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On Special Relativity: Incompatibility of the Light Speed Postulate with the Coordinate's Transformation Symmetry Assumption

Radwan M. Kassir[1]

Abstract

In this paper the speed of light postulate is closely examined from the perspective of two inertial reference frames—unprimed and primed—in relative motion, revealing that the resulting inverse space-time transformation is in disagreement with the corresponding Lorentz transformation following from the light speed postulate and the coordinate's transformation assumed symmetry with respect to the reference frames. It is demonstrated that the speed of light postulate actually requires length and time contraction with respect to the unprimed reference frame, length and time dilation with respect to the primed frame, resulting in the frames being at rest with respect to each other! When the coordinate's transformation symmetry assumption is applied on the direct transformation resulting from the light speed postulate—which is shown incompatible with this assumption—, the Lorentz transformation and its inverse are erroneously obtained; it is shown to be restricted to certain coordinate relations, resulted in mathematical contradictions, and thus demonstrated to be unviable.

Keywords: Special Relativity; Speed of Light Postulate; Principle of Relativity; Coordinates Transformation Symmetry; Lorentz Transformation; Length Contraction; Length Expansion; Time Contraction; Time Dilation

1. Introduction

The Lorentz transformation, providing interrelation between the coordinates of two inertial reference frames in relative motion, forms the heart of the Special Relativity Theory. Einstein[1] mainly derived the transformation on the basis of two principles: 1- the principle of relativity, stating that the laws of physics are the same in all inertial reference frames, and 2- the speed of light principle, postulating that the speed of light in vacuum is invariant with respect to all inertial frames of reference.

Yet, another essential tool used in the Lorentz transformation derivation is that the direct and inverse transformations exhibit mutually symmetrical property; that is, the inverse transformation equation can be deduced from the direct one by swapping the coordinates and reversing the velocity sign. This is essentially the result of the isotropic property of space, combined with the first principle of the special relativity. This assumption is rather intuitive. However, in this paper, it is demonstrated that the speed of light principle deviates from this "law" of transformation symmetry. That is, the speed of light

[1] D.A.H. (S & P), Mechanical Dept., Verdun St., PO Box 11-7159, Beirut 1107 2230, Lebanon

principle consequent direct transformation from the perspective of one frame is not symmetrical relative to the corresponding inverse transformation from the perspective of the other frame in relative translational motion with respect to the first frame. It is shown that this fact has a fatal outcome in regard to the coherence of the special relativity.

The erroneous transformation resulting from the application of the law of symmetry essentially leads to contradictions, and consequently to the refutation of the Lorenz transformation, in agreement with the findings of earlier studies. [2, 3]

2. The Speed of Light Postulate Consequent Transformation

Let $K(x, y, z, t)$ be a coordinate system attached to a reference frame K, and let $K'(x', y', z', t')$ be another coordinate system attached to a reference frame K' in relative translational motion at a uniform velocity v, with respect to K.

A light ray is emitted, when the two frames are overlying at the instant of time $t = 0$, from a point at the coinciding frame origins, in the relative motion direction. After period of time t with respect to K, corresponding to t' with respect to K', has elapsed, the light ray tip will have travelled a distance x with respect to K, x' with respect to K'.

Since, according to the special relativity's second postulate, the speed of light is the same with respect to both frames, the light ray trajectory drawn independently in K and K' would appear as shown in Fig. 1 in solid lines. However, the light ray tip point L' is actually perceived as point L, with respect to K. Hence, the distance x' must be contracted with respect to K in order for point L' to coincide with point L. Suppose the distance x' is contracted by a factor of $(1/\gamma < 1)$, as shown in Fig. 1a with the gray dashed line, the following expression is inferred from Fig.1a, relative to K.

$$\begin{aligned} &\frac{x'}{\gamma} + vt = x, \\ &x' = \gamma(x - vt), \end{aligned} \qquad (1)$$

where vt is the distance travelled by K' with respect to K during the travel time t.

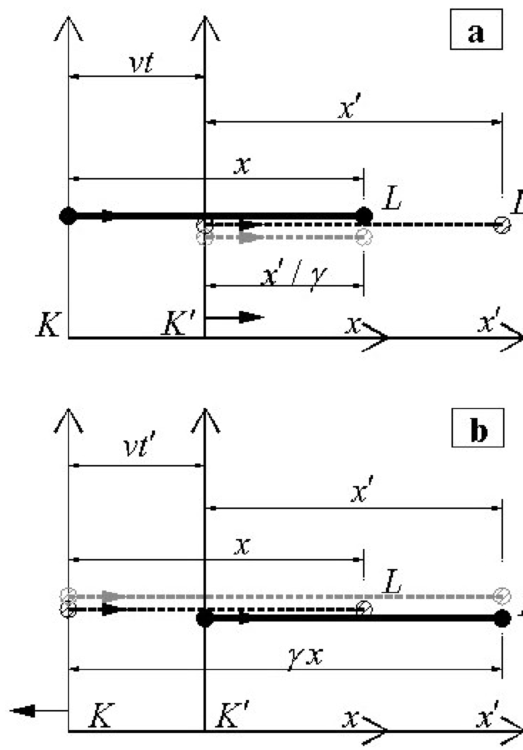

Fig.1 Light ray tip point path from the perspective of K (a), and K' (b).

On the other hand, the light ray tip point L is actually perceived as point L', with respect to K'. Hence, the distance x must then be expanded with respect to K' in order for point L to coincide with point L'. By reciprocity, the distance x must be expanded by the factor of $\gamma > 1$, as shown in Fig. 1b with the gray dashed line. Hence, the following expression is inferred from Fig.1b, relative to K'.

$$\begin{aligned} &\gamma x - vt' = x', \\ &x = \frac{1}{\gamma}(x' + vt'). \end{aligned} \tag{2}$$

Equations (1) and (2) lead to

$$\gamma x = x' + \gamma vt,$$

and

$$\gamma x = x' + vt',$$

resulting in

$$t' = \gamma t. \tag{3}$$

In fact, equation (3) can be readily deduced from Fig. 1. With respect to K (Fig. 1a), since the light ray tip has travelled a contracted distance (x'/γ) in K' at the same speed as in K, the perceived travel time in K must be contracted (by the same distance contraction factor, i.e., $t = t'/\gamma$, or $t' = \gamma t$) so as to agree with the time perception in K.

Similarly, with respect to K' (Fig. 1b), since the light ray tip has travelled an expanded distance γx in K at the same speed as in K', the perceived travel time in K' must be dilated (by the same distance expansion factor, i.e., $t' = \gamma t$) so as to agree with the time perception in K'.

Now, dividing both sides of equations (1) and (2) by the speed of light c yields

$$t' = \gamma t\left(1 - \frac{v}{c}\right), \tag{4}$$

$$t = \frac{t'}{\gamma}\left(1 + \frac{v}{c}\right). \tag{5}$$

Substituting equation (3) in equations (4) and (5), returns

$$v = 0. \tag{6}$$

It follows that the constancy of the speed of light results in the two reference frames being at rest with respect to each other.

3. The Special Relativity Blunder

Using the isotropic property of space, and the Special Relativity first postulate stating that the laws of physics are the same in all inertial reference frames, the coordinate transformation with respect to the unprimed frame K, given by equation (1)—obtained from the constancy of the speed of light postulate—would represent the inverse transformation (i.e., with respect to the primed frame K'), had we swapped in the equation the unprimed and the primed coordinates, and reverse the sign of the relative velocity (as K is traveling in the opposite direction with respect to K'). This will lead to the following transformation equation and its inverse.

$$x' = \gamma(x - vt); \tag{7}$$

$$x = \gamma(x' + vt'). \tag{8}$$

Obviously, equation (8) is inconsistent with the speed of light principle, as it is not in line with equation (2) required by this principle.

Now, dividing both sides of equations (7) and (8) by c, the speed of light, the following time transformation equations are obtained.

$$t' = \gamma t\left(1 - \frac{v}{c}\right), \tag{9}$$

$$t = \gamma t'\left(1 + \frac{v}{c}\right). \tag{10}$$

Substituting equation (9) into equation (10) leads after simple simplification to

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\ . \tag{11}$$

Replacing equation (10) in equation (7), and equation (9) in equation (8),returning, respectively

$$x = \gamma\left[x'\left(\frac{1}{\gamma^2} + \frac{v^2 t'}{cx'}\right) + vt'\right];$$

and

$$x' = \gamma\left[x\left(\frac{1}{\gamma^2} + \frac{v^2 t}{cx}\right) - vt\right],$$

requiring $x' = ct'$ and $x = ct$, to yield the transformation equations (8) and (7), respectively. When this requirement (i.e., $x' = ct'$ and $x = ct$) is applied to equations (9) and (10), the following equations are returned.

$$t' = \gamma\left(t - \frac{vx}{c^2}\right); \tag{12}$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right). \tag{13}$$

It follows that, equations (7), (8), (12), and (13), which are nothing but the Lorentz transformation equations, are restricted to $x = ct$ and $x' = ct'$, which leads to various contradictions.

In fact, when $t' = 0$, Lorentz transformation (12) leads to $t = vx/c^2$. But, as shown above, $x = ct$ in equation (12), yielding the contradiction $t = vct/c^2$, or $v = c$.

Similarly, Lorentz transformation (13) can lead to a similar contradiction for $t = 0$ (i.e. $v = -c$).

Furthermore, substituting equation (12) into equation (13), returns

$$t = \gamma\left(\gamma\left(t - \frac{vx}{c^2}\right) + \frac{vx'}{c^2}\right), \tag{14}$$

which can be simplified to

$$t\left(\gamma^2 - 1\right) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma x'}{x}\right). \tag{15}$$

Since, as shown earlier, equations (12) and (13) require $x = ct$; $x' = ct'$, then equation (15) can be written as

$$t\left(\gamma^2 - 1\right) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma t'}{t}\right). \tag{16}$$

Now, for time $t' = 0$, the transformed t-coordinate with respect to K would be $t = vx/c^2$, according to equation (12). Consequently, for $t \neq 0$, equation (16) would reduce to

$$t\left(\gamma^2 - 1\right) = t\gamma^2,$$

yielding the contradiction,

$$\gamma^2 - 1 = \gamma^2 \text{ , or } 0 = 1 .$$

It follows that the conversion of the time coordinate $t' = 0$ to $t = vx / c^2$, for $x \neq 0$, by Lorentz transformation equation (12), is proved to be invalid, since it leads to a contradiction when used in equation (16), resulting from the Lorentz transformation equations for $t \neq 0$ (i.e. beyond the initial overlaid-frames instant satisfying $t = 0$ for $t' = 0$).

A similar contradiction is obtained by substituting equation (13) into equation (12), and applying equation (13) for the conversion $t = 0$; $t' = -vx' / c^2$.

In addition, substituting equation (7) into equation (8), yields

$$x = \gamma\left(\gamma\left(x - vt\right) + vt'\right);$$

$$x\left(\gamma^2 - 1\right) = \gamma v\left(\gamma t - t'\right);$$

$$x\left(\gamma^2 - 1\right) = \gamma vt\left(\gamma - \frac{t'}{t}\right). \tag{17}$$

Since equations (7) and (8)—along with equations (12) and (13)—require $x = ct;\ x' = ct'$, equation (17) can be written as

$$x\left(\gamma^2 - 1\right) = \gamma vt\left(\gamma - \frac{x'}{x}\right). \tag{18}$$

Now, for $x' = 0$, the transformed x-coordinate with respect to K would be $x = vt,$ according to equation (7). Consequently, for $x \neq 0$, equation (18) would reduce to

$$x\left(\gamma^2 - 1\right) = x\gamma^2,$$

$$\gamma^2 - 1 = \gamma^2, \quad \text{or} \quad 0 = 1.$$

It follows that the conversion of the space coordinate $x' = 0$ of K' origin to $x = vt$, at time $t > 0$, with respect to K by Lorentz transformation equation, is invalid, since it leads to a contradiction when used in equation (18), resulting from Lorentz transformation equations, for $x \neq 0$ (i.e. beyond the initial overlaid-frames position satisfying $x = 0$ for $x' = 0$).

A similar contradiction would follow upon substituting equation (8) into equation (7), and applying equation (8) for the conversion $x = 0;\ x' = -vt'.$

4. Conclusions

Considering two internal reference frames—unprimed and primed— in relative motion, the direct coordinate transformation and its inverse were easily deduced from the constancy of the speed of light principle, using simple diagrams for a light ray travel path from the perspective of each of the two frames. The direct transformation was found to be in agreement with the corresponding Lorentz transformation.

However, counterintuitively, and unlike the Lorenz transformation case, the deduced inverse transformation was not symmetrical with respect to the direct transformation. The direct transformation—from the perspective of the unprimed frame—required that the space and time in the primed frame be contracted with respect to that of the unprimed frame, while the inverse transformation—from the perspective of the primed frame—showed the inverse relation for the space and time (i.e., the space and time in the unprimed frame were dilated with respect to the primed frame). It followed that the constancy of the speed of light principle required that the two frames be at rest with respect to each other. Moreover, further analysis of the Lorentz transformation, following from the coordinate transformation symmetry assumption, showed fatal mathematical contradictions leading to its refutation.

References

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