaluated in percentage by the difference between  $a_{vts}$  and  $a_{vte}$  relative to  $a_{vte}$ :

$$D_a = \frac{a_{vts} - a_{vte}}{a_{vte}} = \frac{a_{vts}}{a_{vte}} - 1 = 100 \left( \frac{\sqrt{1-N_t^2}}{1+N_t^2} - 1 \right) \%$$

TABLE 5 presents results of calculations of  $a_{vts}$  and  $a_{vte}$ , and their relative difference  $D_a$  expressed in percentage. The factor  $\frac{F}{m}$  is assigned as  $\frac{F}{m} = 1$ .

TABLE 5

$N_t$	$a_{vts}$	$a_{vte}$	$D_a\%$
0	1	1	0
0.1	0.97	0.985	-1.52
0.2	0.886	0.941	-5.84
0.3	0.76	0.868	-12.44
0.4	0.608	0.77	-21.04
0.5	0.45	0.65	-30.77
0.6	0.301	0.512	-41.21
0.7	0.175	0.364	-51.92
0.8	0.079	0.216	-63.43
0.9	0.02	0.083	-75.9
0.99	0	0.003	-100

Fig.5 presents plots of  $a_{vts}$  and  $a_{vte}$  in the  $N_t$  range 0 - 0.99, according to Table 5.

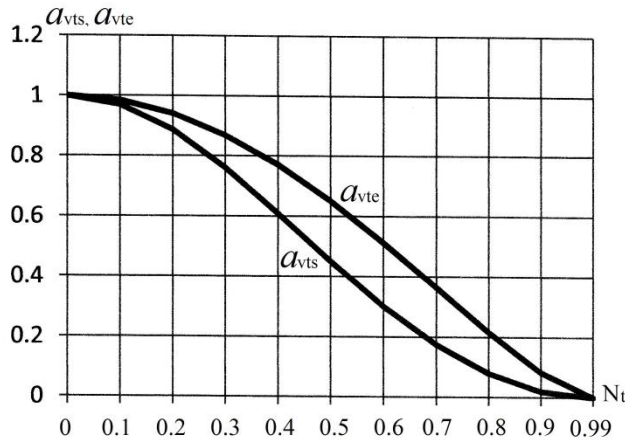


Fig.5

#### 4. Summation

The major physical mechanical equations of the s-relativity and their Einstein's relativistic counterparts are presented in a summarizing table. The speed variables in the equations are expressed in the normalized form  $N = \frac{v}{c}$  representing speed as fractions of speed of light.

The time dependant variables are subscript marked as (<sub>t</sub>).

#### Summarizing table

Description	S-relativity definition	Einstein relativity definition
Mechanical momentum	$\mathbf{p} = mc \frac{N}{1-N^2}$	$\mathbf{p} = mc \frac{N}{\sqrt{1-N^2}}$
Full energy of the moving body	$E = mc^2 \left[ \frac{1}{1-N^2} + \frac{1}{2} \ln(1 - N^2) \right]$	$E = \frac{mc^2}{\sqrt{1-N^2}}$
Energy of the body at rest	$E_r = mc^2$	$E_r = mc^2$
Kinetic energy of the moving body	$E_k = mc^2 \left[ \frac{N^2}{1-N^2} + \frac{1}{2} \ln(1 - N^2) \right]$	$E_k = mc^2 \left( \frac{1}{\sqrt{1-N^2}} - 1 \right)$
External force required to maintain constant acceleration of the moving body	$F_t = ma_v \frac{1+N_t^2}{(1-N_t^2)^2}$	$F_t = ma_v \frac{1}{\left( \sqrt{1-N_t^2} \right)^3}$
Speed acceleration of the moving body under applied constant force	$a_{vt} = \frac{F}{m} \frac{(1-N_t^2)^2}{1+N_t^2}$	$a_{vt} = \frac{F}{m} \left( \sqrt{1 - N_t^2} \right)^3$

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