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VARIATIONS OF THE RELATIVISTIC TRANSFORMATION - 2

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This is a corrected version of the previous article of the same title with clarifications to some statements and reformatting of the equations presentation.

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VARIATIONS OF THE RELATIVISTIC TRANSFORMATION

ABSTRACT
In this research the transformation equations for inertial systems carrying propagation of light are derived in various configurations. The derived equations are different from the currently known ones. The transformation equations for inertial systems carrying objects traveling at any of physically achievable speeds are newly developed and evaluated. This research concludes that the Lorentz transformation is applicable for the inertial systems carrying only spherical propagation of light. The relativistic factor ϒ is found to be not necessary for the transformation, and its validity is compromised, since in some cases it causes violation of conservation of constant velocity of light. The derived transformation equations can be viable foundation for special relativity without specific relativistic **ϒ** factor.

Keywords: Inertial systems; Coordinates transformation; Directing equations; Transformation factor; Direct coordinates ratio; Lorentz Relativistic Factor..

1. Basic derivation

The Principle of Special Relativity stipulates, that all inertial systems, which move with constant linear velocity relative to each other, are equal in ranks over any performances, and all laws of nature are valid within such systems and upon their interaction.

As long as inertial systems do not interact, their internal performances are independent within their frame of reference. But if such systems carrying inner moving objects become connected, either materially or informational, then certain spatial and timing transformation procedure has to be applied in order to reconcile mutual appearance of local conditions to both systems.

There are two known transformation procedures: the Galilean Transformation and the Lorentz Transformation. These transformations define interrelation of time and spatial coordinates between inertial reference frames as it appears to each other.

The following research is a mathematical analysis of spatial and timing relationship between uniformly moving inertial systems with axiomatic consideration of absolute invariance of speed of light.

Consider two inertial reference frames (Fig.1): system **K**, carrying space-time coordinates (**X, Y, Z, t**) originated at zero point **O**, and system **K’,** carrying coordinates (**X’, Y’, Z’, t’**) originated at point **O’**. Both systems arranged in such a way that abscissa **X’** of system **K’** coincides with abscissa **X** of system **K** (the **y** offset of **X’** from **X** on Fig.1 is shown for illustrative purpose). It’s been conditionally assumed that system **K** is stationary and system **K’** is moving with velocity **v** along positive direction of axis **X**. The moment when the origin **O’** coincides with the origin **O** is considered as time zero point (**t=t’=0**), and at this moment a light pulse is generated in system **K’** from the origin **O’** along positive direction of axis **X’**. Upon elapsing of local time **t’** the light pulse will reach point **B’** in moving system **K’**, traveling distance **x’=O’B’,** and the same pulse, viewed from stationary system **K**, will reach point **B**, coinciding with B’, traveling distance **x=OB**. At the same period of time system **K'** will advance along axis **X** for distance **vt**. It is clearly seen from Fig. 1 the difference in length of tracks **OB** and **O'B'**: **OB>O'B'**. Since velocity of light is considered to be fundamentally the same in all inertial systems, this difference in length has to be attributed to difference in rating of time **t'** in system **K'** relative to time **t** in system **K** from prospective of stationary system **K**. Therefore, different timing rates: **t'** for system **K'** and **t** for system **K** are to be assigned.

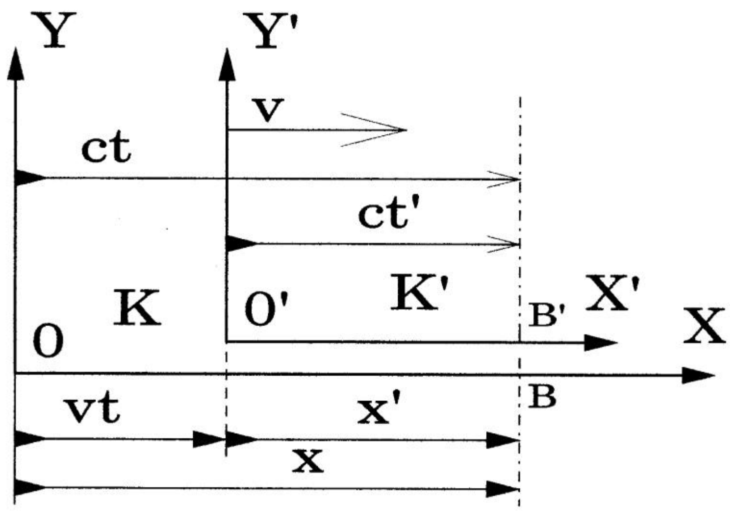

Fig. 1

Then the distances **x** and **x',** which are traveled by the same light pulse in systems **K** and **K',** will be:

$$x'=ct' \qquad (1\text{-}1)$$

$$x=ct \qquad (1\text{-}2)$$

Expressions (1-1) and (1-2) are the major directing equations establishing the inertial systems' coordinate relationship with regards to the light propagation within the moving system. These equations assert the elementary transformational pattern of propagation of light in the moving system relative to the stationary system.

As it is seen from Fig. 1 the relationship between coordinates of the moving light pulse in systems **K'** and **K** is:

$$x'= x - vt \qquad (1\text{-}3)$$

Making substitution of **x'** and **x** from (1-1) and (1-2) into (1-3) gives:

$$ct'= ct - vt \qquad (1\text{-}4)$$

Equation (1-4) can be rearranged:

$$t' = t - vt/c \qquad (1\text{-}5)$$

Equation (1-5) shows time relationship between two inertial systems. It shows that there is a time communication link **vt/c** between the systems. The term **vt** is the distance between the stationary origin **O** and the moving origin **O'**, so the **vt/c** is the time required for communication between these systems at speed of light.

In order to make the communication link referenced to abscissa of stationary system **K,** make substitution of **t** from equation (1-2) (as **t=x/c)** in the item **vt/c** of equation (1-5):

$$t' = t - vx/c^2 \qquad (1\text{-}6)$$

Expression (1-6) represents timing relationship between two inertial frames where the timing is referenced to the position of the light pulse relative to abscissa of system **K**.
Equations (1-3) and (1-6) describe transformational relationship of spatial and timing coordinates between systems **K'** and **K** related to propagation of the same light pulse along **X** axis. These equations are sufficient for cross-transformation of the reference coordinates between these inertial systems. It may be reasonably assumed, that since there is no activities perpendicular to the relative **X** motion of the systems then **y'=y**, and **z'=z**.

It can be summarized: the equations required and sufficient for the space-time coordinates transformation of the light propagation within the inertial system moving in **X** direction along the stationary system, are:

$$\begin{aligned} x' &= x - vt \\ y' &= y \qquad (1\text{-}7) \\ z' &= z \\ t' &= t - vx/c^2 \end{aligned}$$

For further discussion the equations (1-7) and their derivatives will be referred as s-transformation.
Inverting the moving and stationary systems in the s-transformation can be achieved by reversing direction of velocity **v** (**v→-v**) in formulas (1-7):

$$\begin{aligned} x &= x' + vt' \\ y &= y' \qquad (1\text{-}8) \\ z &= z' \\ t &= t' + vx'/c^2 \end{aligned}$$

The s-transformation equations (1-7) are different from the Galilean transformation only by the item $\mathbf{vx/c^2}$ in the time transformation equation. It is historically understandable, since at the Galilean times he was familiar with mechanical arrangements of the things, but completely unaware of the properties of light. He intuitively assumed that the speed of light is infinitely high and all communications applying light occur instantaneously. Therefore, if in the time equation of the set (1-7) assume that the speed of light is infinite then $\mathbf{vx/c^2=0}$, and equations (1-7) will turn into the original Galilean transformation equations. The s-transformation equations (1-7) are also different from the Lorentz transformation by absence of the relativistic factor $\Upsilon = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. The relativistic factor did not naturally appear during the performed inertial systems analysis which produced the s-transformation equations. More instances of derivation of the transformation equations are presented in the ADDENDUM.

In order to verify validity of the derived s-transformation equations (1-7) the cross-system transformation is exercised next.

As stated above, the system **K** is stationary and system **K'** moves along **X** direction with speed **v**. When the origins **O'** and **O** in systems **K'** and **K** coincide at time **t=t'=0** an isotropic light pulse is generated from **O'**, forming spreading spherical front in system **K'** originated at **x'=0**, **y'=0**, **z'=0**. Since the speed of light is the same in both systems, the spherical front of the light pulse must satisfy the equations for both systems: **K'** and **K**:

$$x'^2+y'^2+z'^2=(ct')^2 \qquad (1\text{-}9)$$

$$x^2+y^2+z^2=(ct)^2 \qquad (1\text{-}10)$$

In order to do transformation of coordinates of the light front in the moving frame to coordinates of the stationary frame substitute **x'** and **t'** from equations (1-7) into equation (1-9):

$$(x-vt)^2+y^2+z^2=c^2(t-vx/c^2)^2 \qquad (1\text{-}10a)$$

Expand (1-10a):

$$x^2-2xvt+v^2t^2+y^2+z^2=c^2t^2-2xvt+v^2x^2/c^2 \qquad (1\text{-}10b)$$

Substitute **x=ct** from (1-2) into $\mathbf{v^2x^2/c^2}$ of (1-10b), then the equation (1-10b) will be:

$$x^2-2xvt+v^2t^2+y^2+z^2=c^2t^2-2xvt+v^2t^2 \qquad (1\text{-}10c)$$

Canceling items $\mathbf{-2xvt+v^2t^2}$ in both parts of the equation (1-10c) gives:

$$x^2+y^2+z^2=c^2t^2 \qquad (1\text{-}10d)$$

Equation (1-10d) is identical to the equation (1-10) of the light propagation referenced to the stationary system.

The above performed example of s-transformation confirms that the equations (1-7) are sufficient for the inertial systems coordinates transformation related to propagation of light. It appears that no additional factors to the **x'** and **t'** equations of the s-transformation (1-7) are required.

2. Combining (adding) velocities

If there is an object traveling in the moving frame in **X'** direction at speed **u** relative to **X',** then its speed **w** relative to the stationary frame can be determined by taking derivatives of equations (1-8):

$$dx=dx'+vdt'$$

$$dt=dt'+vdx'/c^2$$

The derivative **dx/dt** will be:

$$\frac{dx}{dt}=\frac{\frac{dx'}{dt'}+v}{1+\frac{v\frac{dx'}{dt'}}{c^2}}$$

Note that **dx/dt=w** and **dx'/dt'=u**. Then an expression for combining (adding) velocities, as it is judged from the stationary system, will be:

$$w = \frac{u+v}{1+\frac{uv}{c^2}} \qquad (2\text{-}1)$$

For further discussion a concept of Normalized Velocity "**N**" will be introduced, which represents velocity of the moving object expressed as fraction of the velocity of light:

$$N_w=w/c; \quad N_v=v/c; \quad N_u=u/c \qquad (2\text{-}2)$$

Then formula (2-1) for combined velocities can be written in normalized form as follows:

$$N_w = \frac{N_u+N_v}{1+N_uN_v} \qquad (2\text{-}1a)$$

Using the same approach for the speed determination, the expression for combined speed of more than two objects can be derived. If system **A** moves with speed **v** relative to system **B**, and system **B** has speed **u** relative to system **C**, and system **C** has speed **w** relative to stationary system **D**, then the speed **q** of system **A** relative to stationary system **D** will be:

$$q = \frac{v+u+w+\frac{vuw}{c^2}}{1+\frac{uv}{c^2}+\frac{vw}{c^2}+\frac{uw}{c^2}} \qquad (2\text{-}3)$$

Or in normalized form formula (2-3) will be:

$$N_q = \frac{N_v+N_u+N_w+N_vN_uN_w}{1+N_vN_u+N_vN_w+N_uN_w} \qquad (2\text{-}3a)$$

As it follows from expressions (2-1, 2-1a, 2-3, 2-3a) any combinations of the added speeds in the range between **0** and the speed of light **c** cannot exceed the speed of light.

3. Direct coordinates correlation

The spatial and timing coordinates of the object traveling in the moving system can be transformed to the coordinate frame of the stationary system by direct coordinates correlation. If the traveling object is a light pulse then the fundamental physical property of speed of light will be the governing factor for transformation. For the inertial systems carrying light the transformation is govern by the directing equations (1-1) and (1-2).

The direct coordinates correlation can be determined from equations of the s-transformation.
From equation (1-2) substitute **t** as **t=x/c** into the first equation of (1-7), and substitute **x** as **x=ct** into the forth equation of (1-7):

$$x' = x - vx/c = x(1-v/c) \qquad (3\text{-}1)$$

$$t' = t - vct/c^2 = t(1-v/c) \qquad (3\text{-}1a)$$

Expressions (3-1) and (3-1a) are the s-transformation equations in the direct coordinates correlation form. This transformation relates to the particular direction of movement of the moving system relative to the stationary system.. Therefore, the parameters related to the moving items should carry the index indicating the direction of movement. Since the preceding analysis was performed exclusively in relation to the moving activity along **X** axis, the **X** direction should be marked as a subscript note to the items moving in this direction.

For the system moving along **X** coordinate such marking will be:
$v \to v_x$; $t' \to t'_x$; $v_x/c \to N_x$.
Then the s-transformation expressions (3-1) and (3-1a) will be:

$$x' = x(1 - v_x/c) \qquad (3\text{-}2)$$

$$t'_x = t(1 - v_x/c) \qquad (3\text{-}2a)$$

Or in normalized form equations (3-2) and (3-2a) will be:

$$x' = x(1 - N_x) \qquad (3\text{-}3)$$

$$t'_x = t(1 - N_x) \qquad (3\text{-}3a)$$

In the stationary system the time **t** is isotropic, therefore no need to reference it to coordinates. Timing in the stationary reference frame is always $\mathbf{t_x = t_y = t_z = t}$. The time distribution in the stationary frame is $\mathbf{t_x^2 + t_y^2 + t_z^2 = t^2}$.

In (3-2), (3-2a) and (3-3), (3-3a) the equations for **x'** and t'_x have identical transformation factors, so in the case of light propagation they can be combined into one unified expression, which further will be referred as the direct correlation factor $\boldsymbol{\eta_x}$ related to axis **X:**

$$\eta_x = x'/x = t'_x/t = (1 - v_x/c) = (1 - N_x) \qquad (3\text{-}4)$$

For the **Y** and **Z** directions the equations for the direct correlation factor will be similar:

$$\eta_y = y'/y = t'_y/t = (1 - v_y/c) = (1 - N_y) \qquad (3\text{-}4a)$$

$$\eta_z = z'/z = t'_z/t = (1 - v_z/c) = (1 - N_z) \qquad (3\text{-}4b)$$

The direct correlation factor $\boldsymbol{\eta}$ renders static form of the dynamic equations, expressing moving activities of the inertial systems.

It can be summarized:
For two inertial systems where one system, carrying propagation of light, moves along **X** axis of the stationary system with relative speed $\mathbf{v_x}$ the ratio of coordinates **x'/x** of the moving light pulse and ratio of time durations t'_x/t, as they appear to stationary system, are both the same and equal $\mathbf{(1 - v_x/c)}$**.**

As it follows from (3-4) the range of variation of $\boldsymbol{\eta_x}$, depending on $\mathbf{N_x}$, is:

when $\quad -1 \le N_x \le 1$

then $\quad 2 \ge \eta_x \ge 0 \qquad (3\text{-}5)$

The following is physical interpretation of conditional equations (3-5):
As it follows from Fig. 1 the combined speeds of the moving system and the light beam measured within the moving system must always appear to the stationary system as speed of light **c:**

$$v_x t + ct' = ct$$

or it can be rearranged: $\quad v_x + \frac{t'}{t} c = c$

Substitute $\frac{t'}{t}$ as η_x from (3-4): $\quad v_x + \eta_x c = c \qquad (3\text{-}5a)$

In the right part of equation (3-5a) is the speed of light **c** measured in the stationary system; and in the left part of the equation is the speed of light **c** measured in the moving system. In this framework the direct correlation factor η_x signifies the timing dilation in the moving system as it appears to the stationary system.

In accordance with (3-5a):

if $v_x=c$ ($N_x=1$), then η_x has to be 0 in order to conserve speed of light in the stationary system;
if $v_x=-c$ ($N_x=-1$), then η_x has to be 2 for the same reason.

η_x cannot be negative.
η_x cannot be $\eta_x>2$, otherwise it would've meant that speed of light measured in the stationary system is higher than speed of light measured in the moving system, but this would've been violation of absolute invariance of speed of light.
This interpretation explains meaning of conditional equations (3-5).

4. Assessment and comparing of the Lorentz transformation

The Lorentz equations for transformation of **x-t** inertial coordinates are:

$$x' = \Upsilon (x-v_x t)$$
$$t' = \Upsilon (t-v_x x/c^2)$$
$$\Upsilon=\frac{1}{\sqrt{1-\frac{v_x^2}{c^2}}}$$

The Lorentz transformation has the same functional part as the s-transformation, but differs by incorporating of the relativistic factor $\Upsilon=\frac{1}{\sqrt{1-\frac{v_x^2}{c^2}}}$.

For further evaluation of the Lorentz transformation equations they are converted to the direct coordinates correlation form by using substitutions **t=x/t** and **x=ct** from the directing equation (1-2):

$$x' = (x-v_x t)\ \Upsilon=(x-v_x x/c)/\sqrt{1-\frac{v_x^2}{c^2}} = x(1-v_x/c)/\sqrt{1-\frac{v_x^2}{c^2}} = x\sqrt{\frac{1-\frac{v_x}{c}}{1+\frac{v_x}{c}}}$$

$$t' = (t-v_x x/c^2)\ \Upsilon= (t-v_x ct/c^2)/\sqrt{1-\frac{v_x^2}{c^2}} = t(1-v_x/c)/\sqrt{1-\frac{v_x^2}{c^2}} = t\sqrt{\frac{1-\frac{v_x}{c}}{1+\frac{v_x}{c}}}$$

Combining the above equations to form unified coordinates correlation factor for the Lorentz transformation gives:

$$\eta_x[L]=x'/x=t'/t=\sqrt{\frac{1-\frac{v_x}{c}}{1+\frac{v_x}{c}}} \qquad (4\text{-}1)$$

or in normalized form:

$$\eta_x[L]=x'/x=t'/t=\sqrt{\frac{1-N_x}{1+N_x}} \qquad (4\text{-}1a)$$

As it follows from equations (4-1) and (4-1a) the Lorentz transformation is sensitive to the direction of the

moving system's velocity $\mathbf{v_x}$, which affects variations of the unified correlation factor.
The range of $\boldsymbol{\eta_x}$**[L]** as function of variation of $\pm\mathbf{N_x}$ is:

$$\text{when} \quad -1 \le N_x \le 1,$$
$$\text{then} \quad \infty \ge \eta_x[L] \ge 0 \qquad (4\text{-}2)$$

According to (4-2) the high limit for the direct correlation factor $\boldsymbol{\eta_x}$**[l]** corresponding to the extreme negative speed (N_x=-1) is infinity. But, as it was explained in section 3 in the interpretation to similar formula (3-5), the highest limit for the correlation factor at the normalized speed $\mathbf{N_x=-1}$ cannot exceed 2 ($\mathbf{2\ge\eta_x}$) in order to comply with the requirement for fundamental constancy of the speed of light in the stationary and moving systems. Not complying with this requirement indicates misconception in the Lorentz transformation equations, particularly applying the relativistic factor **ϒ**, which causes this erroneous outcome.

In order to evaluate difference between the Lorentz and the s-transformation a comparison of the coordinates correlation factors $\boldsymbol{\eta_x}$ at normalized speeds in the range $\mathbf{-0.9\le N_x\le 0.9}$ is performed. Table 1 shows results of calculations of $\boldsymbol{\eta_x}$ for the Lorentz transformation (**L**) and for the s-transformation (**S**), and the relative differences **E=(L-S)/S** between the Lorentz transformation and the s-transformation in absolute units and in percentage.

TABLE 1

N	S	L	E	E%
-0.9	1.9	4.359	1.294	129.4
-0.8	1.8	3	0.667	66.7
-0.7	1.7	2.38	0.4	40
-0.6	1.6	2	0.25	25
-0.5	1.5	1.732	0.155	15.5
-0.4	1.4	1.528	0.091	9.1
-0.3	1.3	1.363	0.048	4.8
-0.2	1.2	1.225	0.021	2.1
-0.1	1.1	1.106	0.005	0.5
0	1	1	0	0
0.1	0.9	0.905	0.006	0.6
0.2	0.8	0.816	0.02	2
0.3	0.7	0.734	0.049	4.9
0.4	0.6	0.655	0.092	9.2
0.5	0.5	0.577	0.154	15.4
0.6	0.4	0.5	0.25	25
0.7	0.3	0.42	0.4	40
0.8	0.2	0.333	0.665	66.5
0.9	0.1	0.229	1.29	129

Fig. 2 presents plots according to Table1.
As seen from the plots of Fig. 2, the s-transformation plot is linear in all range $\mathbf{-0.9\le N_x\le 0.9}$, but the Lorentz transformation graph is non-linear in this range and shows the higher deviation from linearity at higher values of $\mathbf{N_x}$.
Three values of deviation **E** at particular values of $\mathbf{N_x}$ are shown below for illustration:
At N_x=±0.1 (v_x=±30,000km/sec) – E=0.5%;
At N_x=±0.2 (v_x=±60,000km/sec) – E=2%;
At N_x=±0.9 (v_x=±270,000km/sec) – E=129%.

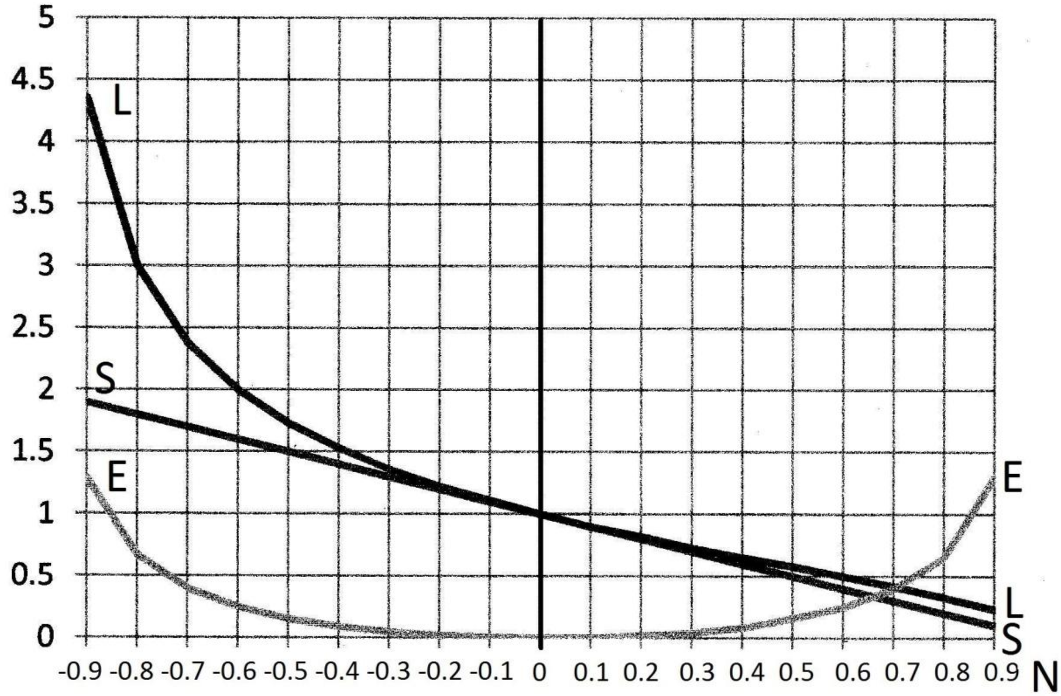

Fig. 2

5. Derivation and evaluation of the Lorentz relativistic factor

In the course of current research number of transformation procedures on the inertial systems carrying propagation of light were performed, and none of the transformations required the Lorentz relativistic factor ϒ. Consequently it was concluded that the Lorentz relativistic factor was not necessary for the inertial systems' transformation. Even more, in some cases the Lorentz relativistic factor causes violation of conservation of the speed of light. Still, the relativistic factor is commonly accepted as fundamental parameter in the Special Theory of Relativity and in other fields of physics, dealing with the relativistic conditions.
In order to reconcile such contradiction it would be instrumental to define and understand the origin of the derivation of the Lorentz relativistic factor.

Derivation of the Lorentz transformation is associated with spherical propagation of light pulse in the moving inertial system. The equation for such event in the moving system is:

$$x'^2+y'^2+z'^2=(ct')^2 \qquad (5\text{-}1)$$

Since the speed of light is considered to be the same in the moving and stationary systems the light pulse front has to satisfy similar equation in the stationary system as well:

$$x^2+y^2+z^2=(ct)^2 \qquad (5\text{-}1a)$$

Applying the s-transformation for equation (5-1) gives:

$$(x-vt)^2+y^2+z^2=c^2(t-vx/c^2)^2 \qquad (5\text{-}2)$$

Expand it:

$$x^2-2xvt+v^2t^2+y^2+z^2=c^2t^2-2xvt+v^2x^2/c^2 \qquad (5\text{-}2a)$$

Cancelling identical items **2xvt** in both parts of the equation (5-2a) gives:

$$x^2+v^2t^2+y^2+z^2=c^2t^2+v^2x^2/c^2 \qquad (5\text{-}3)$$

Rearrange equation (5-3):

$$x^2(1-v^2/c^2)+y^2+z^2=c^2t^2(1-v^2/c^2) \qquad (5\text{-}3a)$$

In equation (5-3a) the unwanted terms **$(1-v^2/c^2)$** are attached to items **x^2** and **c^2t^2**, which makes equation (5-3a) different from the expected equation (5-1a). These unwanted terms are identical. It happens because the entity, which travels within the moving system, is traveling with speed of light, otherwise these terms would not be identical [as it is concluded in the Section 6, expressions (6-12), (6-13), (6-14). The unwanted terms are supposed to be eliminated, and it can be done by multiplying of both parts of equation (5-3a) by the compensating inverse term **$1/(1-v^2/c^2)$**, but this operation will affect coordinates **y** and **z**. This problem is technically circumvented by attaching square root of the inversed unwanted term as a factor to the transformation equations for **x'** and **t'.** This compensating term is known as the Lorentz relativistic factor **$\Upsilon=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$**.

The Lorentz relativistic factor is not universal: it derived and worked only for the moving inertial systems carrying specifically spherical propagation of light. In order to test such limitation consider the coordinates transformation for the moving system carrying propagation of not spherical, but linear light front. The general form of equation for such event in the moving system is:

$$x'+y'+z'=ct' \qquad (5\text{-}4)$$

Transformation of the linear light front (5-4) is supposed to produce in the stationary system equation:

$$x+y+z=ct \qquad (5\text{-}4a)$$

Applying s-transformation equations to (5-4) gives:

$$(x-vt)+y+z=c(t-xv/c^2) \qquad (5\text{-}5)$$

Rearrange equation (5-5):

$$x(1+v/c)+y+z=ct(1+v/c) \qquad (5\text{-}6)$$

Equation (5-6) contains common unwanted term **(1+v/c)**, attached to the items **x** and **ct**. The Lorentz relativistic factor **ϒ** cannot resolve the problem for this linear equation. In this case the compensating factor to be attached to the **x'** and **t'** equations of the Lorentz transformation should be **1/(1+v/c)**, which is specifically pertinent to this type of equation. Therefore, it can be concluded that introducing of the relativistic factor **ϒ**, as it is done for the Lorentz transformation, is a limited way to resolve the transformation procedure for the systems, carrying only isotropic spherical propagation of light. If the derivation of the Lorentz transformation were based on the linear propagation of the light front, then the relativistic factor **ϒ** would have been **1/(1+v/c)**.

It may be noted that applying substitution of the directing equation **x=ct** from (1-2) into **xv/c^2** of the equation (5-5) properly completes the s-transformation for the linear light front without the **ϒ** factor:

$$x+y+z=ct$$

The conclusion is: due to limited and specific character of the relativistic factor **ϒ** its consideration as the fundamental universal space-time relativistic characteristic is questionable.

The s-transformation can be valid foundation for the special relativity version without incorporation of the relativistic factor.

6. Generalized sg-transformation for the arbitrary speeds

All previous discussions were related to the inertial systems, where the moving system carried propagation of light. The unique feature of such arrangement is that the object traveling in the moving system is fundamentally invariable entity moving with absolute constant velocity **c**. This feature is the foundation of using the directing equations (1-1) and (1-2) as they stated in Section 1. However, in case if the objects travel in the moving system at arbitrary speeds, the directing equations (1-1) and (1-2) become invalid, and another directing equations containing speed of the object traveling within the moving system have to be applied.

Returning back to Fig. 1 consider that in the moving system a material object travels along axis **X'** at speed $\mathbf{u_x}$, instead of the light pulse**.** In this case the speed $\mathbf{u_x}$ will appear to the stationary system as combined speed $\mathbf{u_x}$ and $\mathbf{v_x}$. According to formula (2-1) such combined speed will be $\mathbf{(u_x+v_x)/(1+u_xv_x/c^2)}$. Thus the coordinates **x** and **x'** will be defined as:

$$x' = t'_x u_x \qquad (6\text{-}1)$$

$$x = t\frac{u_x+v_x}{1+\frac{u_x v_x}{c^2}} \qquad (6\text{-}2)$$

Equations (6-1) and (6-2) are the directing equations for the generalized case of s-transformation, further referred as sg-transformation, and they cover all range of the physically achievable relative velocities of the inertial systems and the objects traveling within the moving system.

Returning to Fig. 1, distance **x'** traveled by the object in system K' will be:

$$x' = x - v_x t \qquad (6\text{-}3)$$

Substitute **x** and x' from (6-1) and (6-2) into (6-3):

$$u_x t'_x = t\frac{u_x+v_x}{1+\frac{u_x v_x}{c^2}} - v_x t \qquad (6\text{-}4)$$

Expanding and rearranging equation (6-4) gives direct correlation timing transformation equation:

$$t'_x = t\frac{1-\frac{v_x^2}{c^2}}{1+\frac{u_x v_x}{c^2}} \qquad (6\text{-}5)$$

For determining the direct transformation equation for X coordinates rewrite equation (6-3):

$$x' = x(1 - v_x t/x) \qquad (6\text{-}6)$$

Substitute **x** in brackets of equation (6-6) from (6-2):

$$x' = x\left(1 - v_x\frac{1+\frac{u_x v_x}{c^2}}{u_x+v_x}\right)$$

After expanding and rearranging it will be:

$$x'=x\frac{1-\frac{v_x^2}{c^2}}{1+\frac{v_x}{u_x}} \qquad (6\text{-}7)$$

Equations (6-7) and (6-5) constitute the sg-transformation in direct correlation form, where the moving system, traveling at speed $\mathbf{v_x}$, carries an object traveling at speed $\mathbf{u_x}$:

$$x'=x\frac{1-\frac{v_x^2}{c^2}}{1+\frac{v_x}{u_x}}$$

$$t'_x=t\frac{1-\frac{v_x^2}{c^2}}{1+\frac{u_x v_x}{c^2}} \qquad (6\text{-}8)$$

$$x=t\frac{u_x+v_x}{1+\frac{u_x v_x}{c^2}}$$ - the directing equation

The functional form of the timing equation can be defined by manipulation of the timing equation (6-5):

$$t'_x=t\frac{1-\frac{v_x^2}{c^2}}{1+\frac{u_x v_x}{c^2}}=t-\left(t-t\frac{c^2-v_x^2}{c^2+u_x v_x}\right)$$

Expanding and rearranging of the above equation gives:

$$t'_x=t-\frac{c^2t+u_xv_xt-c^2t+v_x^2t}{c^2+u_xv_x}=t-\frac{v_x}{c^2}\left(t\frac{u_x+v_x}{1+\frac{u_xv_x}{c^2}}\right)$$

The term in brackets in the above equation is the **x** directing equation (6-2). After substituting the directing equation in brackets as **x** the above expression for $\mathbf{t_x'}$ will be:

$$t'_x=t-\frac{xv_x}{c^2} \qquad (6\text{-}9)$$

Equations (6-3) and (6-9) constitute functional form of the sg-transformation equations. These equations coincide with equations of the s-transformation. The difference between the functional form of the s-transformation and sg-transformation is in structure of the directing equations.

The generalized sg-transformation in functional form will be:

$$x'=x-v_xt$$

$$t'_x=t-\frac{xv_x}{c^2} \qquad (6\text{-}10)$$

$$x=t\frac{u_x+v_x}{1+\frac{u_xv_x}{c^2}}$$ – the directing equation

The sg-transformation equations (6-8) and (6-10) are equally applicable to **Y** and **Z** coordinates by replacing of all **X** designations by the corresponding coordinates designations.

If $\mathbf{v_y=0}$ and $\mathbf{v_z=0}$, then: **y'=y**, **z'=z**, $\mathbf{t_y'=t}$, $\mathbf{t'_z=t}$.

The following is review of equations (6-8) at particular values of $\mathbf{u_x}$ and $\mathbf{v_s}$**:**

a) $v_x=c$ (the moving system travels at speed of light)

$x'=0$

$t'_x=0$

$x=tc$

b) $v_x=0$ (the moving system is motionless)

$$x'=x$$
$$t'_x=t$$
$$x=tu_x$$

c) $v_x<<c$ (the moving system moves very slow in comparison to speed of light)

$$x'=x\frac{u_x}{u_x+v_x}$$
$$t'_x=t$$
$$x=t(u_x+v_x)$$

d) $u_x=c$ (moving system carries propagation of light pulse):

$$x'=x(1-v_x/c)$$
$$t'_x=t((1-v_x/c)$$
$$x=tc$$

e) $u_x=0$ (no X activity in the moving system):

$$x'=0$$
$$t'_x=t(1-v_x^2\,/c^2)$$
$$x=t_xv_x$$

Here are comments on the above review:

a) If, hypothetically, the moving system moves at speed of light, then all coordinates and activities in the moving system have to appear to the stationary system as zeroes in order to maintain combined speed of all involved entities not exceeding velocity of light. The directing equation is defined by the velocity of light.

b) If the moving system is motionless relative to the stationary system, then all coordinates in the "moving" and stationary systems are identical, and the directing equation is defined by the velocity of the object within the moving system.

c) In this case the **x'** equation can be rewritten as $\frac{\mathbf{x'}}{\mathbf{u_x}}=\frac{\mathbf{x}}{\mathbf{u_x+v_x}}$. Both parts of this equation represent timing: **t'=t**, which characterizes the non-relativistic condition. Thus, if the moving system is at very slow motion comparative to speed of light, then the relativistic effects become extremely weak and the coordinates relationship approximates Galilean non-relativistic correlation.

The directing equation is defined by the sum of velocities of the moving system and the object within the moving system.

d) When the travelling object in the moving system is a light pulse, then the sg-transformation turns to s-transformation equations (3-2), (3-2a). Therefore, the s-transformation is a limited case of the sg-transformation when $\mathbf{u_x=c}$, and the directing equation is defined by the velocity of light.

e) When no objects travel in the moving system then coordinate **x’** stays at **0** and the directing equation points to **x** position of the origin **O’** relative to the stationary system.

This case is special and needs to be evaluated specially.

The timing equation in this case represents timing relationship of two moving bodies **K** and **K'**. Since neither of the bodies carries any moving entities, there is no basis for distinction of the stationary and the moving system, which makes the bodies absolutely equal. This feature allows direct evaluating of relationship of such moving bodies without resorting to additional reference frame. In this case equation $\mathbf{x=t_x v_x}$ expresses the current distance between two moving bodies on the same path. The timing equation affirms that the internal timing rate within each uniformly moving body, as it appears to another body, depends on their relative velocities and is always contracted by factor $\boldsymbol{\eta_x=1-v_x^2/c^2}$. This timing state does not depend on direction of the bodies' movement as long as they are on the same path. It means:

$$t'_x=t(1-v_x^2/c^2) \qquad (6\text{-}11)$$
$$t=t'_x(1-v_x^2/c^2)$$

Equations (6-11) express natural ultimate limitation of two bodies' relative speed**:** the velocity $\mathbf{v_x}$ cannot exceed speed of light **c**, otherwise the relative timing rate in each body becomes negative, which is physically unacceptable.

Relative contraction of the timing rate entails relative contraction of the length in this direction, since length is defined by the timing required by light to pass through this length.

There is a special point in usage of the sg-transformation: $\mathbf{u_x=-v_x}$. At this point the traveling object in the moving system "freezes" at position **x=0**, and denominator in the **x'** equation (6-8) becomes 0. As a result the **x'** equation becomes undetermined of the **0/0** type. In this case the functional type of the **x'** equation (6-10) should be used with assigning **x=0**. Then the result for the **x'** will be: $\mathbf{x'=-v_x t}$.

For the sg-transformation the direct coordinate correlation factor **η** will be determined from equations (6-8).

The direct correlation factor $\boldsymbol{\eta_x}$ for the **X** coordinate is:

$$\eta_x=x'/x=\frac{1-\frac{v_x^2}{c^2}}{1+\frac{v_x}{u_x}} \qquad (6\text{-}12)$$

The direct correlation factor $\boldsymbol{\eta_{tx}}$ for the timing along X coordinate is:

$$\eta_{tx}=t'_x/t=\frac{1-\frac{v_x^2}{c^2}}{1+\frac{u_x v_x}{c^2}} \qquad (6\text{-}13)$$

Equations (6-12) and (6-13) in normalized form will be:

$$\eta_x=x'/x=\frac{1-N_{vx}^2}{1+\frac{N_{vx}}{N_{ux}}} \qquad (6\text{-}12a)$$

$$\eta_{tx}=t_x'/t=\frac{1-N_{vx}^2}{1+N_{vx}N_{ux}} \qquad (6\text{-}13a)$$

As it follows from expressions (6-12) and (6-13) the direct correlation factors $\boldsymbol{\eta_x}$ are different for the spatial and timing transformation equations at arbitrary speed of the object traveling within the moving system. The only point where the direct correlation factors are merged together is when the moving system carries light. At this point $\mathbf{u_x=c}$, and equations (6-12) and (6-13) become identical, turning into the unified s-transformation equation (3-3):

$$\text{when } u_x=c \ (N_{ux}=1), \ \text{then: } \eta_x=\eta_{tx}=1-v_x/c=1-N_{vx} \qquad (6\text{-}14)$$

Expressions (6-12), (6-13) and (6-14) reveal the basis of why the Lorentz relativistic factor is the same for the spatial and timing coordinates: because the Lorentz transformation handles exclusively propagation of light in the moving system. If the object within the moving system travels with speed other than speed of light, then, according to (6-12) and (6-13), the Lorentz relativistic factors for the spatial and timing equations have to be different.

Table 2 presents results of calculation of η_x and η_{tx} as a function of speed of the object traveling within the moving system. The speed of the moving system is arbitrary chosen at **60%** of speed of light.

TABLE 2

$N_u=v_x/c$	η_x	η_{tx}
0	0	0.64
0.1	0.091	0.604
0.2	0.16	0.571
0.3	0.213	0.542
0.4	0.256	0.516
0.5	0.291	0.492
0.6	0.32	0.471
0.7	0.345	0.451
0.8	0.366	0.432
0.9	0.384	0.416
1	0.4	0.4

$N_{vx}=0.6$

Fig. 3 displays plots according to Table 2, showing relationship of the spatial and timing correlation factors depending on the speed $\mathbf{u_x}$ of the object traveling within the moving system. The spatial and timing correlation factors are merging together only when the moving system carries propagation of light: $\mathbf{N_{ux}=1}$.

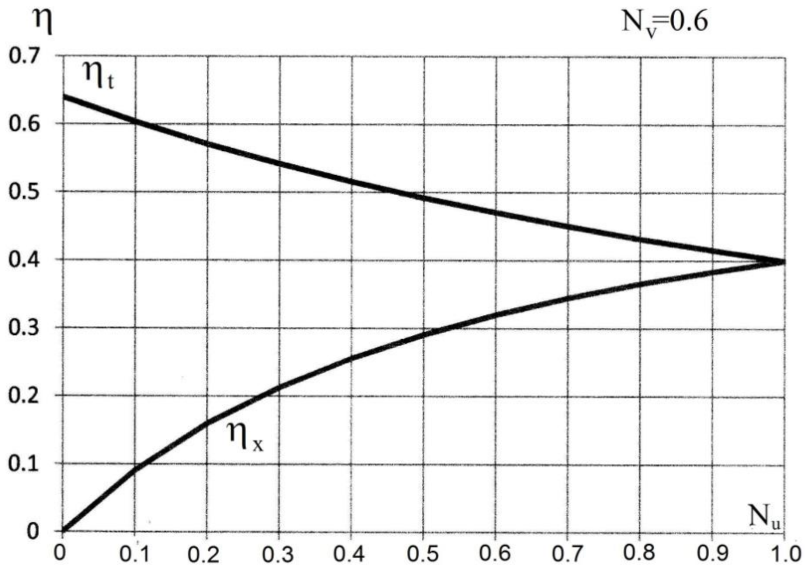

Fig. 3

7. Summary

-The generalized sg-transformation establishes spatial and timing relationship between inertial systems carrying objects moving at any physically achievable speeds (page 12).

-The generalized sg-transformation in functional form, where the moving system moves at speed $\mathbf{v_x}$ relatively to the stationary system and carries an object traveling at arbitrary speed $\mathbf{u_x}$ relative to the moving reference frame, is (page 13):

$$x' = x - v_x t$$

$$t'_x = t - \frac{x v_x}{c^2}$$

$$x = t\frac{u_x + v_x}{1 + \frac{u_x v_x}{c^2}} \quad \text{– the directing equation}$$

The **X** transformation equations are equally applicable to **Y** and **Z** coordinates by replacing of all **x** designations by the corresponding coordinate designation.

If $\mathbf{v_y=0}$ and $\mathbf{v_z=0}$, then **y'=y** and **z'=z**.

- The generalized sg-transformation in direct coordinates correlation form is (page 13):

$$x'=x\frac{1-\frac{v_x^2}{c^2}}{1+\frac{v_x}{u_x}}$$

$$t'_x=t\frac{1-\frac{v_x^2}{c^2}}{1+\frac{u_x v_x}{c^2}}$$

$$x=t\frac{u_x+v_x}{1+\frac{u_x v_x}{c^2}}$$ - the directing equation

- The s-transformation is a limited case of the sg-transformation where the object traveling within the moving system moves at speed of light. Assigning velocity $\mathbf{u_x}$ as the speed of light $\mathbf{c}$ ($\mathbf{u_x=c}$) turns equations of the sg-transformation into the s-transformation equations (page 4):

$$x'=x-v_x t_x$$

$$t'_x=t-xv_x/c^2$$

$$x=tc$$ - the directing equation

- The s-transformation in direct coordinates correlation form is (page 6):

$$x'=x(1-v_x/c)$$

$$t'_x=t(1-v_x/c)$$

$$x=tc$$ - the directing equation

- In two inertial systems moving at speed $\mathbf{v_x}$ relative to each other the timing and the length in each system will appear contracted at factor $\boldsymbol{\eta_x=1-\frac{v_x^2}{c^2}}$ in the direction of their movement. This factor expresses natural ultimate limitation on the bodies' relative speed**:** the velocity $\mathbf{v_x}$ cannot exceed speed of light $\mathbf{c}$, otherwise the relative timing rate and lengths in each body will appear negative, which is physically unacceptable (page 15).

-The relativistic factor **ϒ** of the Lorentz transformation appears to be unnecessary for the inertial systems transformation, and when it is applied its applicability is limited only to the cases, where the moving system carries spherical propagation of isotropic light (page 11).

- The Lorentz relativistic factor ϒ in some cases brings the results incompatible with fundamental property of invariance of speed of light, therefore validity of the relativistic factor is compromised (page 9).

- The s-transformation can be viable foundation for the version of special relativity without incorporating of the relativistic factor (page 12).

Various versions of derivation of the relativistic transformation

1. Relativistic derivation from the Galilean transformation equations

Consider two inertial systems: stationary system **K** with spatial and timing coordinates (**X,Y,Z,t**), originated at point **O**, and system **K'** with spatial and timing coordinates (**X',Y',Z',t'**), originated at point **O'**. Axis **X'** coincides with axis **X**. System **K'** uniformly moves along positive coordinate **X** of stationary system. When points **O** and **O'** coincide at time **t=t'=0** an isotropic spherical light pulse is generated at **x=x'=0**, **y=y'=0**, **z=z'=0**. Since the speed of light is the same (**=c**) in both systems, the light pulse front has to satisfy both equations:

$$x'^2+y'^2+z'^2=(ct')^2 \qquad (1\text{-}1)$$

$$x^2+y^2+z^2=(ct)^2 \qquad (1\text{-}2)$$

The Galilean transformation equations are:

$$x'=x-vt$$
$$y'=y$$
$$z'=z$$
$$t'=t$$

In order to transform equation (1-1) to (1-2) substitute the Galilean transformation into equation (1-1):

$$x^2-2xvt+v^2t^2+y^2+z^2=c^2t^2 \qquad (1\text{-}3)$$

Equation (1-3) in comparing with (1-2) contains two additional unwanted items: $\mathbf{-2xvt+v^2t^2}$, therefore the Galilean transformation is not applicable in this case. In order to rectify this problem the Galilean transformation has to be modified to accommodate special property of the light speed.

The solution for eliminating of unwanted items in the left part of equation (1-3) is to include an extra term in the timing part of equation (1-3), which will compensate the unwanted terms. It can be achieved by adding a compensating item to the time equation of the Galilean transformation. Assign this compensating item as "**m**", then the modified Galilean transformation will be:

$$x'=x-vt$$
$$y'=y$$
$$z'=z \qquad (1\text{-}4)$$
$$t'=t+m$$

Substitute transformation equations (1-4) into equation (1-1):

$$x^2+y^2+z^2 \;\; -2xvt+v^2t^2=c^2t^2 \;\; +2c^2tm +c^2m^2 \qquad (1\text{-}5)$$

In equation (1-5) the extra items, which are subjects for elimination, are:

$$-2xvt+v^2t^2 = 2c^2tm+c^2m^2 \qquad (1\text{-}6)$$

In the stationary system the directing equation for propagation of light pulse along **X** axis is:

$$x=ct \qquad (1\text{-}7)$$

Make substitution (1-7) into equation (1-6):

$$-2cvt^2+v^2t^2 = 2c^2tm+c^2m^2 \qquad (1\text{-}8)$$

Rearranging equation (1-8) gives:

$$m^2+2tm+2vt^2/c-v^2t^2/c^2=0 \qquad (1\text{-}9)$$

Solution for equation (1-9) is:

$$m=-t\pm\sqrt{t^2-\frac{2vt^2}{c}+\frac{v^2t^2}{c^2}}=-t\pm t(1-v/c) \qquad (1\text{-}10)$$

There are two solutions for equation (1-9):

$$m_1=-t+t-vt/c=-vt/c; \quad m_2=-t-t+vt/c=-2t+vt/c \qquad (1\text{-}10a)$$

Substitute (1-10a) into timing equation of the transformation equations (1-4):

$$t'_1=t-vt/c; \quad t'_2=t-2t+vt/c=-t+vt/c$$

The solution $\mathbf{t'_2}$ should be discarded, since negative time is not accepted, therefore **m=-vt/c** is accepted, and the solution for the transformation timing equation is:

$$t'=t-vt/c \qquad (1\text{-}11)$$

Applying substitution **t=x/c** from the directing equation (1-7) into **vt/c** of (1-11) gives:

$$t'=t-vx/c^2 \qquad (1\text{-}12)$$

Considering (1-12) the derived transformation equations replacing the Galilean transformation in order to comply with absolute invariance of the light speed, will be:

$$\begin{aligned} x'&=x-vt \\ y'&=y \\ z'&=z \\ t'&=t-vx/c^2 \end{aligned} \qquad (1\text{-}13)$$

The transformation equations (1-13) are in fact the s-transformation.

To exercise another approach, the modification of the Galilean transformation will be performed on the basis of linear propagation of the light front. The general equation for linear propagation of front of light in the moving system is:

$$x'+y'+z'=ct' \qquad (1\text{-}14)$$

Upon transformation of the same front of light to the stationary system it is expected to be expressed by the equation:

$$x+y+z=ct \qquad (1\text{-}15)$$

To determine the time correcting term **m** in Galilean transformation substitute equations (1-4) into (1-14):

$$x-vt+y+z=ct+cm \qquad (1\text{-}16)$$

In order to make equation (1-16) identical to the equation (1-15) the following condition in equation (1-16) must be satisfied:

$$-vt=cm$$

Then the term **m** can be determined as:

$$m=-vt/c \qquad (1\text{-}17)$$

or making substitution **t=x/c** from (1-7) into (1-17), the term **m** will be defined as:

$$m=-vx/c^2 \qquad (1\text{-}18)$$

Substituting (1-18) into (1-4) gives:

$$\begin{aligned} x' &= x-vt \\ y' &= y \\ z' &= z \qquad (1\text{-}19) \\ t' &= t-vx/c^2 \end{aligned}$$

Equations (1-19) are the s-transformation.
This confirms consistency of modification of the timing equation of the Galilean transformation, regardless of the propagation mode of the light front within the moving inertial system.

2. Linear equations as the basis for the transformation derivation

It is reasonable to seek linear transformation of (**x',t'**) to (**x,t**) in form of general linear equations:

$$x'=a_1x+a_2t \qquad (2\text{-}1)$$

$$t'=b_1x+b_2t \qquad (2\text{-}2)$$

Coefficients $\mathbf{a_1}$, $\mathbf{a_2}$, $\mathbf{b_1}$, $\mathbf{b_2}$ are constants that can depend only on the speeds **v** and **c**.
In order to determine the coefficients the substitution of (2-1) and (2-2) into equation (1-1) will be applied, and the result should be equation (1-2).

Before making the substitutions it should be noted that the origin **O'** of the moving frame (**x'=0**) is a point that moves with speed **v** along stationary axis **X**, and its location in the stationary frame is always **x=vt**. Therefore, when **x'=0** the equation (2-1) has to satisfy condition:

$$0=a_1vt+a_2t$$

Determine $\mathbf{a_2}$:

$$a_2=-a_1v \qquad (2\text{-}3)$$

Substitute $\mathbf{a_2}$ into equation (2-1):

$$x'=a_1x-a_1vt \qquad (2\text{-}1a)$$

Substitute equations (2-1a) and (2-2) into equation (1-1):

$$[a_1x-a_1vt)]^2+y^2+z^2=c^2[b_1x+b_2t]^2 \qquad (2\text{-}4)$$

Expand equation (2-4):

$$a_1^2x^2-2a_1^2xvt+a_1^2v^2t^2+y^2+z^2=c^2x^2b_1^2+2c^2xtb_2b_1+c^2t^2b_2^2 \qquad (2\text{-}4a)$$

In equation (2-4a) assign the coefficients, which attached to the items forming the expected equation (1-2), as **1** in order to clear the expected target equation:

$$a_1=1, \quad b_2=1$$

then equation (2-4a) can be rewritten as follows:

$$x^2+y^2+z^2 -2xvt+v^2t^2=c^2t^2 +c^2x^2b_1^2+2c^2xtb_1 \qquad (2\text{-}5)$$

Comparing equation (2-5) with equation (1-2), the equation (2-5) contains unwanted items in the left and right parts. These items are supposed to be mutually canceled by proper choosing of the coefficient $\mathbf{b_1}$. Extracting these items from equation (2-5) gives:

$$-2xvt+v^2t^2 = c^2x^2b_1^2+2c^2xtb_1 \qquad (2\text{-}6)$$

In the stationary system the directing equation for propagation of light pulse along **X** axis is:

$$x=ct$$

Make this substitution into equation (2-6):

$$-2cvt^2+v^2t^2 = c^4t^2b_1^2+2c^3t^2b_1$$

Canceling $\mathbf{t^2}$ and rearranging gives:

$$b_1^2+(2/c)b_1+2v/c^3-v^2/c^4=0 \qquad (2\text{-}7)$$

Solution for equation (2-7) is:

$$b_1=-\frac{1}{c}\pm\sqrt{\frac{1}{c^2}-\frac{2v}{c^3}+\frac{v^2}{c^4}}=-1/c\pm(1/c-v/c^2)$$

There are two solutions for equation (2-7):

1) $b_{1(1)}=-1/c+1/c-v/c^2=-v/c^2$; 2) $b_{1(2)}=-1/c-1/c+v/c^2=-2/c+v/c^2$

In order to make selection both solutions are to be substituted into equation (2-2):

(1) $t'=-xv/c^2+t$

(2) $t'=-2x/c+xv/c^2+t=-2t+xv/c^2+t=-t+xv/c^2$

Solution (2) is supposed to be discarded, since negative time is not accepted, therefore remains $\mathbf{b_1=-v/c^2}$.

Considering (2-3), all coefficients for the linear transformation equations are determined as:

$$a_1=1;\ a_2=-v;\ b_1=-v/c^2;\ b_2=1 \qquad (2\text{-}8)$$

Substituting values (2-8) into (2-1) and (2-2) and attaching coordinates **y'=y** and **z'=z** produces the transformation equations:

$$\begin{aligned} x'&=x-vt\\ y'&=y\\ z'&=z \qquad (2\text{-}9)\\ t'&=t-vx/c^2 \end{aligned}$$

The derived transformation (2-9) is the s-transformation.

For another approach the coefficients for the equations (2-1a) and (2-2) will be defined on the basis of propagation of the linear light front in the moving system. As it was stated above, the general equation for linear propagation of front of light in the moving system is equation (1-14):

$$x'+y'+z'=ct'$$

Substituting linear equations (2-1a) and (2-2) into the above equation gives:

$$a_1x-a_1vt+y+z=cb_1x+cb_2t \qquad (2\text{-}10)$$

In equation (2-10) the coefficients, which are attached to the items forming the expected equation (1-15), will be assigned as 1 in order to clear the expected equation:

$$a_1=1, \quad b_2=1$$

then equation (2-10) can be rewritten as follows:

$$x+y+z-vt=ct+cb_1x \qquad (2\text{-}11)$$

Comparing equation (2-11) with the expected equation (1-15) the equation (2-11) contains unwanted items in the left and right parts. These items are supposed to be mutually canceled by proper choosing of the coefficient $\mathbf{b_1}$. Extracting these items from equation (2-11) gives:

$$-vt=cb_1x$$

Define $\mathbf{b_1}$:

$$b_1=-vt/cx$$

Substitute **x** by the directing equation (1-7) **x=ct**:

$$b_1=-v/c^2 \qquad (2\text{-}12)$$

The coefficients for the linear transformation equations are determined as:

$$a_1=1; \; a_2=-v; \; b_1=-v/c^2; \; b_2=1 \qquad (2\text{-}13)$$

Substituting values (2-13) into (2-1) and (2-2) and attaching coordinates **y'=y** and **z'=z** produces the transformation equations:

$$x'=x-vt$$
$$y'=y$$
$$z'=z \qquad (2\text{-}14)$$
$$t'=t-vx/c^2$$

The derived transformation (2-14) is the s-transformation.

It can be summarized:

Four different approaches were applied for derivation of the relativistic transformation equations for inertial systems, where the moving system carries propagation of light. All of these approaches arrived to the same result: the s-transformation. None of these approaches called for any relativistic factors to be added to the equations.

References:

1. Albert Einstein. "RELATIVETY. The Special and the General Theory" , Crown Publisher, Inc., One Park Avenue, New York, N.Y.10016, 1952