

Doppler Effect
on Light Reflected from Revolving Mirrors:
A Brief Review of Majorana's 1918 Experiment

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Abstract:

In this review of Majorana's 1918 experiment, amplified Doppler shifts in the wavelength of light by multiple reflections from moving mirrors are recalculated on the basis of the undulatory theory, the new-source theory, and the elastic-impact theory respectively, and compared to the experimental result and to the theoretical predictions computed by Q. Majorana.

Introduction:

Majorana's experiment is designed to check for variations of the first order in the wavelength of light reflected a number of times from moving mirrors.

Based on his calculations of the Doppler effect, Q. Majorana expects to find out, in this experiment, either the wavelength of reflected light varies with the velocity of the light source, as predicted by the ethereal hypothesis, or remains constant, as computed by him in accordance with the emissive or ballistic hypothesis.

On the face of it, the expected result of the experiment, under discussion, is a forgone conclusion. If the frequency of reflected light varies with the velocity of the mirror as predicted by all theories, then the wavelength of the same light must vary with the velocity of the mirror as well.

Nonetheless, Q. Majorana has constructed his ingeniously conceived experiment to verify that expected result experimentally; even though the emissive or ballistic hypothesis, with such a calculated prediction of fixed wavelengths, doesn't really have much of a chance of passing successfully and with flying colors such an experimental test.

But does the emissive or ballistic hypothesis actually predict the unchanged wavelength of light with the velocity of the light source?

If one calculates the shifted frequency of light by the Doppler effect, using the emissive or ballistic theory, and inserts that result, along with the speed resultant of light and the speed of the light source, into the wave equation of the ethereal hypothesis, then one must arrive at Majorana's conclusion that the emissive or ballistic hypothesis predicts the non-varying wavelength of light with the velocity of the light source; i.e.

$$\lambda' = \lambda .$$

However, there are, at least, four available arguments, within the theoretical framework of the emissive or ballistic hypothesis, that can be advanced against the correctness of the above Majorana's conclusion: Two arguments based on the Galilean principle of relativity; one argument based on the analogy with the changing wavelength of light in refractive media; and one argument is based upon the derivation of the wave equation for light assumed to travel at the speed of $c \pm v$ in vacuum.

Firstly, according to the Galilean principle of relativity, upon which the Galilean transformation is founded, the case of a moving light source and a light detector at rest is equivalent to the case of a moving light detector and a light source at rest. And since the emissive or ballistic hypothesis and the ethereal hypothesis use exactly the same equations and the same computational procedures to calculate shifted frequencies and wavelengths of light, in the case of moving light detectors and light sources at rest; and both hypotheses arrive at the same conclusion that the wavelength of light varies with the velocity of the light detector; then it must be concluded, based on the Galilean principle of relativity, that, in the case of a moving light source and a light detector at rest, the wavelength of light varies with the velocity of the light source in accordance with the emissive or ballistic hypothesis.

Secondly, according to the Galilean principle of relativity, any calculations done, in the reference frame of the stationary light detector, can be carried out in the same way as well in the reference frame of the moving light source, where the velocity of the light source is assigned to the light detector, and the velocity of light, in the reference frame of the moving source, is always equal to c . And so, the conclusion once again is that the wavelength of light varies with the velocity of the light source according to the emissive or ballistic hypothesis.

And thirdly, calculations of Doppler effect due to the motion of light detectors and light sources inside refractive media, to be worked out correctly, must take into account the change of the effective wavelength of light and apply the necessary corrections by inserting the refractive index into the wave equation of light assumed to travel at the speed of c in vacuum. In the same manner, calculations of Doppler effect, due to the motion of the light source, must take into account the change of the effective wavelength of light and apply the necessary corrections by inserting the correct wavelength, the correct frequency, and the correct speed resultant into the wave equation of light assumed to travel in vacuum at the speed of c .

And finally, by inspecting the wave equation, which is derived by assuming that light travels in vacuum at the speed of c , according to the ethereal hypothesis:

$$c = f\lambda$$

similar wave equations can be derived by assuming, for example, that light travels in vacuum at the speed of $c' = 4c$:

$$c' = 4f\lambda$$

where the frequency remains the same, as in the wave equation of the ethereal hypothesis, because the wave period T is fixed by the emitting source; while the wavelength changes by a factor of 4, because, in this case, light travels during the fixed wave period of T a distance greater by a factor of 4 than a similar distance covered by light travels at the speed of c .

In the same way, an equivalent wave equation can be derived, on the basis of the emissive or ballistic hypothesis, by assuming that light travels in vacuum at the speed of $c \pm v$:

$$c' = f\lambda \left(1 \pm \frac{v}{c} \right)$$

where the frequency remains the same, as given by the wave equation of the ethereal hypothesis, because the wave period T is fixed by the internal dynamics of the emitting source; but the wavelength changes by a factor of:

$$\left(1 \pm \frac{v}{c} \right)$$

because light, in this case, travels a distance different by the above factor from that covered by light traveling at the speed of c , during the same wave period T .

Accordingly, the Doppler effect on wavelengths and frequencies, due to the motion of the light source, must apply only to the compatible wave equation with the theoretical assumptions of each hypothesis. In other words, if light is assumed to travel in vacuum at the speed of c , then the Doppler effect, due to the motion of the light source, necessarily transforms the non-shifted wave parameters in this equation:

$$c = f\lambda$$

to the shifted wave parameters in this equation:

$$c = f'\lambda'$$

By contrast, if light is assumed to travel in vacuum at the speed of $c \pm v$, the Doppler effect, caused by the motion of the light source, in this case, must transform the non-shifted wave parameters in this

equation:

$$c' = f\lambda \left(1 \pm \frac{v}{c} \right)$$

to the shifted wave parameters in this equation:

$$c' = f'\lambda' \left(1 \pm \frac{v}{c} \right)$$

The conclusion, therefore, is that the emissive or ballistic hypothesis, as Q. Majorana prefers to call it, does predict shifted wavelengths as well as shifted frequencies that vary with the velocity of the light source and differ only by amounts of the second order from shifted wavelengths and frequencies predicted by the ethereal hypothesis.

A Summary of Majorana's Experimental Report:

A. Majorana's Apparatus:

In this experiment, a horizontal brass wheel R (35 cm in diameter and 6 mm. Thick), which can be made to revolve clockwise and counterclockwise with a maximum angular velocity of 80 turns per second, carries on its periphery ten plane mirrors M's.

The mirrors are fixed firmly at equal intervals on the periphery of the wheel and inclined to the radius from R passing through the center of each of them at an angle $\alpha = 29^\circ$.

The support for the bearings of the axle of R carries also fixed plane mirrors F's.

The position of the F's and M's is such that a parallel beam of light emitted at L, after a certain number of reflections from the F's and M's may be received at L' when the wheel R has determinate angular positions.

The source of light, in this experiment, is a mercury arc in vacuo, the light of which is filtered by solutions of chromate of potassium and chloride of nickel to absorb the violet and yellow rays.

To obtain the value of the wavelength λ , the light, received at L', is examined with Michelson's interferometer, and observed with the telescope C {Ref. #1.a}.

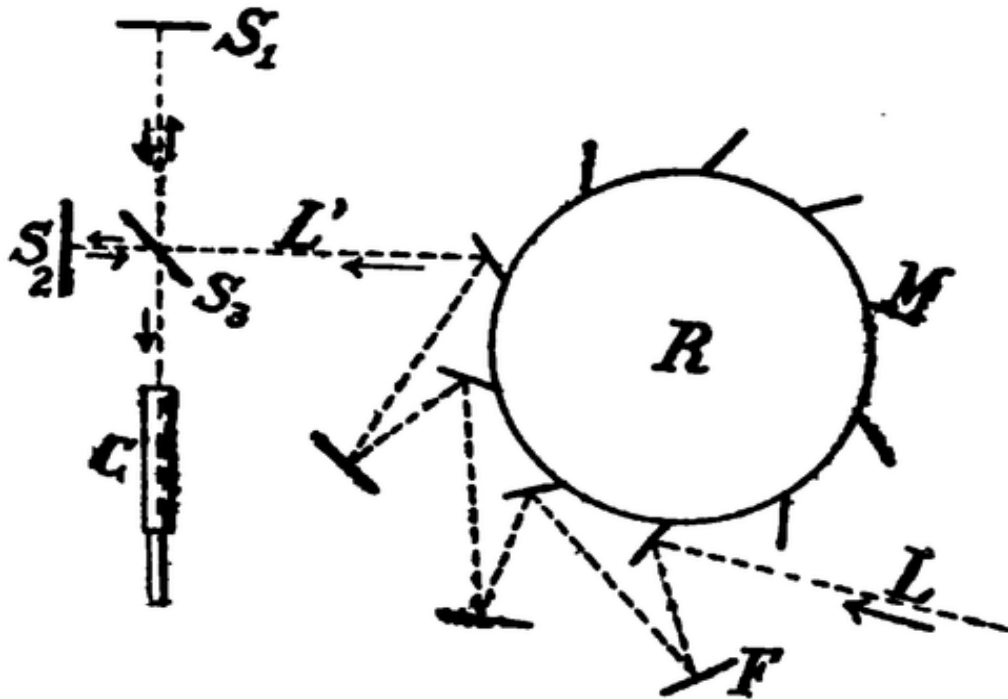


Illustration 1: Majorana's apparatus

B. Majorana's Measurements:

Q. Majorana, before stating the measurement of the displacement observed, calculates the expected amount on the assumption that the velocity of the light reflected from a mirror is the same as that of the incident light, as follows:

Let g be the number of revolutions of R per second, and d the diameter, reckoned between the centers of two opposite mirrors M 's. Then πdg is the instantaneous velocity of translation of the latter.

Since the mirrors are inclined at an angle α to the radius of the wheel passing through each of them, the component of the given velocity in the direction normal to the plane of each mirror is:

$$v = \pi dg \cos \alpha .$$

And therefore,

$$n' = n \left(1 + \frac{2k\pi dg \cos \alpha \cos \theta}{c} \right)$$

where n and n' are the emitted and observed frequencies;

and, by the hypothesis of the invariability of c ,

$$\lambda' = \lambda \left(1 - \frac{2k\pi dg \cos\alpha \cos\theta}{c} \right)$$

where λ and λ' are the emitted and observed wavelengths.

If l is the difference of path of two interfering rays in Michelson's apparatus, the number of fringes which are seen to cross the micrometer thread of the telescope, when λ becomes λ' , is

$$f = \frac{l}{\lambda} \left(\frac{2k\pi dg \cos\alpha \cos\theta}{c} \right)$$

If the observation is made by noting the position of the fringes when the wheel turns in one sense with the velocity g , and that corresponding to an equal and contrary velocity, the number of fringes crossing the micrometer thread will be $2f$.

And since, in the above apparatus, $d = 38$ cm., $\alpha = 29^\circ$, $\theta = 27^\circ$, $k = 4$. If λ is put equal to 0.546μ (green mercury line), $l = 13$ cm., $c = 3 \times 10^{10}$ cm., and $g = 60$ (turns of R per second in one sense and afterward in the other), the expected fringe displacement, according to the preceding formula, is about $2f = 0.71$.

The experiment gives, for the case mentioned, a fringe displacement of between 0.7 and 0.8 fringes; and it is not possible, for reasons of visibility, to carry the precision of the observations further {*Ref. #1.a & #1.b*}.

Majorana's Theoretical Predictions:

A. Majorana's Calculations:

Q. Majorana states, at the beginning of his computations, that, according to the ethereal hypothesis, the speed of light is independent of the speed of the light source. And then he carries out the following calculations:

Let the source S move towards an observer fixed at O, and emit light of wavelength λ and of frequency n .

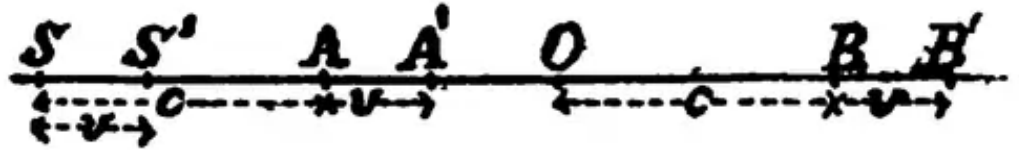


Illustration 2: A Moving Source and Stationary Observer

If it's assumed that the light waves are transmitted through a stationary æther, then waves emitted in a second by S will be distributed over the segment:

$$S'A = c - v.$$

At the same time, all the n waves distributed in the segment:

$$OB = c$$

will have passed through O; and we have, therefore,

$$\frac{c - v}{n} = \frac{c}{n}$$

or

$$n' = n \left(\frac{c}{c - v} \right)$$

And let

$$\frac{v}{c} = \beta$$

and neglect terms of higher order than the first in β ,

$$n' = n(1 + \beta)$$

The new wavelength is obtained by the relations:

$$c = n\lambda = n'\lambda' :$$

$$\lambda' = \lambda(1 - \beta).$$

Next, Majorana assumes that the speed of light is dependent on the speed of the light source, in accordance with the ballistic or emissive hypothesis; and works out the following computations:

In one second the n waves emitted by S will be distributed over the segment:

$$S'A' = c$$

At the same time, there will pass through O, n' waves which will be distributed over the segment:

$$OB' = c + v.$$

And therefore,

$$\frac{c}{n} = \frac{c + v}{n'},$$

or

$$n' = n(1 + \beta).$$

And since

$$c = n\lambda$$

and

$$c + v = n'\lambda'$$

then, in this case,

$$\lambda' = \lambda.$$

And so, Majorana concludes that with regard to the frequency, the conclusion, with the exception of the terms in β^2 , is the same, for the æthereal and the ballistic hypothesis.

But, based on the above calculations, different values of the wavelength that differ by a term of the first order in β are obtained from the two hypotheses, under discussion. And, therefore, if the Doppler effect is measured, by observation of the wavelength, different results should be obtained.

As for light reflected from a mirror in uniform motion, according to Majorana; if v is the velocity of the mirror, normal to its plane, reckoned as positive towards the source; c the velocity of the light in vacuum which makes the angle of incidence θ with the mirror; n , and n' the frequencies of the ray before and after reflection; then n' can be obtained from this equation:

$$n' = n \left(\frac{1 + 2\beta \cos\theta + \beta^2}{1 - \beta^2} \right)$$

which, upon neglecting the terms in β^2 , reduces to

$$n' = n(1 + 2\beta \cos\theta).$$

And when it's assumed that the light is reflected with the angle of incidence θ , k times from several mirrors in motion with the velocity v , the equation, for computing frequencies, becomes:

$$n' = n(1 + 2k\beta \cos\theta).$$

Therefore, according to the ethereal hypothesis, after neglecting the terms in β^2 , the equation for the wavelength is:

$$\lambda' = \lambda(1 - 2k\beta \cos\theta).$$

And finally, if it's assumed that the velocity of the reflected light is variable, the sum of c and the component of the velocity of the image in the direction of the ray, is:

$$c' = c + 2kv \cos\theta.$$

And since

$$c' = n'\lambda'$$

and

$$c = n\lambda$$

then, according to the emissive or ballistic hypothesis {*Ref. #1.a & #1.b*}:

$$\lambda' = \lambda.$$

B. Remarks on Majorana's Calculated Predictions:

The following remarks can be made with regard to Majorana's calculations:

1. Q. Majorana makes no distinction, in the above calculations, between Michelson's two ballistic theories: The new-source theory and the elastic-impact theory {*Ref. #2.a*}. These two theories treat reflection from moving mirrors differently; and hence, an explicit distinction between the two theories is an essential step for working out correctly computations of this sort.
2. Also, Q. Majorana uses only this equation:

$$c' = c + 2kv \cos\theta.$$

for the elastic-impact theory;

and makes, in his calculations, no reference to this equation:

$$c' = c + v \cos \theta$$

for the new-source theory.

The distinction between these two cases is important, in this context; because in the former case the speed of light increases by a factor of $2v \cos \theta$ indefinitely with repeated reflections from moving mirrors; while in the latter case, it increases only by a factor of $v \cos \theta$ and only once.

3. Majorana's conclusion that the ballistic hypothesis predicts:

$$\lambda' = \lambda .$$

for light from a moving source, is based upon the wrong wave equations:

$$c = n\lambda$$

$$c' = n'\lambda'$$

and subsequently, it's incorrect.

These are the correct wave equations for light assumed to travel in vacuum at the speed resultant of $c \pm v$:

$$c' = n\lambda (1 \pm \beta) ,$$

$$c' = n'\lambda' (1 \pm \beta)$$

within the framework of the ballistic hypothesis;

and the correct conclusion, therefore, is:

$$\lambda' = \lambda \left(1 \pm \frac{\beta}{1 \pm \beta} \right)$$

where λ' is the Doppler-shifted wavelength due to the motion of the light source.

4. With regard to Doppler effect by reflection from a moving mirror, and contrary to the aforementioned Majorana's calculated prediction, the shifted wavelength λ' , on the basis of the ballistic hypothesis, to be tested in the current experiment, varies with the velocity of a moving mirror, in accordance with this Doppler formula:

$$\lambda' = \lambda \left(1 \pm \frac{2v}{c + 2v} \right)$$

which can be obtained by using the Galilean principle of relativity, or from these wave equations of the ballistic hypothesis for moving sources:

$$c' = f\lambda \left(1 \pm \frac{v}{c} \right),$$

$$c' = f'\lambda' \left(1 \pm \frac{v}{c} \right)$$

5. According to the Galilean principle of relativity, upon which the Galilean transformation is based, the case of a moving light source and a stationary detector is equivalent to the case of a moving detector and a stationary light source. And hence, it's possible, regarding the wavelength computations above, to do those calculations in the reference frame of the moving light source, in which the light source is always at rest, the relative motion is always assigned to the detector, and the speed of light is always equal to c . And the calculated result, if everything is done correctly, should come out exactly the same as the calculated result in the reference frame of the detector; i.e. according to the emissive or ballistic hypothesis:

$$\lambda' = \lambda \left(1 \pm \frac{\beta}{1 \pm \beta} \right)$$

where λ' is the Doppler-shifted wavelength due to the motion of the light source.

6. It's also possible to arrive at the same calculated wavelength result, by using the wave equation of the emissive or ballistic hypothesis.

Every conceivable theory, in this field, has its own wave equation based upon its basic assumption concerning the speed of light in vacuum.

For instance, if it's assumed that light travels in vacuum at the speed of $c' = c$, then, in this case, its speed c' , its frequency f , and its wavelength λ , are related by this wave equation:

$$c' = f\lambda.$$

And in the same way, if it's assumed that light travels in vacuum at the speed of $c' = 2c$, then, in this case, its speed c' , its frequency f , and its wavelength λ are related by this wave equation:

$$c' = 2f\lambda,$$

where the frequency remains the same; but the wavelength increases by a factor of 2.

And now, if it's assumed that light travels in vacuum at the speed of $c' = c + v$, then, in this case, its speed c' , its frequency f , and its wavelength λ must form this wave equation:

$$c' = f\lambda \left(1 + \frac{v}{c}\right),$$

where the frequency remains the same; but the wavelength increases by a factor of:

$$\left(1 + \frac{v}{c}\right).$$

And, of course, if it's assumed that light travels in vacuum at the speed of $c' = c - v$, then, in this case, its speed c' , its frequency f , and its wavelength λ are related by this wave equation:

$$c' = f\lambda \left(1 - \frac{v}{c}\right),$$

where the frequency remains the same; but the wavelength decreases by a factor of:

$$\left(1 - \frac{v}{c}\right).$$

And generally, if it's assumed that light travels in vacuum at the speed of $c' = c \pm nv$, where n stands for a real number, then its speed c' , its frequency f , and its wavelength λ necessarily form the following wave equation:

$$c' = f\lambda \left(1 \pm n \frac{v}{c}\right),$$

where the frequency remains the same; but the wavelength changes by a factor of:

$$\left(1 \pm n \frac{v}{c}\right).$$

7. It's imperative, therefore, that in calculations of Doppler effect due to the space motion of a light source, on the basis of any physical theory, one uses only the wave equation of that theory based upon its assumption about the speed of light in vacuum. If, for instance, that theory assumes the speed of light in vacuum to be $c' = c$, then the Doppler effect, according to that same theory, operates only on this wave equation:

$$c' = f\lambda.$$

And, in all cases, if it's assumed that the speed of light in vacuum to be in equal to $c' = c \pm nv$, then the Doppler effect must operate on this generalized wave equation:

$$c' = f\lambda \left(1 \pm n \frac{v}{c} \right).$$

And so, it's clear that, in the aforementioned computations by Q. Majorana, the use of this wave equation for light assumed to be traveling in vacuum at c :

$$c' = f\lambda$$

to calculate the Doppler effect on light assumed to travel in vacuum at $c' = c + v$, has led him to the incorrect conclusion that the ballistic hypothesis predicts, in the experiment under discussion, no Doppler shift in the wavelength.

In brief, the right wave equation, for the emissive or ballistic hypothesis, is this equation:

$$c' = f\lambda \left(1 + \frac{v}{c} \right),$$

which, upon shifting the wave parameters by the Doppler effect due to a moving source, takes this form:

$$c' = f'\lambda' \left(1 + \frac{v}{c} \right).$$

And therefore, based on this latter wave equation, the ballistic hypothesis predicts the following amount of Doppler shift in the wavelength of light assumed to be traveling in vacuum at the speed of $c' = c + v$:

$$\lambda' = \frac{\lambda}{1 + v/c}.$$

A Detailed Account of Doppler Effect by Reflection from Moving Mirrors:

A moving mirrors plays the role of a moving light detector and the role of a moving light source at the same time. Consequently, the calculations of Doppler effect by moving mirrors require firstly the use of the Doppler formulas for moving detectors to obtain the Doppler effect on the incident light, and secondly the use of the Doppler formulas for moving sources to compute the Doppler effect on the reflected light. And as such, those calculations, on the basis of any theory, are somewhat time-consuming and extensive.

The physical theories, under discussion, are three: One theory is labeled by Q. Majorana as the

'æthereal hypothesis'; and two theories are labeled by him as the '*emissive or ballistic hypothesis*'. But it's more convenient, here, to follow the classification scheme employed by A. Michelson and to call these theories respectively: The undulatory theory, the new-source theory, and the elastic-impact theory {Ref. #2.a & #2.b}.

It's a basic requirement, in this Majorana's experiment, to compute theoretical predictions of Doppler effect amplified several times by repeated reflections from moving mirrors.

In the calculations, to follow, v denotes the the velocity of the mirror:

$$v = \pi dg \cos\alpha \cos\theta$$

as defined by Q. Majorana above, along with the angle of incidence θ .

The method of computing Doppler effect, on wave frequencies and wavelengths, by computing the wave period is applied throughout the following calculations; since it's convenient, clear, and concise. In addition, the computed wave period, by this method, can be converted to the corresponding frequency or wavelength in just one single step.

Also, in these computations of Doppler effect, by multiple reflections from moving mirrors, it's necessary to calculate the wave period of light directly, at least in three successive reflections, on the basis of each theory, under investigation, in order to arrive, by mathematical induction, at the general formula of the amplified Doppler effect, within the framework of that theory.

A. The Prediction of the Undulatory Theory:

On the basis of the undulatory theory, incident light with the speed of c , frequency f , and wavelength λ is always reflected from a moving mirror with the same speed of c , but with shifted frequency f' and shifted wavelength λ' due to Doppler effect.

I. Doppler Effect by Multiple Reflections from Approaching Mirrors:

According to the undulatory theory, light emitted, with the wave period of T , by the stationary source S , in this Majorana's experiment, travels at the speed of c towards the approaching mirror M_1 . And therefore, the wave period T'_1 with respect to the approaching mirror M_1 is:

$$T'_1 = \frac{cT - vT'_1}{c} = T \left[\frac{c}{c + v} \right]$$

The approaching mirror M_1 reflects the incident light towards the stationary mirror F_1 , with the speed of c and the wave period of T'_2 :

$$T'_2 = \frac{cT'_1 - vT'_1}{c} = T'_1 \left[\frac{c-v}{c} \right] = T \left[\frac{c}{c+v} \right] \left[\frac{c-v}{c} \right] = T \left[\frac{c-v}{c+v} \right]$$

The stationary mirror F_1 reflects the incident light, with the wave period of T'_2 and the speed of c , towards the approaching mirror M_2 . And so, the wave period T'_3 with respect to the approaching mirror M_2 :

$$T'_3 = \frac{cT'_2 - vT'_3}{c} = T'_2 \left[\frac{c}{c+v} \right] = T \left[\frac{c-v}{c+v} \right] \left[\frac{c}{c+v} \right]$$

The approaching mirror M_2 reflects the incident light towards the stationary mirror F_2 with the speed of c and the wave period of T'_4 :

$$T'_4 = \frac{cT'_3 - vT'_3}{c} = T'_3 \left[\frac{c-v}{c} \right] = T \left[\frac{c-v}{c+v} \right]^2$$

The stationary mirror F_2 reflects the incident light, with the wave period of T'_4 and the speed of c , towards the approaching mirror M_3 . And hence, the wave period T'_5 with respect to the approaching mirror M_3 is:

$$T'_5 = \frac{cT'_4 - vT'_5}{c} = T'_4 \left[\frac{c}{c+v} \right] = T \left[\frac{c-v}{c+v} \right]^2 \left[\frac{c}{c+v} \right]$$

The approaching mirror M_3 reflects the incident light towards the stationary mirror F_3 with the speed of c and the wave period of T'_6 :

$$T'_6 = \frac{cT'_5 - vT'_5}{c} = T'_5 \left[\frac{c-v}{c} \right] = T \left[\frac{c-v}{c+v} \right]^3$$

From the common aspects of the calculated results above, it follows, therefore, that the general Doppler formula for computing the shifted wave period (T'_n) by multiple reflections from approaching mirrors, on the basis of the undulatory theory, is:

$$T'_n = T \left[\frac{c-v}{c+v} \right]^n$$

where n is the number of reflections.

And so, for any given number of reflections from approaching mirrors, in this experiment, the undulatory theory predicts a shifted period T'_n :

$$T'_n = T \left[\frac{c - v}{c + v} \right]^n = T \left[1 - \frac{2v}{c + v} \right]^n$$

Therefore, by using these two wave equations of the undulatory theory:

$$\begin{aligned} c' &= f\lambda \\ c' &= f'\lambda' \end{aligned}$$

the blue-shifted wavelength λ'_n can be obtained:

$$\lambda'_n = c'T'_n = cT'_n = \lambda \left[1 - \frac{2v}{c + v} \right]^n$$

For any given number of reflections from approaching mirrors, therefore, the undulatory theory predicts the amplified Doppler blue shift in the wavelength λ'_n of light by multiple reflections from approaching mirrors to be less by an amount of about:

$$\left(\frac{2nv^2/c^2}{1 + v/c} \right) \lambda$$

than the blue-shifted wavelength calculated by Q. Majorana.

II. Doppler Effect by Multiple Reflections from Receding Mirrors:

According to the undulatory theory, light emitted, with the wave period of T , by the source S , in Majorana's experiment, travels at the speed of c towards the receding mirror M_1 . And subsequently, the wave period T'_1 with respect to the receding mirror M_1 :

$$T'_1 = \frac{cT + vT'_1}{c} = T \left[\frac{c}{c - v} \right]$$

The receding mirror M_1 reflects the incident light towards the stationary mirror F_1 , with the speed of c and the wave period of T'_2 :

$$T'_2 = \frac{cT'_1 + vT'_1}{c} = T'_1 \left[\frac{c + v}{c} \right] = T \left[\frac{c}{c - v} \right] \left[\frac{c + v}{c} \right] = T \left[\frac{c + v}{c - v} \right]$$

And the stationary mirror F_1 reflects the incident light, with the wave period of T'_2 and the speed of c , towards the receding mirror M_2 . And so, the wave period T'_3 with respect to the receding mirror M_2 :

$$T'_3 = \frac{cT'_2 + vT'_3}{c} = T'_2 \left[\frac{c}{c-v} \right] = T \left[\frac{c+v}{c-v} \right] \left[\frac{c}{c-v} \right]$$

The receding mirror M_2 reflects the incident light towards the stationary mirror F_2 with the speed of c and the wave period of T'_4 :

$$T'_4 = \frac{cT'_3 + vT'_3}{c} = T'_3 \left[\frac{c+v}{c} \right] = T \left[\frac{c+v}{c-v} \right]^2$$

The stationary mirror F_2 reflects the incident light, with the wave period of T'_4 and the speed of c , towards the receding mirror M_3 . Accordingly, the wave period T'_5 with respect to the receding mirror M_3 is:

$$T'_5 = \frac{cT'_4 + vT'_5}{c} = T'_4 \left[\frac{c}{c-v} \right] = T \left[\frac{c+v}{c-v} \right] \left[\frac{c}{c-v} \right]$$

The receding mirror M_3 reflects the incident light towards the stationary mirror F_3 with the speed of c and the wave period of T'_6 :

$$T'_6 = \frac{cT'_5 + vT'_5}{c} = T'_5 \left[\frac{c+v}{c} \right] = T \left[\frac{c+v}{c-v} \right]^3$$

From the common characteristics of the calculated results above, it follows, therefore, that the general Doppler formula for computing the shifted wave period (T'_n) by multiple reflections from receding mirrors, on the basis of the undulatory theory, is:

$$T'_n = T \left[\frac{c+v}{c-v} \right]^n$$

where n is the number of reflections.

Accordingly, for any given number of reflections from receding mirrors, in this experiment, the undulatory theory predicts a shifted period T'_n :

$$T'_n = T \left[\frac{c+v}{c-v} \right]^n = T \left[1 + \frac{2v}{c-v} \right]^n$$

And therefore, by using these two wave equations of the undulatory theory:

$$c' = f\lambda$$

$$c' = f'\lambda'$$

the red-shifted wavelength λ'_n can be obtained:

$$\lambda'_n = c'T'_n = cT'_n = \lambda \left[1 + \frac{2v}{c-v} \right]^n$$

For any given number of reflections from receding mirrors, therefore, the undulatory theory predicts the amplified Doppler red shift in the wavelength λ'_n of light by multiple reflections from receding mirrors to be higher by an amount of about:

$$\left(\frac{2nv^2/c^2}{1-v/c} \right) \lambda$$

than the red-shifted wavelength computed by Q. Majorana.

B. The Prediction of the New-Source Theory:

On the basis of the new-source theory, incident light with the speed of c , frequency f , and wavelength λ is always reflected from a moving mirror with the speed resultant of c and the velocity of the mirror, and shifted frequency f' and shifted wavelength λ' due to Doppler effect.

I. Doppler Effect by Multiple Reflections from Approaching Mirrors:

According to the new-source theory, light emitted with the wave period of T , by the stationary source S , in the Majorana's experiment, travels at the speed of c towards the approaching mirror M_1 . And therefore, the wave period T'_1 with respect to the approaching mirror M_1 is:

$$T'_1 = \frac{cT - vT'_1}{c} = T \left[\frac{c}{c+v} \right]$$

The approaching mirror M_1 reflects the incident light towards the stationary mirror F_1 , with the speed resultant of $(c+v)$ and the wave period of T'_2 :

$$T'_2 = \frac{T'_1(c + v) - vT'_1}{c + v} = T'_1 \left[\frac{c}{c + v} \right] = T \left[\frac{c}{c + v} \right]^2$$

The stationary mirror F_1 reflects the incident light, with the wave period of T'_2 and the speed of c , towards the approaching mirror M_2 . And so, the wave period T'_3 with respect to the approaching mirror M_2 :

$$T'_3 = \frac{cT'_2 - vT'_3}{c} = T'_2 \left[\frac{c}{c + v} \right] = T \left[\frac{c}{c + v} \right]^3$$

The approaching mirror M_2 reflects the incident light towards the stationary mirror F_2 with the speed resultant of $(c + v)$ and the wave period of T'_4 :

$$T'_4 = \frac{T'_3(c + v) - vT'_3}{c + v} = T'_3 \left[\frac{c}{c + v} \right] = T \left[\frac{c}{c + v} \right]^4$$

The stationary mirror F_2 reflects the incident light, with the wave period of T'_4 and the speed of c , towards the approaching mirror M_3 . Consequently, the wave period T'_5 with respect to the approaching mirror M_3 is:

$$T'_5 = \frac{cT'_4 - vT'_5}{c} = T'_4 \left[\frac{c}{c + v} \right] = T \left[\frac{c}{c + v} \right]^5$$

The approaching mirror M_3 reflects the incident light towards the stationary mirror F_3 with the speed resultant of $(c + v)$ and the wave period of T'_6 :

$$T'_6 = \frac{T'_5(c + v) - vT'_5}{c + v} = T'_5 \left[\frac{c}{c + v} \right] = T \left[\frac{c}{c + v} \right]^6$$

By inspecting the common patterns of the calculated results above, it follows, therefore, that the general Doppler formula for computing the shifted wave period (T'_n) by multiple reflections from approaching mirrors, on the basis of the new-source theory, is:

$$T'_n = T \left[\frac{c}{c + v} \right]^{2n}$$

where n is the number of reflections.

Accordingly, for any given number of reflections from approaching mirrors, in this experiment, the new-source theory predicts a shifted period T'_n :

$$T'_n = T \left[1 - \frac{v}{c + v} \right]^{2n}$$

And since, in order to obtain the value of the wavelength λ'_n , in this experiment, Q. Majorana uses a stationary Michelson's interferometer to examine the reflected light (as shown in the figure #1), this light, according to the new-source theory, is reflected and transmitted by the stationary beam splitter of Michelson's interferometer towards the telescope C, with the shifted period T'_n but at the speed of c .

And therefore, the shifted wavelength λ'_n can be obtained, in this particular case, simply by multiplying the shifted period T'_n by the speed c :

$$\lambda'_n = cT'_n = \lambda \left[1 - \frac{v}{c + v} \right]^{2n}$$

For any given number of reflections by approaching mirrors, in Majorana's experiment, therefore, the new-source theory predicts an amplified Doppler blue shift in the wavelength λ'_n of light by multiple reflections from approaching mirrors to be less by an amount of about:

$$\left(\frac{nv^2/c^2}{1 + v/c} \right) \lambda$$

than the blue-shifted wavelength calculated by Q. Majorana.

II. Doppler Effect by Multiple Reflections from Receding Mirrors:

According to the new-source theory, light emitted, with the wave period of T , by the source stationary S , in Majorana's experiment, travels at the speed of c towards the receding mirror M_1 . And therefore, the wave period T'_1 with respect to the receding mirror M_1 is:

$$T'_1 = \frac{cT + vT'_1}{c} = T \left[\frac{c}{c - v} \right]$$

The receding mirror M_1 reflects the incident light towards the stationary mirror F_1 , with the speed resultant of $(c - v)$ and the wave period of T'_2 :

$$T'_2 = \frac{T'_1(c - v) + vT'_1}{c - v} = T \left[\frac{c}{c - v} \right]^2$$

The stationary mirror F_1 reflects the incident light, with the wave period of T'_2 and the speed of c , towards the receding mirror M_2 . And hence, the wave period T'_3 with respect to the receding mirror

M_2 :

$$T'_3 = \frac{cT'_2 + vT'_3}{c} = T \left[\frac{c}{c-v} \right]^3$$

The receding mirror M_2 reflects the incident light towards the stationary mirror F_2 with the speed resultant of $(c - v)$ and the wave period of T'_4 :

$$T'_4 = \frac{T'_3(c-v) + vT'_3}{c-v} = T \left[\frac{c}{c-v} \right]^4$$

The stationary mirror F_2 reflects the incident light, with the wave period of T'_4 and the speed of c , towards the receding mirror M_3 . And therefore, the wave period T'_5 with respect to the receding mirror M_3 is:

$$T'_5 = \frac{cT'_4 + vT'_5}{c} = T \left[\frac{c}{c-v} \right]^5$$

The receding mirror M_3 reflects the incident light towards the stationary mirror F_3 with the speed resultant of $(c - v)$ and the wave period of T'_6 :

$$T'_6 = \frac{T'_5(c-v) + vT'_5}{c-v} = T \left[\frac{c}{c-v} \right]^6$$

By examining the common characteristics of the calculated results above, it follows, therefore, that the general Doppler formula for computing the shifted wave period (T'_n) by multiple reflections from receding mirrors, on the basis of the new-source theory, is:

$$T'_n = T \left[\frac{c}{c-v} \right]^{2n}$$

where n is the number of reflections.

For any given number of reflections from receding mirrors, in this experiment, the new-source theory predicts a shifted period T'_n :

$$T'_n = T \left[1 + \frac{v}{c-v} \right]^{2n}$$

And since, in this experiment, in order to obtain the value of the wavelength λ'_n , Q. Majorana uses a

stationary Michelson's interferometer to examine the reflected light, this light, according to the new-source theory, is reflected and transmitted by the stationary beam splitter of the Michelson's interferometer towards the telescope C, with the shifted period T'_n but at the speed of c .

And therefore, the shifted wavelength λ'_n can be obtained, in this particular case, through the multiplication of the shifted period T'_n by the speed c :

$$\lambda'_n = cT'_n = \lambda \left[1 + \frac{v}{c-v} \right]^{2n}$$

For any given number of reflections from receding mirrors, in Majorana's experiment, therefore, the new-source theory predicts an amplified Doppler red shift in the wavelength λ'_n of light by multiple reflections from approaching mirrors to be higher by an amount of about:

$$\left(\frac{nv^2/c^2}{1-v/c} \right) \lambda$$

than the red-shifted wavelength calculated by Q. Majorana.

C. The Prediction of the Elastic-Impact Theory:

On the basis of the elastic-impact theory, incident light with the speed of c , frequency f , and wavelength λ is always reflected from a moving mirror with the speed resultant of its initial speed and twice the velocity of the mirror, shifted frequency f' and shifted wavelength λ' due to Doppler effect.

I. Doppler Effect by Multiple Reflections from Approaching Mirrors:

According to the elastic-impact theory, light emitted, with the wave period of T , by the stationary source S , in the Majorana's experiment, travels at the speed of c towards the approaching mirror M_1 . And therefore, the wave period T'_1 with respect to the approaching mirror M_1 is:

$$T'_1 = \frac{cT - vT'_1}{c} = T \left[\frac{c}{c+v} \right]$$

The approaching mirror M_1 reflects the incident light towards the stationary mirror F_1 , with the speed resultant of $(c + 2v)$ and the wave period T'_2 :

$$T'_2 = \frac{T'_1(c + 2v) - vT'_1}{c + 2v} = T'_1 \left[\frac{c + v}{c + 2v} \right] = T \left[\frac{c}{c + 2v} \right]$$

The stationary mirror F_1 reflects the incident light, with the wave period T'_2 and the same speed resultant $(c + 2v)$, towards the approaching mirror M_2 . And hence, the wave period T'_3 with respect to the approaching mirror M_2 :

$$T'_3 = \frac{T'_2(c + 2v) - vT'_3}{c + 2v} = T'_2 \left[\frac{c + 2v}{c + 3v} \right] = T \left[\frac{c}{c + 3v} \right]$$

The approaching mirror M_2 reflects the incident light towards the stationary mirror F_2 with the speed resultant $(c + 4v)$ and the wave period of T'_4 :

$$T'_4 = \frac{T'_3(c + 4v) - vT'_3}{c + 4v} = T'_3 \left[\frac{c + 3v}{c + 4v} \right] = T \left[\frac{c}{c + 4v} \right]$$

The stationary mirror F_2 reflects the incident light, with the wave period of T'_4 and the speed resultant of $(c + 4v)$, towards the approaching mirror M_3 . And accordingly, the wave period T'_5 with respect to the approaching mirror M_3 is:

$$T'_5 = \frac{T'_4(c + 4v) - vT'_5}{c + 4v} = T'_4 \left[\frac{c + 4v}{c + 5v} \right] = T \left[\frac{c}{c + 5v} \right]$$

The approaching mirror M_3 reflects the incident light towards the stationary mirror F_3 with the speed resultant $(c + 6v)$ and the wave period of T'_6 :

$$T'_6 = \frac{T'_5(c + 6v) - vT'_6}{c + 6v} = T'_5 \left[\frac{c + 5v}{c + 6v} \right] = T \left[\frac{c}{c + 6v} \right]$$

From the common characteristics of the computed results above, it follows, therefore, that the general Doppler formula for calculating the shifted wave period (T'_n) by multiple reflections from approaching mirrors, on the basis of the elastic-impact theory, is:

$$T'_n = T \left[\frac{c}{c + 2nv} \right]$$

where n is the number of reflections.

For any given number of reflections from approaching mirrors, in this experiment, the elastic-impact theory predicts a shifted period T'_n :

$$T'_n = T \left[1 - \frac{2nv}{c + 2nv} \right]$$

And therefore, by using the following wave equations of the elastic-impact theory for light assumed to travel in vacuum at the speed resultant $(c + v)$:

$$c' = f\lambda \left(1 + \frac{v}{c} \right),$$

$$c' = f'\lambda' \left(1 + \frac{v}{c} \right).$$

the blue-shifted wavelength λ'_n can be obtained:

$$\lambda'_n = \frac{c'T'_n}{1 + v/c} = \lambda \left[1 - \frac{2nv}{c + 2nv} \right]$$

For any given number of reflections by approaching mirrors, therefore, the elastic-impact theory predicts the amplified Doppler blue shift in the wavelength λ'_n of light by multiple reflections from approaching mirrors to be less by an amount of about:

$$\left(\frac{4n^2 v^2 / c^2}{1 + 2nv/c} \right) \lambda$$

than the blue-shifted wavelength calculated by Q. Majorana.

II. Doppler Effect by Multiple Reflections from Receding Mirrors:

According to the elastic-impact theory, light emitted, with the wave period of T , by the stationary source S , in Majorana's experiment, travels at the speed of c towards the receding mirror M_1 . And therefore, the wave period T'_1 with respect to the receding mirror M_1 is:

$$T'_1 = \frac{cT + vT'_1}{c} = T \left[\frac{c}{c - v} \right]$$

The receding mirror M_1 reflects the incident light towards the stationary mirror F_1 with the speed resultant $(c - 2v)$ and the wave period of T'_2 :

$$T'_2 = \frac{T'_1(c - 2v) + vT'_1}{c - 2v} = T'_1 \left[\frac{c - v}{c - 2v} \right] = T \left[\frac{c}{c - 2v} \right]$$

The stationary mirror F_1 reflects the incident light, with the wave period of T'_2 and the same speed resultant of $(c - 2v)$, towards the receding mirror M_2 . And so, the wave period T'_3 with respect to the receding mirror M_2 :

$$T'_3 = \frac{T'_2(c - 2v) + vT'_3}{c - 2v} = T'_2 \left[\frac{c - 2v}{c - 3v} \right] = T \left[\frac{c}{c - 3v} \right]$$

The receding mirror M_2 reflects the incident light towards the stationary mirror F_2 with the speed resultant $(c - 4v)$ and the wave period of T'_4 :

$$T'_4 = \frac{T'_3(c - 4v) + vT'_4}{c - 4v} = T'_3 \left[\frac{c - 3v}{c - 4v} \right] = T \left[\frac{c}{c - 4v} \right]$$

The stationary mirror F_2 reflects the incident light, with the wave period of T'_4 and the speed resultant of $(c - 4v)$, towards the receding mirror M_3 . And hence, the wave period T'_5 with respect to the receding mirror M_3 is:

$$T'_5 = \frac{T'_4(c - 4v) + vT'_5}{c - 4v} = T'_4 \left[\frac{c - 4v}{c - 5v} \right] = T \left[\frac{c}{c - 5v} \right]$$

The receding mirror M_3 reflects the incident light towards the stationary mirror F_3 with the speed resultant $(c - 6v)$ and the wave period of T'_6 :

$$T'_6 = \frac{T'_5(c - 6v) + vT'_6}{c - 6v} = T'_5 \left[\frac{c - 5v}{c - 6v} \right] = T \left[\frac{c}{c - 6v} \right]$$

From the shared aspects of the calculated results above, it follows, therefore, that the general Doppler formula for computing the shifted wave period (T'_n) by multiple reflections from receding mirrors, on the basis of the elastic-impact theory, is:

$$T'_n = T \left[\frac{c}{c - 2nv} \right]$$

where n is the number of reflections.

And so, for any given number of reflections by receding mirrors, in this experiment, the elastic-impact theory predicts a shifted period T'_n :

$$T'_n = T \left[1 + \frac{2nv}{c - 2nv} \right]$$

Therefore, by using these wave equations of the elastic-impact theory for light travels in vacuum at the speed resultant of $(c - v)$:

$$c' = f\lambda \left(1 - \frac{v}{c} \right),$$

$$c' = f'\lambda' \left(1 - \frac{v}{c} \right)$$

the red-shifted wavelength λ'_n can be obtained:

$$\lambda'_n = \frac{c' T'_n}{1 - v/c} = \lambda \left[1 + \frac{2nv}{c + 2nv} \right]$$

For any given number of reflections from receding mirrors, therefore, the elastic-impact theory predicts the amplified Doppler red shift in the wavelength λ'_n of light by multiple reflections from receding mirrors to be higher by an amount of about:

$$\left(\frac{4n^2 v^2 / c^2}{1 - 2nv/c} \right) \lambda$$

than the red-shifted wavelength calculated by Q. Majorana.

Conclusion:

As demonstrated above, calculated predictions, within the framework of each theory in this experimental test, show only differences of the second order in the shifted wavelength by multiple reflections from moving mirrors.

And since, it's, initially, designed to check for differences of the first order only, the Majorana's experiment, under discussion, cannot, in practice, provide any experimental evidence for or against any of the three theories in this experimental test.

However, the current experiment, by Q. Majorana, has the potential, upon minor modifications, to measure differences of the second order in the shifted wavelength, and to provide experimental evidence in this regard.

The computed predictions of the undulatory theory, the new-source theory, and the elastic-impact theory have differences of the second order in the following ratio: 2 : 1 : 4, respectively. And because these differences of the second order in the Doppler shift can be amplified to any desirable level by repeated reflections from moving mirrors, this Majorana's experiment has, indeed, a real potential of distinguishing effectively between these theories and providing decisive evidence for or against the independent speed of light of the speed of the light source.

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