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Unveiling the Conflict of the Speed of Light Postulate: Outlined Mathematical Refutation of the Special Relativity

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This paper reveals the mathematical contradictory aspects of Einstein's speed of light postulate and the Lorentz transformation (LT) equations. Essential analyses of the equations, leading to the intelligible refutation of the mathematical foundation of the Special Relativity Theory (SRT), are emphasized in an outlined structure.

Introduction

The e-print <http://vixra.org/abs/1306.0098>, by the author, is a formal dissertation on LT and SRT, dealing with the mathematical and theoretical inconsistencies of the speed of light principle and the LT equations. This communication, also posted at viXra.com (http://vixra.org/abs/1306.0185), just outlines the performed mathematical deductions, prominently leading to the refutation of the SRT.

Lorentz Transformation

Consider two inertial frames of reference, $K(x, y, z, t)$ and $K'(x', y', z', t')$, in translational relative motion with speed v.

LT equations [1–3]:

$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \end{aligned} \tag{1}$$

$$\begin{aligned} x &= \gamma(x' + vt') \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \end{aligned} \tag{2}$$

$$\begin{aligned} y &= y' \\ z &= z' \end{aligned} \tag{3}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

Equations (1) and (2) result in the following relativistic velocity transformation equations:

$$\begin{aligned} u' &= \frac{u - v}{1 - \frac{uv}{c^2}} \\ u &= \frac{u' + v}{1 + \frac{u'v}{c^2}} \end{aligned} \tag{5}$$

Where c is the speed of light propagation in empty space, and u and u' are the velocity of a moving body in the x-direction, when measured with respect to K and K', respectively.

Constancy of the Speed of Light

Equations:

In line with [2], for the inertial frames $K(x, y, z, t)$ and $K'(x', y', z', t')$ in relative motion, the space-time coordinates must obey the light sphere equation transformation:

$$x^2 + y^2 + z^2 = c^2 t^2, \tag{6}$$

$$x'^2 + y'^2 + z'^2 = c^2 t'^2. \tag{7}$$

Subtracting equation (7) from equation (6), given that the y and z coordinates remain unaltered, we get

$$x^2 - x'^2 = c^2 t^2 - c^2 t'^2. \tag{8}$$

Analysis:

Lorentz transformation equations (1) can lead to

$$x'^2 = \gamma^2 (x^2 + v^2 t^2 - 2xvt), \tag{9}$$

$$c^2 t'^2 = \gamma^2 \left(c^2 t^2 + \frac{v^2 x^2}{c^2} - 2xvt \right). \tag{10}$$

Eliminating the term $2xvt$ from equations (9) and (10), yields

$$x^2 + v^2 t^2 - \frac{x'^2}{\gamma^2} = c^2 t^2 + \frac{v^2 x^2}{c^2} - \frac{c^2 t'^2}{\gamma^2}. \tag{11}$$

Similarly, Lorentz transformation equations (2) bring about the following expression;

$$-x'^2 - v^2 t'^2 + \frac{x^2}{\gamma^2} = -c^2 t'^2 - \frac{v^2 x'^2}{c^2} + \frac{c^2 t^2}{\gamma^2}. \tag{12}$$

Adding equations (11) and (12) will lead to the following expression;

$$x^2 \left(1 + \frac{1}{\gamma^2}\right) - x'^2 \left(1 + \frac{1}{\gamma^2}\right) + v^2 (t^2 - t'^2) = c^2 t^2 \left(1 + \frac{1}{\gamma^2}\right) - c^2 t'^2 \left(1 + \frac{1}{\gamma^2}\right) + \frac{v^2}{c^2}(x^2 - x'^2);$$

which can be simplified to

$$(x^2 - x'^2)\left(1 + \frac{1}{\gamma^2} - \frac{v^2}{c^2}\right) = c^2 (t^2 - t'^2)\left(1 + \frac{1}{\gamma^2} - \frac{v^2}{c^2}\right);$$

$$\mathbf{x^2 - x'^2 = c^2 (t^2 - t'^2)}, \tag{13}$$

returning actually the speed of light principle equation (8); thus validating equations (11) and (12) from the perspective of the special relativity. It should be noted that equation (8) is obtained from the Lorentz transformation equations without any restriction on the value of γ (i.e. γ can be replaced in the Lorentz transformation equations by an arbitrary constant, while equation (8) can still be obtained from the invalid resulting equations).

Whereas, the subtraction of equation (12) from equation (11), results in

$$(x^2 + x'^2)\left(1 - \frac{1}{\gamma^2} - \frac{v^2}{c^2}\right) = c^2 (t^2 + t'^2)\left(1 - \frac{1}{\gamma^2} - \frac{v^2}{c^2}\right).$$

Now, if we assume for the time being the following equality (as suggested by the above equation)

$$x^2 + x'^2 = c^2 (t^2 + t'^2), \tag{14}$$

then quations (13) and (14) will readily reduce to

$$\mathbf{x^2 = c^2 t^2} \tag{15}$$

$$\mathbf{x'^2 = c^2 t'^2} \tag{16}$$

which satisfy both equations (11) and (12), as well as equation (8)—when x^2 and x'^2 are replaced with c^2t^2 and $c^2t'^2$, respectively—thus validating equation (14) that can also be derived from its consequent equations (15) and (16). Therefore, the verified equation (14) makes equations (15) and (16) the only solution for the constancy of the speed of light equation (8).

—It should be noted that equations (15) and (16) can be evidently inferred from equations (11) and (12).

Consequently, the light sphere equations (6) and (7) are collapsed to the line equations (15) and (16):
– When equations (15) and (16) are substituted into equations (6) and (7), they result in the vanishing of y, z, $y'$and z'.
– This can be reconfirmed by adding equations (6) and (7), and using equation (14).

- **First flaw: The constancy of the speed of light equations (6) and (7) are preliminarily restricted to one-dimentional light propagation (i.e. y and z coordinates in both frames are forced to be zero), parallel to the direction of the relative motion.**

Now, dividing equation (15) by equation (16) yields

$$\left(\frac{x}{x'}\right)^2 = \left(\frac{ct}{ct'}\right)^2,$$

or

$$\frac{x}{x'} = \pm\frac{ct}{ct'}. \tag{17}$$

For $c > v$, x and x' will always have the same sign (positive or negative)—whether the light beam is emitted in the positive or negative $x-$direction with respect to K and K' origins. Therefore,

$$\frac{x}{x'} \geq 0,$$

and given that

$$\frac{ct}{ct'} \geq 0,$$

equation (17) becomes

$$\frac{x}{x'} = \frac{ct}{ct'}. \tag{18}$$

Hence, equation (18) combined with equations (15) and (16), leads to

$$\mathbf{c = \frac{x}{t} = \frac{x'}{t'}}. \tag{19}$$

Outcome: the constancy of the speed of light can be expressed by equation (19).

The Implication

Assuming the space-time is preserved (i.e. cannot be modified), the coordinates x and x' (Fig. 1) would then be related by the following equation with respect to K, in accordance with the Galilean transformation;

$$x' = x - vt. \tag{20}$$

Whereas, with respect to K', the same coordinates (Fig. 2) would be related by the following equation

$$x = x' + vt'. \tag{21}$$

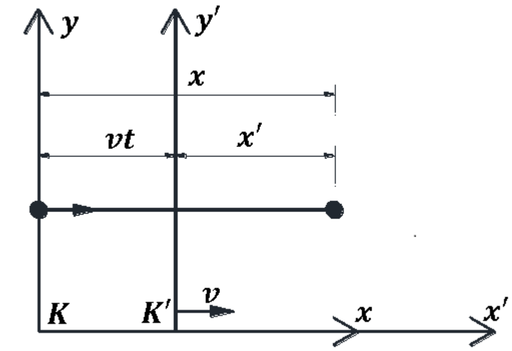

Fig. 1: x-coordinate with respect to K.

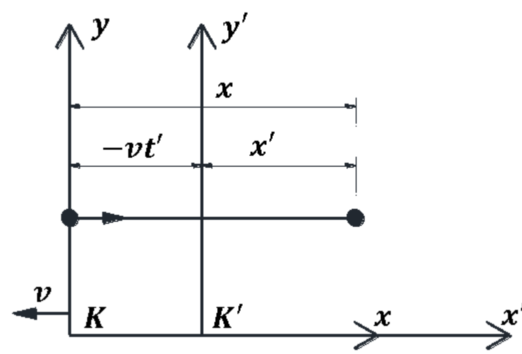

Fig. 2: x'-coordinate with respect to K'.

Substituting equation (20) into equation (21), we get

$$t = t'. \tag{22}$$

Dividing both sides of equations (20) and (21) by c, and applying the speed of light constancy principle as determined above ($c = x/t = x'/t'$), the following expressions are obtained;

$$t' = t - \frac{vx}{c^2}, \tag{23}$$

$$t = t' + \frac{vx'}{c^2}. \tag{24}$$

Substituting equation (23) in equation (24), we get

$$x = x'. \tag{25}$$

and replacing equation (25) in equations (20) and (21) leads to the conflicting result of $v = 0$ at any $t > 0$.

It follows that the set of equations (20), (21), (23) and (24)—which will be referred to as (S1)—resulting from the Galilean transformation applied under the principle of the constancy of the speed of light, leads to the only conflicting solution $v = 0$, $x = x'$, and $t = t'$, binding the two reference frames together, although the relative motion of the reference frames is set as the main condition under which the equation set (S1) is derived. Consequently, the light speed constancy principle is unviable, at least in the case of no space-time distorting transformation.

On the other hand, although the equation set (S1) requires the conflicting binding of the two reference frames, it leads to the constancy of the speed of light general criteria given by equation (8), implying that the frames-binding requirement of the equation set (S1) remains applicable to equation (8).

- **Second flaw: The speed of light constancy principle is incompatible with reference frames in relative motion.**

In fact, equations (20) and (23) lead to

$$x'^2 = x^2 + v^2t^2 - 2xvt,$$

and

$$c^2t'^2 = c^2t^2 + \frac{v^2x^2}{c^2} - 2xvt.$$

Eliminating $2xvt$ from the above two equations yields

$$x^2 + v^2t^2 - x'^2 = c^2t^2 + \frac{v^2x^2}{c^2} - c^2t'^2. \tag{26}$$

Similarly, equations (21) and (24) can lead to

$$-x'^2 - v^2t'^2 + x^2 = -c^2t'^2 - \frac{v^2x'^2}{c^2} + c^2t^2. \tag{27}$$

Adding equations (26) and (27), returns equation (8):

$$x^2 - x'^2 = c^2t^2 - c^2t'^2.$$

Indeed, the addition of equations (26) and (27) results in the following expressions,

$$2(x^2 - x'^2) + v^2(t^2 - t'^2) = 2c^2(t^2 - t'^2) + \frac{v^2}{c^2}(x^2 - x'^2);$$

$$(x^2 - x'^2)\left(2 - \frac{v^2}{c^2}\right) = c^2(t^2 - t'^2)\left(2 - \frac{v^2}{c^2}\right);$$

yielding the speed of light constancy principle equation,

$$x^2 - x'^2 = c^2(t^2 - t'^2).$$

Furthermore, the equation set (S1), resulting in the contradictory frames binding, also generate the relativistic velocity equations (5)—by dividing equation (20) by equation (23), and equation (21) by equation (24)—and conversely, working back from equations (5), equations (20), (21), (23) and (24) can be deduced.

- **Third flaw: The Lorentz velocity transformation equations are merely invalid velocity criteria of the speed of light constancy principle.**

Lorenz Transformation Re-derivation

Owing to equation (19), the derivation of the Lorentz Transformation is substantially simplified:

– Assuming a space-time distorting transformation, a length conversion by a factor of β (a positive real number) along the direction of motion is hypothesized.

–This length conversion can therefore be expressed using Figs. 1 and 2 as follows.

$$x = vt + \beta x'. \tag{28}$$

$$x' = -vt' + \beta x. \tag{29}$$

Rearrange equations (28) and (29):

$$x' = \frac{1}{\beta}(x - vt), \tag{30}$$

$$x = \frac{1}{\beta}(x' + vt'). \tag{31}$$

Divide both sides of equations (30) and (31) by c, and apply the speed of light constancy principle equation (19):

$$t' = \frac{1}{\beta}\left(t - \frac{vx}{c^2}\right), \tag{32}$$

$$t = \frac{1}{\beta}\left(t' + \frac{vx'}{c^2}\right). \tag{33}$$

Solving equations (30), (31), (32) and (33) for β results in

$$\beta = \sqrt{1 - \frac{v^2}{c^2}},$$

or

$$\frac{1}{\beta} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma. \tag{34}$$

In fact, for $x' = 0$, equations (30) and (33) yield $x = vt$, and $t' = \beta t$, respectively, reducing equation (32) to

$$\beta t = \frac{1}{\beta}\left(t - \frac{v^2 t}{c^2}\right);$$

therefore

$$\beta = \sqrt{1 - \frac{v^2}{c^2}}.$$

Conversely, for $x = 0$, equations (31) and (32) yield $x' = -vt$, and $t = \beta t'$, respectively, reducing equation (33) to

$$\beta t' = \frac{1}{\beta}\left(t' - \frac{v^2 t'}{c^2}\right);$$

hence

$$\beta = \sqrt{1 - \frac{v^2}{c^2}}.$$

$\beta < 1$, then the hypothesized length conversion is a length contraction.

Equations (30), (31), (32), (33), and (34) are the Lorentz transformation.

Lorentz Transformation Contradictions

—For the origin of $K'(0, 0, 0, t')$, at time $t' \neq 0$, the corresponding K x- and t-coordinates shall satisfy the relation

$$\frac{x}{t} = \frac{x'}{t'}$$

that would yield

$$x = x'\left(\frac{t}{t'}\right) = 0,$$

if t was determined. But, $x = 0$ results in undetermined t:

$$t = t'\left(\frac{x'}{x}\right) = \frac{0}{0},$$

making the above x-equation undetermined as well, thus leading to the set of K origin coordinates

$$\left(x = \frac{0}{0}, 0, 0, t = \frac{0}{0}\right)$$

with undetermined x and t.

—For the time origin of $K'(x', y', z', 0)$, with spatial coordinates $\neq 0$, the corresponding K x- and t-coordinates shall satisfy the relation

$$\frac{x}{t} = \frac{x'}{t'}$$

that would yield

$$t = t'\left(\frac{x}{x'}\right) = 0,$$

if x was determined. But, $t = 0$ results in undetermined x:

$$x = x'\left(\frac{t}{t'}\right) = \frac{0}{0},$$

making the above t-equation undetermined as well, thus leading to the set of K origin coordinates

$$\left(x = \frac{0}{0}, y = y', z = z', t = \frac{0}{0}\right)$$

with undetermined x and t.

- **Fourth flaw: The frames of reference origin-coordinates are undetermined with respect to each other**—except for the set coordinates zero value at the initial overlaid-frames instant.

Consequently, Lorentz transformation, implicitly incorporating equation (19), results in various conflicts.

—For instance, substituting equation (32) into equation (33), returns

$$t = \gamma\left(\gamma\left(t - \frac{vx}{c^2}\right) + \frac{vx'}{c^2}\right). \tag{35}$$

Equation (35) is simplified in the following steps.

$$t = \gamma^2 t - \frac{\gamma^2 vx}{c^2} + \frac{\gamma vx'}{c^2},$$

or

$$t(\gamma^2 - 1) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma x'}{x}\right).$$

Using equation (19) in the above equation, we get

$$t(\gamma^2 - 1) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma t'}{t}\right). \tag{36}$$

With respect to equation (32), for $t' = 0$, the transformed t-coordinate with respect to K is $t = vx/c^2$ (t is undetermined with respect to equation (19), when $t' = 0$, as shown earlier, except at the initial overlaid-frames instant, the value of t and t' are set to zero). Therefore, for $t \neq 0$, equation (36) reduces to

$$t(\gamma^2 - 1) = t\gamma^2, \tag{37}$$

yielding the contradiction,

$$\gamma^2 - 1 = \gamma^2,$$

or

$$0 = 1,$$

It follows that the transformation of $t' = 0$ to $t = vx/c^2$, for $x \neq 0$, by Lorentz transformation equation (32), is invalid, since it leads to a contradiction when used in equation (36), resulting from Lorentz transformation equations, for $t \neq 0$ (i.e. beyond the initial overlaid-frames instant satisfying $t = 0$ for $t' = 0$)—Letting $t = 0$ would satisfy equation (37), but another contradiction would emerge; the reference frames would be locked in their initial overlaid position, and no relative motion would be allowed, since in this case the corresponding coordinate to $t' = 0$ would be $t = vx/c^2 = 0$, yielding $v = 0$, as we're addressing the transformation of $t' = 0$ to $t = vx/c^2$ for $x \neq 0$.

Similar contradiction is obtained by substituting equation (33) into equation (32), using equation (19) in the resulting equation, and applying equation (33) for $t = 0$ ($t' = -vx'/c^2$).

—Furthermore, substituting equation (30) into equation (31), yields

$$x = \gamma\left(\gamma(x - vt) + vt'\right);$$

$$x(\gamma^2 - 1) = \gamma v(\gamma t - t');$$

$$x(\gamma^2 - 1) = \gamma vt\left(\gamma - \frac{t'}{t}\right). \tag{38}$$

Using equation (19) in equation (38), we get,

$$x(\gamma^2 - 1) = \gamma vt\left(\gamma - \frac{x'}{x}\right). \tag{39}$$

With respect to equation (30), for $x' = 0$ (corresponding to k' origin), the transformed x-coordinate with respect to k is $x = vt$ (x is undetermined with respect to equation (19) when $x' = 0$, as shown earlier, except at the initial overlaid-frame position, where the corresponding value to $x' = 0$ is $x = 0$). Therefore, for $x \neq 0$, equation (39) reduces to the following contradiction.

$$x(\gamma^2 - 1) = x\gamma^2, \tag{40}$$

$$\gamma^2 - 1 = \gamma^2,$$

or

$$0 = 1.$$

It follows that the transformation of the x'-coordinate of K' origin ($x' = 0$) to $x = vt$, at time $t > 0$, with respect to K by Lorentz transformation equation (30), is invalid, since it leads to a contradiction when used in equation (39), resulting from Lorentz transformation equations, for $x \neq 0$ (i.e. beyond the initial overlaid-frames position satisfying $x = 0$ for $x' = 0$)—Letting $x = 0$ would satisfy equation (40), but another contradiction would emerge; the reference frames would be locked in their initial overlaid position, and no relative motion would be allowed, since in this case the corresponding coordinate to $x' = 0$ would be $x = vt = 0$, yielding $v = 0$, as we're addressing the transformation of $x' = 0$ to $x = vt$ for $t > 0$.

Similar contradiction would follow upon substituting equation(31) into equation (30), using equation (19), and applying equation (31) for $x = 0$, $x' = -vt'$.

- **Fifth flaw: Lorentz Transformation generates mathematical impossibilities.**

The obtained Lorentz transformation contradictions for the particular cases of converting each of the spatial—along the relative motion direction—and time coordinates having a zero value in one reference frame to its corresponding value in the other frame, imply the general unviability of the Lorentz transformation equations.

It follows that, the Lorentz transformation is deemed to be refuted.

Conclusion

Analysis of the Lorentz transformation revealed mathematical restrictions in terms of the deduced, simplified form of the constancy of the speed of light equations residing in the transformation. The Lorentz transformation, readily reconstructed using these basic, restricted light velocity invariance equations, resulted in mathematical contradictions. The principle of the constancy of the speed of light was thus demonstrated to be an unviable assumption, and the ensuing Lorentz transformation was subject to refutation.

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