

General relativity shown to be Newtonian gravitational theory

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By a great deal of difficulty I showed how Einstein had dealt with Galilean relativity incorrectly; and once Galilean relativity is dealt with correctly it is still valid. Now from that Galilean relativity I shall show it leads to General relativity; where General relativity is merely Newtonian gravitational theory with its maths manipulated to make it look different.

1. Galilean Physics works

Recovering “old” ground as dealt with in previous papers:

It just means that Einstein never worked from the proper maths for Galilean relativity; hence he never gave any justification as to why to change from the existing theory.

If we treat SR as a different theory to Galilean relativity; then Einstein never gave any justification for changing from Galilean relativity to SR.

If we look at things another way-- as SR being bodged maths, then correct the maths mistakes with SR then it turns back into Galilean relativity; i.e. SR is just bodged Galilean relativity.

Galilean relativity is as follows-

Given :

$$x - ct = 0$$

$$x' - (c - v) t' = 0$$

(where we are considering the case of only one dimension of space)

the first equation is the unprimed frame and the second equation is the primed frame

The Galilean transform (GT) $x' = x - vt$, $t' = t$

transforms the first equation into the second equation

i.e. subst : $x' = x - vt$, $t' = t$ into $x' - (c - v) t' = 0$

gives:

$x - vt - ct - vt$ and taking note that $x = ct$ this gives $ct - vt - ct - vt = 0$

So when $v = 0$ GT becomes $x' = x$, $t' = t$

and $x' - (c - v) t' = 0$ becomes $x' - ct' = 0$

i.e.

$$\begin{aligned}x - ct &= 0 \\x' - ct' &= 0\end{aligned}$$

is the trivial GT of $v = 0$ and $x = x'$, $t' = t$

However what Einstein does is mistake

$$\begin{aligned}x - ct &= 0 \\x' - ct' &= 0\end{aligned}$$

needing a transform for v non-zero, and he noted that GT treating v as non-zero does not work.

He then went looking for a transform treating v as non-zero.

So Einstein's error is that he does not realise that GT is for:

$$\begin{aligned}x - ct &= 0 \\x' - (c - v) t' &= 0\end{aligned}$$

for v non-zero and $v = 0$.

He gives no justification as to why this transform should be replaced by another transform; hence no need for SR.

Instead he falsely believes GT should be for:

$$x - ct = 0$$

$$x' - ct' = 0$$

for v non-zero, when it isn't.

The whole SR structure is false.

2. Galilean Relativity

The Galilean relativity case for one dimension of space is as follows:

$$x - ct = 0$$

$$x' - (c - v) t' = 0$$

The Galilean transform (GT) $x' = x - vt$, $t' = t$

We can square this and write as:

$$x^2 - c^2 t^2 = 0$$

$$x'^2 - (c - v)^2 t'^2 = 0$$

The Galilean transform (GT) $x' = x - vt$, $t' = t$

(For check see Appendix 3(i))

If we write in the case of 3-D space still with speed v along x direction, we have:

$$x^2 + y^2 + z^2 - c^2 t^2 = 0$$

$$x'^2 + y'^2 + z'^2 - (c - v)^2 t'^2 = 0$$

The Galilean transform (GT) $x' = x - vt$, $t' = t$, $y = y'$, $z = z'$

(For check see appendix 3 (ii), and it also worth making note that in the case under consideration it is light travelling along x axis, not along y and z axis; so for light travelling along x axis $x = ct$. No displacement along y and z so really treating them as $y=y'=z=z'=0$.)

3. Uniform gravitational field

Working from:

$$x^2 + y^2 + z^2 - c^2 t^2 = 0$$

$$x'^2 + y'^2 + z'^2 - (c - v)^2 t'^2 = 0$$

The Galilean transform (GT) $x' = x - vt$, $t' = t$, $y = y'$, $z = z'$

From Newtonian equations of motion we have:

$$v^2 = u^2 + 2as$$

v = final speed (not equating it with previous use of v just yet)

u = initial speed

a = acceleration

s = distance traversed

So this equation describes a point-particle travelling with acceleration a .

If we have gravity we equate this acceleration a with gravitational acceleration g .

The Newtonian gravitational force equation is

$$F = mg$$

gravitational force on point-particle having mass m and acceleration g

Equating a with g

$$\text{We just write: } v^2 = u^2 + 2as$$

$$\text{as } v^2 = u^2 + 2gs$$

if we deal with distance along x direction then $s = x$ giving, taking initial speed u as zero we have:

$$v^2 = 2gx$$

subst this into:

$$\begin{aligned} x^2 + y^2 + z^2 - c^2 t^2 &= 0 \\ x'^2 + y'^2 + z'^2 - (c - v)^2 t'^2 &= 0 \end{aligned}$$

The Galilean transform (GT) $x' = x - vt$, $t' = t$, $y = y'$, $z = z'$

and we have :

$$\begin{aligned} x^2 + y^2 + z^2 - c^2 t^2 &= 0 \\ x'^2 + y'^2 + z'^2 - (c - \text{sqrt}(2gx))^2 t'^2 &= 0 \end{aligned}$$

The Galilean transform (GT) $x' = x - vt$, $t' = t$, $y = y'$, $z = z'$

We can rearrange $(c - \text{sqrt}(2gx))^2 t'^2$

$$\text{as } c^2 (1 - \sqrt{2gx}/c)^2 t'^2$$

So we have:

$$\begin{aligned} x^2 + y^2 + z^2 - c^2 t^2 &= 0 \\ x'^2 + y'^2 + z'^2 - c^2 (1 - \sqrt{2gx}/c)^2 t'^2 &= 0 \end{aligned}$$

The Galilean transform (GT) $x' = x - vt$, $t' = t$, $y = y'$, $z = z'$

we could have written as differentials dx instead of x and so forth then :

$$\begin{aligned} dx^2 + dy^2 + dz^2 - c^2 dt^2 &= 0 \\ dx'^2 + dy'^2 + dz'^2 - c^2 (1 - \sqrt{2gx}/c)^2 dt'^2 &= 0 \end{aligned}$$

The Galilean transform (GT) $x' = x - vt$, $t' = t$, $y = y'$, $z = z'$

Looking at:

$$dx'^2 + dy'^2 + dz'^2 - c^2 (1 - \sqrt{2gx}/c)^2 dt'^2 = 0$$

We could drop the dashes:

$$dx^2 + dy^2 + dz^2 - c^2 (1 - \sqrt{2gx}/c)^2 dt^2 = 0$$

This is then the metric equation in a uniform gravitational field according to Peter M. Brown, except he has $2gx/c^2$ whereas I have $\sqrt{2gx}/c$.
 . (See appendix where I have to deal with Brown's mistakes.)

i.e. we have a General relativity (GR) equation derived from Galilean relativity using Newtonian equations barring the issue of $2gx/c^2$ and $\sqrt{2gx}/c$. The difference being I think that his equation is for gravity directed towards a point, and mine is for parallel lines of gravitational force; an issue I will not be going into. Putting that issue aside:

Galilean relativity with Newtonian equations is Newtonian gravitational theory.

The metric that is supposedly General Relativity conforms to being an equation of Newtonian gravitational theory.

i.e. both theories are really the same mathematically, provided the maths is done correctly. What separates is that Einstein chooses to interpret from non-Euclidean geometry and Newton from Euclidean geometry. However, mathematically both geometries are mathematically consistent as shown by mathematicians; so from a physics point-of-view it is mere choice as to how to physically interpret nature by one geometry or the other.

4. Schwarzschild

This shall be based on Schwarzschild solution as given by standard relativity texts; not the slightly different solution that Schwarzschild himself dealt with.

For Schwarzschild we are considering in spherical coordinates r, θ, ϕ . Spherical symmetry is being considered so can concentrate just on the r direction

So for the earlier case of:

$$dx^2 + dy^2 + dz^2 - c^2 (1 - \sqrt{2gx}/c)^2 dt^2 = 0$$

$$\text{We are now considering } dx^2 + dy^2 + dz^2 = dr^2$$

being symmetrical effect in x, y and z directions.

$$\text{So now } r \text{ instead of } x \text{ in } c^2 (1 - \sqrt{2gx}/c)^2 dt^2$$

Ignoring for the moment θ, ϕ

So we have the equation:

$$dr^2 = c^2 (1 - \sqrt{2gr}/c)^2 dt^2$$

Re-arranging the equation we have:

$$dr^2 / dt^2 = c^2 (1 - \sqrt{2gr}/c)^2$$

This is our velocity squared same as in previous section.

We can do now re-arrange as follows to:

$$(1 - \sqrt{2gr}/c)^{-1} dr^2 = c^2 (1 - \sqrt{2gr}/c) dt^2$$

we can do this because this still gives: $dr^2 / dt^2 = c^2 (1 - \sqrt{2gr}/c)^2$

Bringing into consideration θ, ϕ we would write:

$$(1 - \sqrt{2gr}/c^2)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 = c^2 (1 - \sqrt{2gr}/c^2) dt^2$$

Subtracting right hand side from left hand side:

$$c^2 (1 - \sqrt{2gr}/c) dt^2 - (1 - \sqrt{2gr}/c)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 = 0$$

$ds^2 = 0$ so it is also written:

$$c^2 (1 - \sqrt{2gr}/c) dt^2 - (1 - \sqrt{2gr}/c)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 = ds^2$$

Now using:

$$F = mg = GMm/r^2$$

$$\text{We have: } g = GM/r^2$$

Substitute this into the equation and we have:

$$c^2 (1 - \sqrt{2GM/rc^2}) dt^2 - (1 - \sqrt{2GM/rc^2})^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 = ds^2$$

And this is the Schwarzschild metric except for the issue of $\sqrt{2GM/rc^2}$ and $2GM/rc^2$. Putting that issue aside:

the maths is the same as Newtonian gravitational theory.

For $ds^2 = 0$ the equation describes the path that light takes under gravity as explained by Newtonian gravity theory.

And the thing to note is that this is based on variable light speed.

In our notation here we have:

$$dr/dt = c (1 - \sqrt{2gr/c^2}) = c (1 - \sqrt{2GM/rc^2})$$

Where dr/dt is the variable light speed.

The light speed c is a constant; it is the speed of light in vacuum free of being influenced by fields.

It is unfortunate that Einstein confused the issue with the constancy of light speed, so we now have the difficulty of badly defined words; with c as the constant light speed and dr/dt as the variable light speed (sometimes called

coordinate light speed). Due to Einstein there has been confusion with thinking that light speed is permanently constant, but really it is only the term c that is constant and in general light speed is variable dr/dt .

Thus we have the solutions attributed to General Relativity arises from Newtonian gravitational theory, and when gravity is negligible this reduces to the setup of Galilean relativity.

5. Special Relativity

Special relativity deals with the case of connecting:

$$x^2 - c^2 t^2 = 0$$

with

$$x'^2 - c'^2 t'^2 = 0$$

it assumes $c' = c$

Which is lightspeed constancy, but in the case of GR it has to take on the assumption c does not equal c'

Namely from corrected Brown:

$$v = c (1 + \Phi / c^2)$$

or in notation we are using of c' as v :

$$c' = c (1 + \Phi / c^2)$$

i.e. c' is the variable speed of light.

From this we can derive Schwarzschild metric etc.

The thing to note is we are not deriving from:

$$x^2 - c^2 t^2 = 0$$

with

$$x'^2 - c^2 t'^2 = 0$$

Which would be SR proper as it is known, but instead from:

$$x^2 - c^2 t^2 = 0$$

with

$$x'^2 - c'^2 t'^2 = 0$$

With c' not equal to c i.e. lightspeed variable.

This second case is more what Galilean-Newtonian physics would be working from as regards a variable lightspeed.

Looking at:

$$x^2 - c^2 t^2 = 0$$

with

$$x'^2 - c'^2 t'^2 = 0$$

The use of $c' = c(1 + \Phi/c^2)$

though can seem a bit of a mystery.

Unfortunately I will not be looking at this any further for the moment, other than to make my comment again that I believe it this form when considering gravity directed towards a gravitational source.

Putting that issue aside and going by just:

$$x^2 - c^2 t^2 = 0$$

with

$$x'^2 - c'^2 t'^2 = 0$$

Because we use this for GR instead of:

$$x^2 - c^2 t^2 = 0$$

with

$$x'^2 - c^2 t'^2 = 0$$

I claim GR comes from Newtonian physics not SR.

$x^2 - c^2 t^2 = 0$ with $x'^2 - c^2 t'^2 = 0$ would be SR, and $x^2 - c^2 t^2 = 0$

With $x'^2 - c'^2 t'^2 = 0$ would be Newtonian physics.

Admittedly using $c' = c(1 + \Phi/c^2)$ for:

$$x^2 - c^2 t^2 = 0$$

with

$$x'^2 - c'^2 t'^2 = 0$$

might make it seem a slightly modified Newtonian physics on its way to GR.:

Conclusion

Galilean relativity works. When we correct the maths mistakes in Special Relativity it turns back into Galilean relativity. From Galilean relativity and Newtonian gravitational theory * we can derive General relativity.

(* - note proviso that it might seem a slightly modified Newtonian gravitational theory to some; but the problem of that appearance is caused by me having to look at the numerous mistakes made on the way to GR by Einstein et al. making the issue – if we correct the mistakes made in existing maths of physics is that then modified Newtonian physics or Newtonian physics corrected back to what it should have been.)

So, we have the surprising situation that General relativity with Special relativity as its foundation is built upon bad maths. Throw away the bad maths, instate Galilean relativity as the foundation of General relativity and barring any other mistakes we have mathematical consistency.

As has been noted by some people - the problem with General relativity is its basis of Special relativity; General relativity “they” note works fine until it starts to approximate to Special relativity domain and then all the maths problems begin. Of course the maths problems begin then because the standard maths of Special relativity is bad.

Einstein makes his maths mistakes changes the descriptive from Newton, and he ends up looking like his has created different theories; really his Special relativity when its maths mistakes is corrected is back to Galilean relativity; and his General relativity is Newtonian gravitational theory in disguise.

To be charitable to Einstein what he achieved was he showed that Newtonian physics could be placed into the setting of non-Euclidean geometry (which is his General relativity) not just in the usual setting of Euclidean geometry. It

was just unfortunate that he had so much difficulty with the maths that this fact can get lost in all his mistakes.

There are so many issues involved that I will come back to these topics again in later papers.

Additional

I think it worth taking note of a few things – because of the bad treatment of maths in relativity texts there has been many pitfalls that one can get trapped in. A peculiarity is for the treatment of:

$$x^2 + y^2 + z^2 - c^2 t^2 = 0$$

many texts treat the case of light travelling along x and not along y and z, so effectively y and z are treated as zero; this would be different from the case of light travelling along $\sqrt{x^2 + y^2 + z^2}$. I have adopted that peculiarity of treatment in this text up to section 3.

Another peculiarity to note is many (SR) relativity texts consider the two-way measurement of light where light travels a distance from emitter to mirror and gets reflected back to emitter; the total distance of this two-way travel is then being considered. If both emitter and mirror are moving with velocity v then light travels first (c+v)t distance gets reflected and travels (c-v)t distance

(Assuming it takes same time interval t for both distances; actually maybe different times (but – pass for now).) The distances are then $x_1 = (c+v)t$

$x_2 = (c - v)t$ then we multiply both these together to form:

$$x_1 x_2 = (c - v) t (c + v) t = (c^2 - v^2) t^2$$

We treat $x_1 x_2$ as x^2 then

$$x^2 = (c^2 - v^2) t^2$$

Where x is mean distance

Similarly if deal with $x_1 = (c+v)t_1$ and $x_2 = (c - v)t_2$

with $t^2 = t_1 t_2$

Where t would now be mean time interval.

Introducing y and z gives us:

$$x^2 + y^2 + z^2 = (c^2 - v^2) t^2$$

This equation might be more what they are dealing with in certain SR scenarios (but they have errors confusing the issue) more so than the cases being considered here of the type:

$$x^2 + y^2 + z^2 - (c - v)^2 t^2 = 0$$

$$(Or\ with\ dashes\ as\ x'^2 + y'^2 + z'^2 - (c - v)^2 t'^2 = 0)$$

Which is more the case of light going in one direction not the two-way case as created by reflection.

I hope made this clear – the problem is the different scenarios muddled by standard relativity texts and so forth.

Appendix 1. Speed of Light in a uniform gravitational field.

Brown gives us the Speed of Light in a Uniform Gravitational field as follows; with my corrections:

Brown: The metric in a uniform gravitational field is, according to the equivalence principle, identical to the metric in a uniformly accelerating frame of reference [1]

$$(Eq. 1) \quad ds^2 = c^2 \left(1 + gz / c^2\right)^2 dt^2 - dx^2 - dy^2 - dz^2$$

Where c is the speed of light in a vacuum in a Minkowski frame of reference. Since light moves on null geodesics we set $ds^2 = 0$

me: he is considering here distance being travelled along z

Brown:

$$\begin{aligned} \text{(Eq.2)} \quad 0 &= c^2 \left(1 + gz/c^2\right)^2 dt^2 - dx^2 - dy^2 - dz^2 \\ dx^2 + dy^2 + dz^2 &= c^2 \left(1 + gz/c^2\right)^2 dt^2 \end{aligned}$$

Divide through by dt^2 to obtain

$$\begin{aligned} \text{(Eq.3)} \quad \frac{dx^2 + dy^2 + dz^2}{dt^2} &= c^2 \left(1 + gz/c^2\right)^2 \\ \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 &= c^2 \left(1 + gz/c^2\right)^2 \end{aligned}$$

Now substitute $v_x \equiv dx/dt$, $v_y \equiv dy/dt$, $v_z \equiv dz/dt$, $v^2 \equiv v_x^2 + v_y^2 + v_z^2$

$$\begin{aligned} \text{(Eq.4)} \quad v_x^2 + v_y^2 + v_z^2 &= c^2 \left(1 + gz/c^2\right)^2 \\ v^2 &= c^2 \left(1 + gz/c^2\right)^2 \end{aligned}$$

me: this v is a different v to the one I was using in section 3.

He has the general velocity v as $v^2 \equiv v_x^2 + v_y^2 + v_z^2$

i.e. he is considering distance travelled along x, y and z and not just along z.
So his "z" in

$$\begin{aligned} \text{(Eq.4)} \quad v_x^2 + v_y^2 + v_z^2 &= c^2 \left(1 + gz/c^2\right)^2 \\ v^2 &= c^2 \left(1 + gz/c^2\right)^2 \end{aligned}$$

It should have been written with $r^2 \equiv x^2 + y^2 + z^2$

Then we have instead:

$$\begin{aligned} v_x^2 + v_y^2 + v_z^2 &= c^2 \left(1 + gr/c^2\right)^2 \\ v^2 &= c^2 \left(1 + gr/c^2\right)^2 \end{aligned}$$

Anyway, taking direction of travel along negative x direction not along z and not along r, we have:

$$ds^2 = c^2 \left(1 - gx/c^2\right)^2 dt^2 - dx^2 - dy^2 - dz^2$$

Swap the signs around:

$$ds^2 = -c^2(1 - gx/c^2)^2 dt^2 + dx^2 + dy^2 + dz^2$$

This is the same as we have from section 3:

$$dx^2 + dy^2 + dz^2 - c^2(1 - 2gx/c^2)^2 dt^2 = 0$$

Except Brown has missed out the “2” i.e. not taken into account from:

$$v^2 = 2gx \quad (\text{when } u=0, a=g, a=x)$$

Brown sets $\Phi = gz$, but really for him it is gx .

Note that we choose the arbitrary additive constant that is usually associated with a Newtonian potential, to be zero. Our final result is obtained by taking the square root of both sides of this expression

$$\text{(Eq.5)} \quad v = \left(1 + \Phi/c^2\right) c$$

He then says- This is exactly the result obtained by Einstein in 1907.

That’s when Einstein made the “2” error.

The reference he gives is: **The Principle of Relativity**, Lorentz, Einstein, Minkowski, Weyl, *Dover Pub.* See article which starts on page 99, i.e. **On the Influence of Gravitation on the Propagation of Light**.

There Einstein has some typo error as well, for he writes:

$$c = c_0 (1 + \Phi/c^2)$$

The subscript o has been missed in the third use of c, so should read:

$$c = c_0 (1 + \Phi/c_0^2)$$

c = variable speed of light

c_0 = the speed of light in vacuum that we usually consider constant.

Instead of this Brown writes:

$$\text{(Eq.5)} \quad v = \left(1 + \Phi/c^2\right) c$$

Where v is now the variable speed of light and c is now what Einstein wrote as c_0

The speed of light in Eq. (5) is known as the **coordinate speed of light**, according to Brown which I take to mean his v .

My use of v in section 3 was

$$v = 2gx/c^2$$

$$\text{in } dx^2 + dy^2 + dz^2 - c^2 (1 - 2gx/c^2)^2 dt^2 = 0$$

$$\text{his } v \text{ is } v^2 = c^2 (1 + gx/c^2)^2$$

And for the correction of “2” having distance in negative x direction not along r being corrected to:

$$v^2 = c^2 (1 + 2gx/c^2)^2$$

is something different.

I hope that is clear.

It might have been better if I used subscript 1 for my v and 2 for his v then:

$$v_1 = 2gx/c^2$$

$$v_2^2 = c^2 (1 + 2gx/c^2)^2$$

But there has been so many corrections made already to the mistakes made by Einstein et al.

According to Brown Eq. (4) states that as light rises in a uniform gravitational field, the coordinate speed will increase. If the light travels in the opposite direction the coordinate speed of light will decrease.

It is well known that Einstein was out by a factor of “2” in his 1907 calculation and had to update in subsequent papers, such as in 1915. This was the whole fuss over the 1919 astronomical observation of deflection of light—was it bending by the amount predicted by Einstein’s theory or by Newton’s theory. It was deemed to obey Einstein. But the calculation for Newtonian theory had been done wrong and was out by a factor of “2” when corrected it gave the same value as Einstein’s theory; which is not really a surprise because the maths of Newton theory gives the same as Einstein’s theory; really Einstein’s GR theory **IS** Newton’s theory if we stick with Newtonian descriptive.

Appendix 2. Speed of Light in a Schwarzschild (spherical symmetric) gravitational Field

Brown deals with the Schwarzschild solution much better as follows:

The gravitational field of a spherical body is described by the Schwarzschild metric

$$(Eq. 6) \quad ds^2 = c^2 \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

As above we set $ds^2 = 0$ in Eq. (1) :

$$(Eq. 1) \quad ds^2 = c^2 \left(1 + gz/c^2\right)^2 dt^2 - dx^2 - dy^2 - dz^2$$

gives us:

$$(Eq. 7) \quad 0 = c^2 \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

Divide through by dt^2 , and substitute $v_r \equiv dr/dt$, $v_\theta \equiv d\theta/dt$, $v_\phi \equiv d\phi/dt$

$$(Eq. 8) \quad \frac{1}{1 - \frac{2GM}{rc^2}} \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\theta}{dt}\right)^2 + r^2 \sin^2 \theta \left(\frac{d\phi}{dt}\right)^2 = c^2 \left(1 - \frac{2GM}{rc^2}\right)$$

$$\frac{1}{1 - \frac{2GM}{rc^2}} v_r^2 + r^2 v_\theta^2 + r^2 \sin^2 \theta v_\phi^2 = c^2 \left(1 - \frac{2GM}{rc^2}\right)$$

If the light is radial, i.e. $v_\theta = v_\phi = 0$, then Eq. (7) simplifies to

$$(Eq. 9) \quad \frac{1}{1 - \frac{2GM}{rc^2}} v_r^2 = c^2 \left(1 - \frac{2GM}{rc^2}\right)$$

$$v_r^2 = c^2 \left(1 - \frac{2GM}{rc^2}\right)^2$$

If we now take the square root of both sides of Eq. (9) then we have the speed of light that is travelling radially in Schwarzschild geometry

$$(Eq. 10) \quad v_r = \left(1 - \frac{2GM}{rc^2}\right)c$$

Therefore as light gets closer to the center of the gravitating object the coordinate speed of light slows down.

Appendix 3 (i)

I have realised that I need to check calculations; sometimes it's just too easy to think it looks correct without properly checking again. I think it's the type of error Einstein made.

From section 2:

The Galilean transform (GT) $x' = x - vt$, $t' = t$

We can square this and write as:

$$\begin{aligned} x^2 - c^2 t^2 &= 0 \\ x'^2 - (c-v)^2 t'^2 &= 0 \end{aligned}$$

The Galilean transform (GT) $x' = x - vt$, $t' = t$

Now checking that this is in fact the case:

$$\begin{aligned} x'^2 - (c-v)^2 t'^2 &= (x-vt)^2 - (c-v)^2 t^2 \\ &= x^2 + v^2 t^2 - 2xvt - (c^2 + v^2 - 2cv) t^2 \end{aligned}$$

Now subst $x^2 = c^2 t^2$ and $x = ct$

$$\begin{aligned} c^2 t^2 + v^2 t^2 - 2cv t^2 - c^2 t^2 - v^2 t^2 + 2cv t^2 \\ = 0 \text{ as required.} \end{aligned}$$

okay.

Appendix 3 (ii)

If we write in the case of 3-D space still with speed v along x direction, we have:

$$\begin{aligned} x^2 + y^2 + z^2 - c^2 t^2 &= 0 \\ x'^2 + y'^2 + z'^2 - (c-v)^2 t'^2 &= 0 \end{aligned}$$

The Galilean transform (GT) $x' = x - vt$, $t' = t$, $y = y'$, $z = z'$

Checking :

$$x'^2 + y'^2 + z'^2 - (c - v)^2 t'^2 = (x - vt)^2 + y^2 + z^2 - (c - v)^2 t^2$$

$$= x^2 + v^2 t^2 - 2xvt + y^2 + z^2 - (c^2 + v^2 - 2cv) t^2$$

$$= x^2 + y^2 + z^2 + v^2 t^2 - 2xvt - c^2 t^2 - v^2 t^2 + 2cv t^2$$

$$\text{subst } x^2 + y^2 + z^2 = c^2 t^2$$

$$c^2 t^2 + v^2 t^2 - 2xvt - c^2 t^2 - v^2 t^2 + 2cv t^2$$

$$= -2xvt + 2cv t^2$$

We are considering the case of light travelling along x direction, not along y and z, so really $y=z=0$ (i.e. no displacement along there) and $x = ct$, so that this in above gives:

$$-2ctvt + 2cv t^2 = 0$$

If we were considering the case:

$$x^2 + y^2 + z^2 = c^2 t^2$$

For x, y, z as all non-zero i.e. light travelling in this 3-D space x,y,z not just along the 1-D direction x we would have the scenario different with:

$$\text{sqrt}(x^2 + y^2 + z^2) = +ct \text{ or } -ct$$

References

Speed of Light in a Gravitational Field

http://www.geocities.com/physics_world/gr/c_in_gfield.htm from Physics World website run by: Peter M. Brown at: http://www.geocities.com/physics_world/

[1] **The Principle of Equivalence**, F. Rohrlich, *Annals of Physics*, 22, 160-191 (1963).

[2] **The Principle of Relativity**, Lorentz, Einstein, Minkowski, Weyl, *Dover Pub.* See article which starts on page 99, i.e. **On the Influence of Gravitation on the Propagation of Light**.

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