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The Theory of Static Gravitational Field

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ABSTRACT

The paper discusses in detail assumptions needed for the derivation of metric for the non-rotating centrally gravitating body, which is not based on Einstein field equations. The metric derivation is then generalized to any static mass configuration. It is shown that the Schwarzschild metric, which is the solution of Einstein field equations for the centrally gravitating body, is only a first order approximation of the correct metric and that the Einstein field equation concept is a wrong concept for finding any metric. Several examples of observations in support of the developed theory, which are not as easily explained in the main stream literature, are also presented in this paper.

Key words: Minkowski Space Time, General Relativity Theory, Einstein Field Equations, Black Holes, Schwarzschild Metric, Ricci Scalar, Metric for the Centrally Gravitating Body, Aether.

INTRODUCTION

Any modern theory that has been constructed so far is based on assumptions that are more or less in agreement with observations or are somehow self evident. This does not mean that the theory is absolutely true or that it cannot be later corrected, modified, improved, or proven wrong when more precise data become available for comparison. However, the theory must be clear in defining these assumptions and the assumptions cannot contradict each other or the well established laws of physics. The well known tool to work these problems out is the mathematical formalism, so the math is valuable to resolve such conflicts should they arise. However, the math must be clear and not self-absorbing with many abstract notions that mathematicians like so much to the point that the physics becomes completely lost in it. An example of this problem is the variational principle for the derivation of Einstein field equations followed by the arbitrary assumption about the mass energy tensor. The theory that will be derived in this paper will avoid these pitfalls. However, before we embark on the derivation of the new metric it is also necessary to make few comments about the coordinates.

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SPACE-TIME

There are many misconception in the scientific world today about the space and time. This leads to a confusion and many wrong and imprecise ideas including the various space-time theories. The main reason for this confusion is that the mathematicians have abstracted the coordinates form the actual physical space and use them in their abstract mathematical theories such as, for example, in tensor calculus. This seemingly gives them an independent real existence. However, this is not correct, the coordinates by themselves are just a mathematical tool without the real physical meaning. The coordinates should always be connected to some real entities, which may, for example, be called the space-time. The coordinates by themselves are not the space-time.

This can be compared to numbers. The numbers by themselves do not have any physical meaning until we connect them to some real objects. This is the fundamental difference between the physics and the mathematics, which we must always keep in mind. The real physical objects must also have some other physical properties in addition to just being able to be counted. They must have the size, the weight, the temperature, the intrinsic pressure, the conductivity, the stiffness, the index of refraction, etc. The same is true for the coordinates.

The space that the coordinates describe must also have some other physical properties in addition to its size and distances if it should have a real physical existence. It must have a temperature, the internal pressure, the stiffness, the mass density, etc.. The notion that there is a vacuum between objects with nothing in it cannot be correct. The description of the space-time as is proposed by the General Relativity Theory (GRT) with nothing in the space between the massive objects just an "empty space" around the objects cannot be correct. The space around the central mass must have some other physical properties in addition to distances. It must allow propagation of light, so it must have some stiffness and mass density, it must have a temperature and an internal pressure etc..

Since the space allows the propagation of disturbances in form of waves with a certain velocity, this automatically defines time. So, when we speak about the space-time we must consider that it is a physical entity with real mass density, the stiffness, the temperature, the ability to support the wave propagation etc., not only the abstract distances. It is therefore clear that the space-time must be some physical medium, perhaps aether, which supports all the remaining physical properties that the GRT has conveniently dropped from considerations.

The coordinates by themselves are not the space-time, they only describe one aspect of it.

POTENTIAL DERIVATION

Before embarking on this topic, it is necessary to mention the coordinates once more. In this paper there will be two kinds of coordinates used. The natural coordinates, the ones that we are using for measurement of distances and time by sticks and clocks, and the physical coordinates that will be used in physics formulas. The reason for this is that it will be assumed that the natural coordinates, the space-time we are living in, can be distorted by the gravity. The physical coordinates on the other hand will describe the undistorted environment into which the natural space-time is embedded in. The physical coordinates are thus not directly observable or measurable, they must be calculated from the natural coordinates using some physics formulas, that is why the name physical.

The metric derivation for the centrally gravitating non-rotating body will begin with the definition of potential. It is our experience and observation that there is a gravitational force that attracts all masses. Barring the loss of energy by the gravitational wave radiation, or various frictional losses, it is reasonable to assume that the static gravitational force can be represented by a field that possesses a potential. This also means that the path integral calculating the work of a small test body along a closed path in this field is zero. This, for example, implies that the orbits of planets (viewed as small test bodies) around the Sun are stable and do not significantly decay over the period of many years as we are all well aware of. The potential in such a centrally gravitating system where the trajectories are conserving energy thus has a corresponding field that can be calculated by minimizing the energy stored in this field according to the following variational principle:

$$\delta \int_R \left(\frac{\partial \varphi_n}{\partial \rho} \right)^2 \rho^2 d\rho = 0 , \tag{1}$$

where ρ is a generalized radial physical coordinate to be subsequently defined in more detail. The Euler-Lagrange (EL) equation corresponding to this variational problem is as follows:

$$\frac{d}{d\rho} \left(\frac{\partial \varphi_n}{\partial \rho} \rho^2 \right) = 0 . \tag{2}$$

In these equations the radial coordinate distance r was replaced by a physical distance ρ since it is necessary to include into considerations a possibility that the usual radial coordinate is distorted by the gravity while ρ remains unaffected by it as already mentioned. The field will be actually considered as a distortion and the deformation of the natural space-time we are living in. The coordinate distances and the coordinate time in the curved space-times thus do not have a direct physical meaning as in the flat space-time. The relation between the physical radius and the coordinate radius is expressed using the well known metric coefficient and by the well known formula in the spherically symmetrical coordinate system as follows:

$$d\rho = \sqrt{g_{rr}}\, dr \,. \tag{3}$$

The solution of Eq.2 is easily obtained using the zero boundary condition at infinity and can be written as:

$$\varphi_n = -\frac{\kappa M}{\rho(r)}, \tag{4}$$

where M is the mass of the centrally gravitation body and κ the gravitational constant. This is the famous Newton gravitational potential with the modification of replacing the coordinate radius by the physical radius that was necessary for the curved space-times. However, to proceed further it is also necessary to find the relation between the metric coefficient and the potential using some other physical reasoning since the substitution of the physical radius for the coordinate radius at this point is essentially only a formality.

METRIC DERIVATION

In the Einstein's General Relativity Theory it is assumed that the metric coefficients and not the potential are the field variables and the principle that is used to find them is to find the stationary point of the Lagrangian density: $R_{(c)}\sqrt{-g}$ where $R_{(c)}$ is the Ricci scalar, which depends on the second derivatives of these coefficients. The variational principle is usually written as:

$$\delta \int_{\Omega} R_{(c)}\sqrt{-g}\, dx^4 = 0\,, \tag{5}$$

with g being the coordinate metric determinant. Solutions of this variational principle are the famous Einstein field equations:

$$G_{jk} = R_{jk} - \frac{1}{2} R_{(c)} g_{jk} = -\frac{8\pi\kappa}{c^4} T_{jk}\,, \quad T^{j}_{k|j} = 0\,. \tag{6}$$

While this is a widely accepted concept, believed to be so beautiful that it is beyond any reproach, it is not difficult to raise some obvious objections to it. The Ricci scalar is the consequence, the result of the gravitational field and the presence of mass distorting the coordinate space-time, so minimizing the result instead of the cause, which is the field, does not make much sense in particular when the relation between the field energy and the Ricci scalar may not be linear. The second problem is that the mass energy tensor T_{jk} is not calculated from the formalism and must be determined and added using some other physical reasoning. For the centrally gravitating body it is believed and assumed that it is zero outside of the massive body. This is a strange assumption claiming that in the space where the field is the mass energy tensor is zero. The third problem is that the variational principle in Eq.5 does not guarantee that the field energy will also be at its minimum when the above mentioned Lagrangian density is at its minimum. The well known and the time tested principle of minimum field energy is not used here. It has not been proven that the variational principle introduced in Eq.5 also implies the minimum field energy. Actually, due to the general covariance principle it is some times claimed that there is no energy in the gravitational field or that the field energy is not localized since by transformation of coordinates into a free falling coordinate system the field can be transformed away. Setting the mass energy tensor to zero is not equivalent to finding the minimum value for the field energy. Actually, the zero value for this tensor may not be permissible for the nonzero value of the gravitating mass except perhaps in the weak field limit.

The solution of Eq.6 for the zero mass energy tensor is the famous Schwarzschild metric:

$$ds^2 = g_{tt}(cdt)^2 - g_{tt}^{-1}dr^2 - r^2 d\Omega^2, \tag{7}$$

where the metric coefficient g_{tt} equals to:

$$g_{tt} = 1 - \frac{R_s}{r}, \tag{8}$$

and the Schwarzschild radius R_s is defined as:

$$R_s = \frac{2\ \kappa\, M}{c^2}. \tag{9}$$

The usual angular coordinate term: $\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right)$ used in the Schwarzschild metric line element in Eq.7 was replaced here by a more general angular coordinate term $d\Omega^2$ in order to emphasize the spherical symmetry. However, the Schwarzschild solution produces a coordinate problem at the Schwarzschild radius called the “event horizon”, which then leads to the Black Hole (BH) theories and other unreasonable theoretical creations that may not exist in reality and have yet to be confirmed by observations. It thus seems that the primary source of all these problems and objections is the unusual and nowhere else in physics encountered variational principle of Eq.5 and the Einstein field equations resulting from it with observational proofs of solutions validity only for the weak gravitational fields.

All these problems stem from the fact that there was no physical entity considered behind the natural coordinates. It was considered that the space-time is just an abstract "empty space", a vacuum. However, when we ascribe a physical property to the natural space-time it is immediately obvious that there must be a field energy in that space-time, which originates from the space-time distortion.

In the following sections the Einstein approach to find the metric will therefore be abandoned and the metric for the centrally gravitating body will be found from the more fundamental principles of physics, which have been verified may times before and are without any doubts. The metric will be found by having a small test body moving in the space-time of the studied metric and it will be investigated if the trajectories satisfy the well known principles such as the conservation of angular momentum and the conservation of energy (the trajectory stability). The trajectories that will satisfy these requirements will then be considered correct, corresponding to reality, and the metric that provides them the correct space-time metric. It will also be considered that the deformation of the coordinate space-time by the gravity of the centrally gravitating body is locally isotropic.

The general metric line element of the centrally gravitating body taking an advantage of the spherical symmetry is as follows:

$$ds^2 = g_{tt}(cdt)^2 - g_{rr}dr^2 - g_{\varphi\varphi}d\Omega^2 . \tag{10}$$

The angular coordinate metric coefficient will be obtained by considering for simplicity a small test body orbiting the main gravitating body only in the equatorial plane. The Lagrangian describing such a motion corresponding to the metric line element in Eq.10 is as follows:

$$L = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - g_{rr}\left(\frac{dr}{d\tau}\right)^2 - g_{\varphi\varphi}\left(\frac{d\varphi}{d\tau}\right)^2 . \tag{11}$$

The EL equations of motion are derived from the variational principle:

$$\delta\int_\tau L d\tau = 0 , \tag{12}$$

with their first integrals easily found to be:

$$g_{tt}\frac{dt}{d\tau} = 1 , \tag{13}$$

$$g_{\varphi\varphi}\frac{d\varphi}{d\tau} = \alpha , \tag{14}$$

where α is an integration constant corresponding to the angular momentum. Eliminating $d\tau$ from these formulas leads to the equation for the conservation of angular momentum.

$$\frac{g_{\varphi\varphi}}{g_{tt}}\frac{d\varphi}{dt} = \alpha . \tag{15}$$

From this formula it is then clear that the ratio of the angular metric coefficient and the coordinate time metric coefficient must have a dimension of a radius squared. Therefore, similarly as with the Newton gravitational potential, where the coordinate distance was replaced by the physical distance, the ratio of these two metric coefficients is the physical radius squared instead of the coordinate radius squared. It will thus be considered that it must hold the following:

$$g_{\varphi\varphi} = \rho^2 g_{tt} \,. \tag{16}$$

For the Schwarzschild metric, however, the equivalent of Eq.15 is:

$$\frac{r^2}{(1 - R_s / r)} \frac{d\varphi}{dt} = \alpha \,. \tag{17}$$

This suggests that the angular momentum as is commonly known to be conserved is not conserved here. In particular for the coordinate radius approaching the Schwarzschild radius the angular velocity would approach zero, so no spin interaction could be observed transferred into or out of the BH. This is again not reasonable, since the angular momentum cannot be lost from the matter forming the gravitating body when the BH is purportedly created. In addition it is well known that very compact stars such as the neutron stars spin at very high rates. The neutron stars have their radius close to the Schwarzschild radius and thus their spin should be very slow. This, however, contradicts the observations.

For the derivation of the metric coefficients standing by the radial coordinate and the time coordinate it is useful to consider for simplicity a test body falling only in the radial direction. The Lagrangian describing such a motion is as follows:

$$L = g_{tt} \left(\frac{cdt}{d\tau} \right)^2 - g_{rr} \left(\frac{dr}{d\tau} \right)^2 . \tag{18}$$

The EL equations of motion obtained from the variational principle shown in Eq.12 are as follows:

$$\frac{d}{d\tau} \left(g_{tt} \frac{dt}{d\tau} \right) = 0, \qquad -\frac{d}{d\tau} \left(2 g_{rr} \frac{dr}{d\tau} \right) = \frac{\partial g_{tt}}{\partial r} \left(\frac{cdt}{d\tau} \right)^2 - \frac{\partial g_{rr}}{\partial r} \left(\frac{dr}{d\tau} \right)^2 . \tag{19}$$

The first integrals of these equations for the unity initial condition and the zero boundary condition at infinity ($L = c^2$) can be easily found:

$$g_{tt} \frac{dt}{d\tau} = 1 \,, \tag{20}$$

$$g_{rr} g_{tt} \left(\frac{dr}{d\tau} \right)^2 = c^2 \eta_{tt} - c^2 g_{tt} \,, \tag{21}$$

where $\eta_{tt} = 1$. The forces acting on the falling test body causing acceleration, similar to Newton's inertial and gravitational forces, are then obtained by differentiating Eq.21 with respect to τ and dividing the result by g_{tt}, or directly from the EL equations of motion, with the result:

$$g_{rr}\frac{d^2r}{d\tau^2} = -\left(\frac{c^2}{2}\frac{1}{g_{tt}}\frac{\partial g_{tt}}{\partial \varphi_n}\right)_s \frac{\partial \varphi_n}{\partial r} - \left(g_{rr}\frac{dr}{d\tau}\frac{dr}{d\tau}\right)_s \left(\frac{\partial \ln\sqrt{g_{tt}g_{rr}}}{\partial \varphi_n}\right)_s \frac{\partial \varphi_n}{\partial r}, \tag{22}$$

where the metric coefficients are considered as functions of the Newton gravitational potential. In order to keep the number of covariant quantities in the products on both sides of the equation equal, the terms in the parenthesis must be related to scalar quantities. This is satisfied for the spherical coordinates by the relation:

$$g_{tt}g_{rr} = 1, \tag{23}$$

which also follows from the consideration that the space deformation around the centrally gravitating body is locally isotropic. This means that the deformation in the radial direction and the deformation in any angular direction are the same. The space deformation is, of course, detected in the test body trajectory and the isotropic condition can be expressed as follows:

$$\frac{d\rho}{dr} = \sqrt{g_{rr}} = \frac{\rho}{\sqrt{g_{\varphi\varphi}}}. \tag{24}$$

By substituting into Eq.24 for $g_{\varphi\varphi}$ from Eq.16 the relation in Eq.23, also well known from the Schwarzschild metric, is obtained. Applying this result to Eq.22 then simplifies the acceleration-force equation to read:

$$g_{rr}\frac{d^2r}{d\tau^2} = -\left(\frac{c^2}{2}\frac{1}{g_{tt}}\frac{\partial g_{tt}}{\partial \varphi_n}\right)_s \frac{\partial \varphi_n}{\partial r}. \tag{25}$$

The same result can also be derived from the consideration that the test body in a free fall follows a geodesic trajectory as is derived in the Appendix. Finally to maintain the count of the covariant components equal on both sides of Eq.22 and Eq.25 it is obvious that the metric coefficient for the time coordinate must satisfy the following relation:

$$\frac{c^2}{2}\frac{1}{g_{tt}}\frac{\partial g_{tt}}{\partial \varphi_n} = 1. \tag{26}$$

The solution of this equation provides the desired relation between the metric coefficient and the potential and together with Eq.4 the final result for the coordinate distance:

$$g_{tt} = e^{2\varphi_n/c^2} = e^{-R_s/\rho}, \tag{27}$$

$$r = \int_0^{\rho} e^{\frac{-R_s}{2\rho}} d\rho \,. \tag{28}$$

From this finding it is then concluded that the gravity is compressing the coordinate radius as is shown in Fig.1.

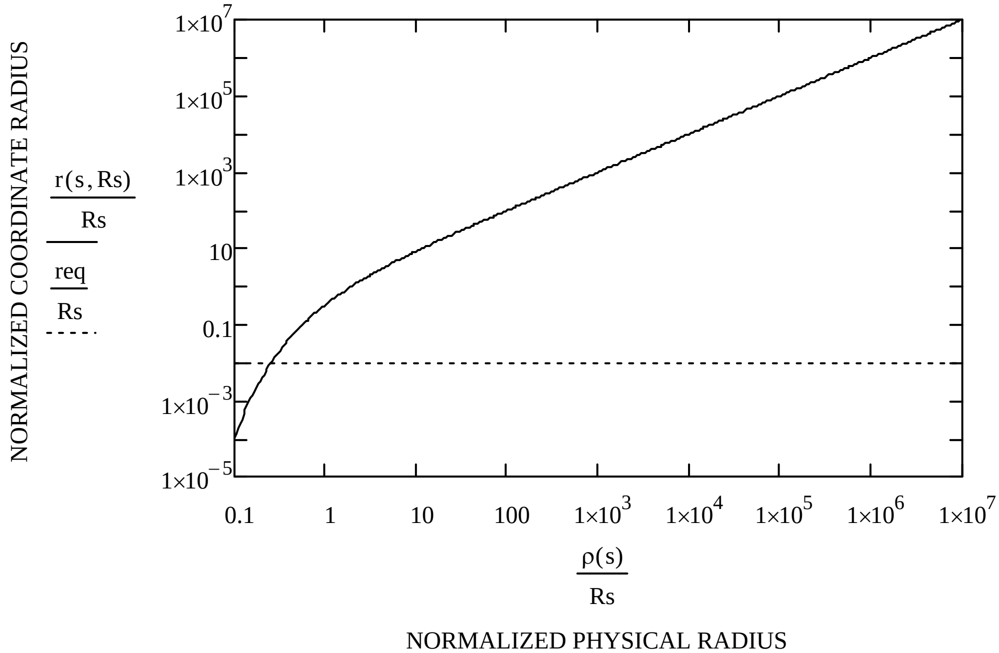

Fig.1. The coordinate radius as a function of the physical radius normalized to R_s. The dotted line represents the minimum radius to which any mass, such as a collapsed star, can be compacted to.

At this point it is also interesting to compare the gravitational potential resulting from the Schwarzschild metric with the gravitational potential derived above by minimizing the field energy. Using Eq.21 and Eq.23 the acceleration-force equation for a test body falling in the radial direction in the space-time of the Schwarzschild metric is:

$$\frac{d^2 r}{d\tau^2} = -\frac{c^2}{2}\frac{R_s}{r^2} = -\left(1 - \frac{R_s}{r}\right)\frac{c^2}{2}\frac{R_s}{r^2 - rR_s} \,. \tag{29}$$

The Schwarzschild metric coefficient had to be factored out from the expression to show that the contravariant quantities are equal on both sides. The Schwarzschild gravitational potential then becomes equal to:

$$\varphi_{sch} = \frac{c^2}{2}\ln\left(1 - \frac{R_s}{r}\right) \approx -\frac{\kappa\, M}{r} \,. \tag{30}$$

It is now clear that this potential cannot result from minimizing the field energy of the gravitational field and that the variational principle introduced in Eq.5 is not reasonable for finding the metrics of a particular mass distributions. The general relationship between the gravitational potential and the metric coefficient as derived in Eq.27 has to be, of course, maintained.

Having thus obtained the expression for the gravitational potential and therefore the expression for the energy of the gravitational field it is possible to find the absolute minimal radius of the centrally gravitating body to which any mass, such as a collapsed star, can be compacted to. This is obtained by equating the energy of the field, which is negative, with the mass energy of the body. The total mass-energy of the field plus the body is thus zero:

$$Mc^2 = \frac{1}{2}\frac{\kappa M^2}{\rho_{eq}}. \tag{31}$$

In this expression the field energy inside of the massive body was for simplicity neglected, since it is more than an order of magnitude smaller than the mass energy of the body. The mass equivalent physical radius then becomes equal to:

$$\rho_{eq} = \frac{R_s}{4}. \tag{32}$$

The corresponding mass equivalent coordinate radius is obtained using Eq. 28:

$$r_{eq} = \int_0^{\rho_{eq}} e^{\frac{-R_s}{2\rho}} d\rho = \frac{R_s}{2}\int_0^{1/2} e^{\frac{-1}{x}} dx = R_s \cdot 0.009384. \tag{33}$$

The absolute minimum of the gravitational potential that any massive body in the Universe can have on its surface is then equal to:

$$\varphi_{n\ \min} = -2c^2. \tag{34}$$

This limit also implies that there is a limit to the intrinsic gravitational red shift, which should be verifiable by observations. The maximum intrinsic gravitational red shift observed in the radiation generated when the in falling matter impacts the surface of the maximally compressed body is thus as follows:

$$Z_{mx} = e^2 - 1 = 6.389. \tag{35}$$

The maximum Z shift measured to date is from the Gamma Ray Burst: GRB050904, equal to: $Z_{mx} = 6.29$ [1] and from the so called “most distant” Quasar: CFHQS J2329-0301 at Z = 6.43, both in a reasonable agreement with expectations. Many other Quasars are also exhibiting large Z shifts but none of them has so far exceeded the value given in Eq.35. The most convincing support for the theory, however, comes from the observation of the pulse length of the long duration GRB explosions. When it is considered that these extremely powerful pulses are generated by explosions of the super massive objects located at the centers of galaxies, which have the estimated mass of approximately equal to $M_G = 3.0 \cdot 10^6 M_S$, the pulse duration is equal to the physical mass equivalent radius obtained from Eq.32 divided by the speed of light and corrected for the maximum gravitational red shift. This leads to the following simple formula:

$$\tau_{GRB} = \frac{\kappa\, M_s 3.0 \cdot 10^6}{2c^3 \sqrt{g_{tt}}} = 54.6 \sec . \tag{36}$$

This result compares favorably with the measured data shown in Fig.2 for which there is presently no reasonable explanation. The presence of the second smaller peak in the graph suggests that there should be another population of super massive compact objects in the Universe, possibly Quasars, with the average mass of approximately equal to: $M_{SM} = 1.6 \cdot 10^4 M_S$, in addition to the central masses of galaxies. The GRBs from even less massive objects, such as the massive stars or ordinary stars, are not detectable due to the low radiation intensity and the fact that they are not compressed to their minimum possible mass equivalent diameter.

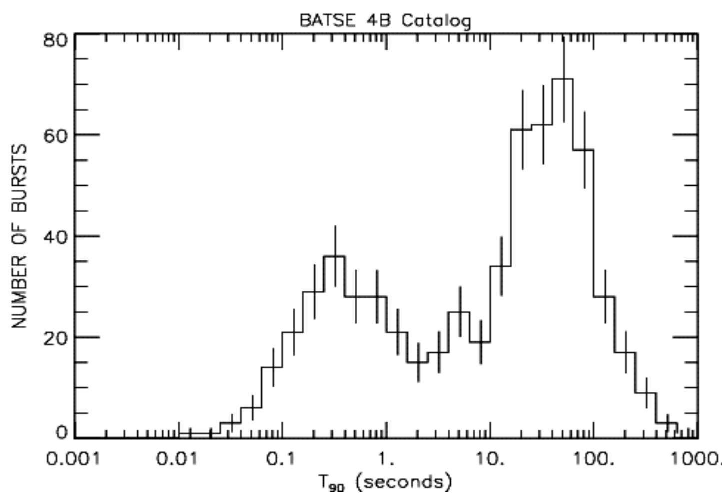

Fig.2. Statistical distribution of the GRB pulse durations as published in the BATSE 4B catalog [2].

For the Schwarzschild metric, on the other hand, there is no limit to the gravitational potential and the potential can attain values approaching $-\infty$. This implies that any value of the intrinsic gravitational red shift should be observed. However, there is no surface to be impacted, so no radiation can actually be generated by impacts of the in falling matter. Other concepts of the radiation generation thus must be conjured up with additional adjustable parameters to explain the observations. Again, this is not reasonable and not physical. The infinite gravitational potential at the Schwarzschild radius also precludes any possibility of explosions of BHs and thus generation of powerful GRBs. Observations of the substantially larger than the maximum intrinsic gravitational red shifts derived in Eq.35 would, however, falsify the presented theory.

Since all the metric coefficients have now been found based on minimizing the gravitational field energy, generating the test body trajectories that conserve the angular momentum along the test body trajectory, and finally maintaining the contravariance of the acceleration-force equation, the correct metric for the centrally gravitating body is as follows:

$$ds^2 = e^{-R_s/\rho}(cdt)^2 - e^{R_s/\rho}dr^2 - \rho^2 e^{-R_s/\rho}d\Omega^2 \,, \qquad d\rho = e^{R_s/(2\rho)}dr \,. \tag{37}$$

There are no BHs in the Universe, only very massive and compact gravitating bodies compressed to their minimum possible sizes with the minimum radius approximately equal to $r_{eq} \approx R_s/100$. The light can always escape from them so they can be seen. Particles can also escape from them having enough initial kinetic energy forming, for example, jets of matter, which are often seen emanating from the centers of many galaxies.

If the variational principle in Eq.5 were founded on the sound physical principles then the metrics in Eq.37 and Eq.7 would be identical.

GENERALIZATION TO ARBITRARY MASS CONFIGURATIONS

To generalize the above derived metric for the centrally gravitating body only a small space-time region will be considered where the gravitational field is uniform. The coordinate system for this case can be oriented such that the field is along the z direction. The metric for such space-time is simplified as follows:

$$ds^2 = g_{tt}(cdt)^2 - dx^2 - dy^2 - g_{zz}dz^2 \,. \tag{38}$$

The metric coefficients along the x and y directions must be unity in order to maintain compatibility with the Lorentz coordinate transformation, which can transform out the gravitational field in the z direction for a free falling coordinate system without any effect on the other two axes. Considering now the test body motion only in the z direction the corresponding EL equations are:

$$\frac{d}{d\tau}\left(g_{tt}\frac{dt}{d\tau}\right) = 0 \,, \tag{39}$$

$$-\frac{d}{d\tau}\left(2g_{zz}\frac{dz}{d\tau}\right) = \left(\frac{1}{g_{tt}}\frac{\partial g_{tt}}{\partial\varphi_n}\right)_s \left(g_{tt}\frac{cdt}{d\tau}\frac{cdt}{d\tau}\right)_s \frac{\partial\varphi_n}{\partial z} - \left(\frac{1}{g_{zz}}\frac{\partial g_{zz}}{\partial\varphi_n}\right)_s \left(g_{zz}\frac{dz}{d\tau}\frac{dz}{d\tau}\right)_s \frac{\partial\varphi_n}{\partial z} \,. \tag{40}$$

Since the dependency of the metric coefficient g_{tt} on the potential is already know from the previous derivations, it is clear that in order to maintain the covariance of Eq.40 on both sides the metric coefficient g_{zz} must be equal to:

$$g_{zz} = e^{2\varphi_n / c^2} . \tag{41}$$

This simplifies Eq.40 as follows:

$$\frac{d}{d\tau}\left(g_{zz} \frac{dz}{d\tau} \right) = -\frac{\partial \varphi_n}{\partial z} . \tag{42}$$

The metric in Eq.38 then becomes equal to:

$$ds^2 = e^{2\varphi_n / c^2} (cdt)^2 - dx^2 - dy^2 - e^{2\varphi_n / c^2} dz^2 . \tag{43}$$

In Eq.40 it is, of course, also possible to consider that $g_{zz} = \exp(-2\varphi_n / c^2)$, however, this choice leads to a disagreement with the Lorentz coordinate transformation and to a variable speed of light in the z direction and thus will not be considered here any further.

In the next step it will prove useful to evaluate and compare metric determinants for various space-time metrics. For the metric in Eq.43 it is clear that the following relation holds:

$$\varphi_n = \frac{c^2}{2} \ln \sqrt{-g} . \tag{44}$$

Considering that the metric determinant of corresponding physical space-time is $g_p = -1$ the relation in Eq.44 can be generalized for any metric and thus for any gravitational field as follows:

$$\varphi_n = \frac{c^2}{2} \ln \sqrt{\frac{g}{g_p}} . \tag{45}$$

Using Eq.27 to eliminate φ_n this formula can be further generalized and written as:

$$g_{tt}^2 = g / g_p . \tag{46}$$

From this result then follows that the gravitational field is compressing the space-time and this is reflected in the corresponding metric determinants. The same relation is also satisfied by the metric derived in Eq.37. Eq.46 thus imposes a certain deformation restriction on the space-time and thus a condition on the metric coefficients similar to condition of Eq.24. For the Schwarzschild metric, however, the metric determinant is unaffected by gravity and the gravitational potential using the formula in Eq.45 is:

$$\varphi_{sch} = 0 . \tag{46}$$

This is a contradiction with the result obtained in Eq.30, which again points to fatal flaws of the

Schwarzschild metric and the variational principle of Eq.5. It seems unreasonable that the metric determinant would not be affected by the gravitational field.

The general procedure for finding the metric for any static mass configuration is then as follows: First solving the Poisson equation for the given mass configuration ρ_m to find the Newton gravitational potential in the physical space:

$$\Delta\varphi_n = \rho_m\left(x^i_{(p)}\right) . \tag{47}$$

This step is followed by finding the metric using Eq.45 in a suitable coordinate system with the coordinate time metric coefficient equal to: $g_{tt} = \exp(2\varphi_n / c^2)$, and with the known relations between the coordinate and physical distances, taking advantage of the mass configuration symmetry and conservation laws of test body trajectories [3]. For the example of the uniform gravity field discussed above this is: $\varphi_n = g_0 z_p$, where g_0 is the gravitational field vector pointing in the negative z direction and $z_p \le 0$ is the physical distance. The metric coefficients and the metric line element are then found by solving Eq.27: $dz_p = \exp(g_0 z_p / c^2)dz$ for the zero initial condition at $z_p = 0$ and valid for $z \le 0$ with the result:

$$ds^2 = \frac{(cdt)^2}{\left(1 - g_0 z / c^2\right)^2} - dx^2 - dy^2 - \frac{dz^2}{\left(1 - g_0 z / c^2\right)^2} . \tag{48}$$

AGREEMENT WITH THE LORENTZ COORDINATE TRANSFORMATION

The metric for the uniform gravitational field has an advantage that its correctness and thus the correctness of the above approach to the theory of the static gravitational field can be easily verified. The effect of the uniform gravitational field should be simply transformed away in a coordinate system that is free falling in the z direction in such a field. From the first integrals of EL equations of motion corresponding to metric in Eq.38 it holds that:

$$g_{tt}\frac{dt}{d\tau} = 1, \qquad\qquad g_{zz}\left(\frac{dz}{d\tau}\right)^2 = \frac{c^2}{g_{tt}} - c^2 . \tag{49}$$

From this result follows that for a coordinate system falling with the test body it holds:

$$\left(\frac{dz}{dt}\right)^2 = c^2 - c^2 g_{tt} . \tag{50}$$

The metric coefficients can then be expressed in terms of the z direction velocity as:

$$g_{tt} = 1 - \frac{v_z^2}{c^2} . \tag{51}$$

It is important to note that the speed of light in the z direction is constant and equal to c, the speed of light in vacuum without any field. The metric in Eq.48 then becomes:

$$ds^2 = \left(1 - v_z^2 / c^2\right)(cdt)^2 - dx^2 - dy^2 - \left(1 - v_z^2 / c^2\right)dz^2 . \tag{52}$$

The remaining step in the proof is to use the Lorentz length contraction and the time dilation differential formulas for the system moving in the z direction with a velocity v_z:

$$dt' = dt_{z'=cnst}\sqrt{1 - v_z^2 / c^2}\ , \qquad dx' = dx\ ,\ dy' = dy\ , \quad dz' = dz_{t'=cnst}\sqrt{1 - v_z^2 / c^2}\ , \tag{53}$$

and obtain the Minkowski flat space-time metric line element of the free falling coordinate system:

$$ds'^2 = (cdt')^2 - dx'^2 - dy'^2 - dz'^2 . \tag{54}$$

From Eq.51 also follows the famous Einstein's inertial mass-energy equivalence formula. By substituting for the metric coefficient and by multiplying the result by the rest mass m_0 of the test body and c^2 the following equation is obtained:

$$\frac{m_0 c^2}{\sqrt{1 - v_z^2 / c^2}} = m_0 c^2 - m_0 g_0 z . \tag{55}$$

The inertial mass-energy of the test body is increased by the energy picked up during the fall in the gravitational field ($z < 0$). It is important to note that the metric in Eq.48 does not satisfy the Einstein field equations, since it does not result in the zero Ricci scalar. The Ricci scalar for this metric is: $R_{(c)} = 2g_0^2 / c^4$ [3]. The agreement with the Lorentz coordinate transformation, however, proves the metric correctness and again invalidates the variational principle approach of Eq.5 in finding the correct space-time metric solutions. Since the gravitational field seems to cause the curvature of the space-time it is reasonable to expect that for the uniform gravitational field the Ricci scalar will not be zero.

The fundamental error by assuming that in a vacuum, away from the massive body, the Ricci scalar and consequently the mass energy tensor T_{jk} are zero could not be more apparent than for the case of the uniform gravitational field. This becomes even more obvious when considering that the weak field approximation boundary condition at infinity cannot be applied to this case. The weak field solutions of Einstein field equations are the only solutions that have been confirmed by observations.

EXPANSION OF METRIC COEFFICIENTS FOR THE CENTRALLY GRAVITATING BODY INTO A POWER SERIES

The first order approximation for the metric coefficient g_{tt} and thus for the metric line element of the centrally gravitating non-rotating body are found using Eq.27 and Eq.28 as follows:

$$r = \int_0^{\rho} e^{\frac{-R_s}{2\rho}} d\rho = \frac{R_s}{2}\int_{\frac{R_s}{2\rho}}^{\infty} \frac{e^{-x}}{x^2} dx = \rho e^{\frac{-R_s}{2\rho}} - \frac{R_s}{2}\int_{\frac{R_s}{2\rho}}^{\infty} \frac{e^{-x}}{x} dx = \rho e^{\frac{-R_s}{2\rho}} + \frac{R_s}{2} Ei\left(-\frac{R_s}{2\rho}\right), \tag{56}$$

where $Ei(x)$ is the Euler exponential integral function. Unfortunately there is no analytic expression for ρ as function of r, so the approximation needs to be found iteratively. For large distances (0 < x << 1), the Euler exponential integral is approximated as:

$$Ei(-x) = \ln(\gamma' x) + \dots, \tag{57}$$

where $\gamma' = 1.781072...$ and $\ln(\gamma') = \gamma$ is the famous Euler constant $\gamma = 0.577215...$. It is therefore possible to write:

$$\rho e^{\frac{-R_s}{2\rho}} = r - \frac{R_s}{2}\ln\left(\frac{\gamma' R_s}{2\rho}\right) + \dots. \tag{58}$$

Rearranging this result as follows:

$$\frac{1}{\rho} = \frac{1}{r} e^{\frac{-R_s}{2\rho}} + \frac{R_s}{2r\rho}\ln\left(\frac{\gamma' R_s}{2\rho}\right) + \dots = \frac{1}{r} - \frac{R_s}{2r\rho} + \frac{R_s}{2r\rho}\ln\left(\frac{\gamma' R_s}{2\rho}\right) + \dots, \tag{59}$$

and substituting for $1/\rho$ from the left hand side of Eq.59 to the right hand side, the iterative expression for $1/\rho$ valid for large distances becomes:

$$\frac{1}{\rho} = \frac{1}{r} - \frac{1}{2}\frac{R_s}{r^2}\left(1 + \ln\left(\frac{2r}{\gamma' R_s}\right)\right) + \dots. \tag{60}$$

From this formula then follows the approximation for g_{tt}:

$$g_{tt} = e^{\frac{-R_s}{\rho}} = 1 - \frac{R_s}{r} + \frac{R_s^2}{r^2}\left(1 + \ln\sqrt{\frac{2r}{\gamma' R_s}}\right) + \dots. \tag{61}$$

In the next step the second order term in Eq.61 can be neglected since the logarithmic function of $\sqrt{r}$ increases very slowly and R_s is for all practical purposes always very much smaller than r. This leads to the familiar formula of the Schwarzschild metric for the metric coefficient g_{tt}:

$$g_{tt} = 1 - \frac{R_s}{r} + \dots . \tag{62}$$

Substituting this approximation into Eq.10, and considering that the logarithmic term multiplied by $R_s / 2$ in Eq.58 can also be for large distances neglected in comparison to r, the formula for the metric line element becomes the celebrated Schwarzschild metric:

$$ds^2 = \left(1 - \frac{R_s}{r}\right)c^2 dt^2 - \left(1 - \frac{R_s}{r}\right)^{-1} dr^2 - r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right). \tag{63}$$

The derived approximation also allows to find a condition that needs to be satisfied to obtain an accurate description of the space-time geometry by this metric $(r > R_s)$. The condition is as follows:

$$\frac{r}{R_s} - \ln\sqrt{\frac{r}{R_s}} >> 1 . \tag{64}$$

From the above derivation there could be no doubt that the Schwarzschild metric is only the first order approximation of the metric introduced in Eq.37 and thus only the first order approximation of reality. The reality is being defined here as a space-time described by a metric that satisfies the contravariance requirement of the test body acceleration-force equation, generates trajectories that conserve the angular momentum, has minimum field energy, and of course also satisfies the four tests of GRT [4]. Since the Schwarzschild metric has been confirmed by observations only for the weak gravitational fields and since it is the first order approximation of the new metric it is, of course, obvious that the new metric also satisfies the weak field tests. The differences will occur only at the strong gravitational fields such as in the vicinity of the neutron stars or the centers of galaxies.

Since the Schwarzschild metric is the correct and unique solution of Einstein field equations for the spherical case, according to the well known Birkhoff theorem [5], there can be only one inescapable conclusion that the Einstein field equations yield only the first order approximations of the correct metric when the energy-momentum tensor T_{jk} is set to zero.

While the study of Einstein field equations and various Einstein Spaces described by these equations can be an interesting and intellectually rewarding experience with a large amount of work already devoted to this topic [6], it is clear that very little of this work can actually be applied to reality. The Einstein field equations and their various derivatives therefore should not be used to search for the metric to model the strong gravitational fields, or the entire Universe.

CONCLUSIONS

In this paper it was shown that the derivation of the Einstein field equations from the variation principle by minimizing the Ricci scalar is not the sound physical concept. There is no proof that the Ricci scalar is directly and linearly connected to the minimum of the field energy and thus the resulting field equations may not correspond to reality. This is reflected in the Schwarzschild solution of these equations, which exhibits several problems such as the BH artifacts, the test particle trajectories that do not conserve angular momentum, existence of the infinite potentials, and so on.

The new metric that replaces the Schwarzschild metric has been derived based on the well tested fundamental principles of physics and does not result in the above mentioned problems. The metric has as its first order weak field approximation the Schwarzschild metric, so it also satisfies the well know tests of GRT. The new metric, however, does not have the event horizon, which allows finding the minimum possible gravitational potential and the minimum possible size of the gravitating body to which any mass can be compacted to instead of the non-physical singularity of the Schwarzschild metric. Several observations in support of the theory were also presented and discussed. Finally the described principle of the metric derivation for the centrally gravitating body was extended to any mass configuration with an example of the metric derivation for the uniform gravitational field.

REFERENCES

1. J. Cottam, F. Paerels, M. Mendez, "Gravitationally redshifted absorption lines in the X-ray burst spectra of a neutron star", Nature 420,51-54 (07 November 2002).
2. http://www.batse.msfc.nasa.gov/batse/
3. J.Hynecek,"New Space-time Metrics for Symmetric Spaces", Physics Essays, vol.22, No. 2, p. 179.
4. J.Hynecek, "New Space-Time Metric, Four Tests of Gravitation Theory, and Newton's Law of Gravitation", Physics Essays, vol.20, No. 2, p. 313.
5. S. Deser, "Schwarzschild and Birkhoff a la Weyl", arXiv:gr-qc/0408067v2.
6. A. Z. Petrov, Einstein spaces, Fizmatgiz, Moscow 1961; English transl., Pergamon Press, Oxford 1969.

APPENDIX

As is well known a free falling body in a gravitational field follows a geodesic trajectory. As a consequence of this fact the covariant derivative of the parameterized (different than coordinate) velocity is zero. The result is equation for the geodesic:

$$\frac{d^2 x^i}{d\tau^2} + \Gamma^i_{jk} \frac{dx^j}{d\tau} \frac{dx^k}{d\tau} = 0\,, \tag{A1}$$

where the Christoffel symbols expressed in terms of metric coefficients are equal to:

$$\Gamma^i_{jk} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mj}}{\partial x^k} + \frac{\partial g_{mk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^m} \right). \tag{A2}$$

Considering that the test body falls only in the radial direction, also considering only the radial coordinate and the spherical symmetry, Eq.A1 simplifies as follows:

$$\frac{d^2 r}{d\tau^2} = g^{rr} \frac{1}{2} \left(-\frac{\partial g_{rr}}{\partial r} \left(\frac{dr}{d\tau} \right)^2 - \frac{\partial g_{tt}}{\partial r} \left(\frac{cdt}{d\tau} \right)^2 \right). \tag{A3}$$

This result can be further simplified by using the first integrals from Eq.20 and Eq.21. This leads to the following:

$$\frac{d^2 r}{d\tau^2} = g^{rr} \frac{c^2}{2} \left(-\frac{\partial g_{rr}}{\partial r} \left(1 - g_{tt}\right) - \frac{\partial g_{tt}}{\partial r} \frac{1}{g_{tt}^2} \right). \tag{A4}$$

Rearranging the right hand side of Eq.A4 using the relation: $g_{rr} g_{tt} = 1$, the following result is obtained:

$$\frac{d^2 r}{d\tau^2} = -g^{rr} \frac{c^2}{2} \frac{1}{g_{tt}} \frac{\partial g_{tt}}{\partial r}\,. \tag{A5}$$

Since the right hand side of Eq.A5 can be considered as being a force term derived from the gradient of some generalized gravitational potential, which approaches the Newton gravitational potential as $r \to \infty$, it is possible to write:

$$\frac{d^2 r}{d\tau^2} = -g^{rr} \frac{\partial \varphi_n}{\partial r}\,. \tag{A6}$$

By equating the terms on the right hand side of Eq.A5 and Eq.A6 the equation for the metric coefficient g_{tt} becomes:

$$\frac{\partial \varphi_n}{\partial r} = \frac{c^2}{2} \frac{1}{g_{tt}} \frac{\partial g_{tt}}{\partial r}\,. \tag{A7}$$

Finally, by integrating Eq.A7, considering the flat space-time at infinity, the result is:

$$g_{tt} = e^{2\varphi_n / c^2}\,. \tag{A8}$$

This is the same formula as in Eq.27, which the metric coefficient g_{tt} must always satisfy. From the above derivation it is also clear that this is a general expression valid for any centrally gravitating body irrespective of the particular form of the gravitational potential.