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NON-RELATIVISTIC AND RELATIVISTIC EXPLANATIONS OF THE RESULT OF FIZEAU'S EXPERIMENT

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Abstract

Fizeau's experiment showed that the speed of light is affected by the motion of medium in which it is propagated. This is regarded as a consequence of relativistic addition of velocities. Another explanation of the result of this experiment is given, on the basis of relative velocities of classical mechanics, without recourse to relativistic mechanics.

Keywords: Experiment, Light, medium, speed.

1 Introduction

Fizeau's experiment was conducted in the 1850s [1] to measure the speed of light, at normal incidence, in moving water. The special theory of relativity gives the speed of light w propagated at normal incidence in a medium of refractive index μ moving with speed v in a vacuum as:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right) \tag{1}$$

An alternative formula is given in this paper, for the speed of light in a moving medium, as:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right) \tag{2}$$

Equation (2) and equation (1), Fresnel's law, are used to give non-relativistic and relativistic explanations of the result of Fizeau's experiment.

2 Laws of Refraction of Light

Figure 1 depicts a ray of light ***SP***, emitted with velocity ***c*** from a stationary source *S*, incident at a point *P* on the boundary of two media, *1* and *2*. ***OPN*** is the normal to the surface at ***P***. A reflected ray ***PR*** is one propagated with velocity ***u*** from the boundary in the same medium *(1)* as the incident ray ***SP***. A refracted ray ***PT*** is one transmitted and propagated with velocity ***w*** in the second medium *(2)*. The angle of incidence is ι, the angle of reflection ρ and the angle of refraction is τ.

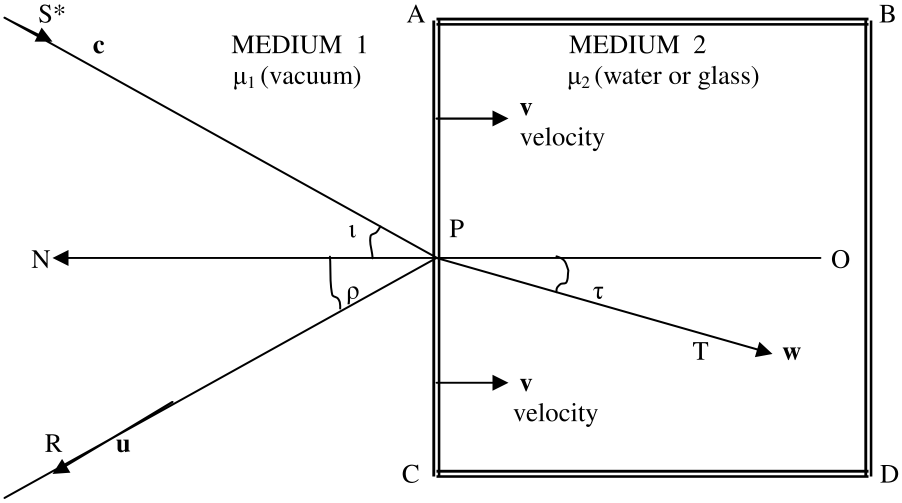

Figure 1 Reflection and refraction of a ray of light emitted from a source *S* with velocity ***c*** in medium *1 (vacuum)*, incident at a point *P* on the surface of medium *2* moving with velocity ***v***. The ray ***PR*** is reflected with velocity ***u*** and the refracted ray ***PT*** transmitted in medium 2 with velocity ***w***.

The laws of refraction of light, for a stationary medium, are:

(i) The incident ray ***SP***, the refracted ray ***PT*** and the normal ***OPN***, at the point of incidence *P*, are coplanar.

(ii) The ratio of the sine of angle of refraction to the sine of angle of incidence is equal to the ratio of the speeds of light *w* and *c* (Snell's law).

The relative indices of refraction μ_1 and μ_2 of *media 1* and *2* respectively, are defined, for a stationary medium, as the ratio of the speed *w* and *s*:

$$\frac{w}{s} = \frac{\sin\tau}{\sin\iota} = \frac{\mu_1}{\mu_2} \qquad (3)$$

3 Speed of light propagated in a moving medium

In Figure 1, let *medium 2* (ABCD) move in *medium 1* (vacuum) with velocity ***v*** along the normal. The ray is transmitted through the medium and propagated with velocity ***w*** at the angle of refraction τ. The refractive indices are defined in terms of the ratio of magnitudes of relative velocities of the refracted ray and the incident ray, with respect to the moving medium and in accordance with the Galilean (classical) relativity. The ratio of magnitudes of the relative velocities, given by $(\mathbf{w}-\mathbf{v})$ and $(\mathbf{c}-\mathbf{v})$, is obtained as the ratio of modulus, thus:

$$\frac{|\mathbf{w}-\mathbf{v}|}{|\mathbf{c}-\mathbf{v}|}=\frac{\mu_1}{\mu_2} \tag{4}$$

With reference to Figure 1, and with $\mu_1 = 1$ for a vacuum and $\mu_2 = \mu$ is the absolute refractive index, equation (4) becomes:

$$\frac{\sqrt{w^2+v^2-2wv\cos\tau}}{\sqrt{c^2+v^2-2cv\cos\iota}}=\frac{1}{\mu} \tag{5}$$

At normal incidence, $\iota = \tau = 0$ and we obtain:

$$\frac{w-v}{c-v}=\frac{1}{\mu}$$

$$w\mu - v\mu = c - v$$

$$w=\frac{c}{\mu}+v\left(1-\frac{1}{\mu}\right) \tag{6}$$

Equation (6) is used to give a non-relativistic explanation of the result of Fizeau's experiment, which measured the speed of light in moving water.

4 Fizeau's experiment

A schematic diagram of the apparatus of Fizeau's experiment [1] is shown in Figure 2. Carried out in the 1850s, it is one of the most remarkable experiments in physics. Light from a source was sent in two opposite directions through transmission and reflection by four half-silvered mirrors M_1 – M_4. One beam travelled downstream (from M_1) through moving water and the other travelled upstream (from M_4) in the same water. By an ingenious arrangement of the mirrors the two beams were made to recombine and be observed in an interferometer. An interference pattern, as observed in the interferometer, resulted from the difference in the time taken for the two beams to travel the same path, partly in moving water.

Various explanations have been given for the result of Fizeau's experiment. One accepted explanation is that the velocity of light in the moving water was increased or decreased in accordance with the *relativistic rule for addition of velocities* based on constancy of the speed of light relative to an observer or object that is stationary or moving. A new explanation is proposed here, on the basis of equation (6), outside the theory of special relativity [2, 3].

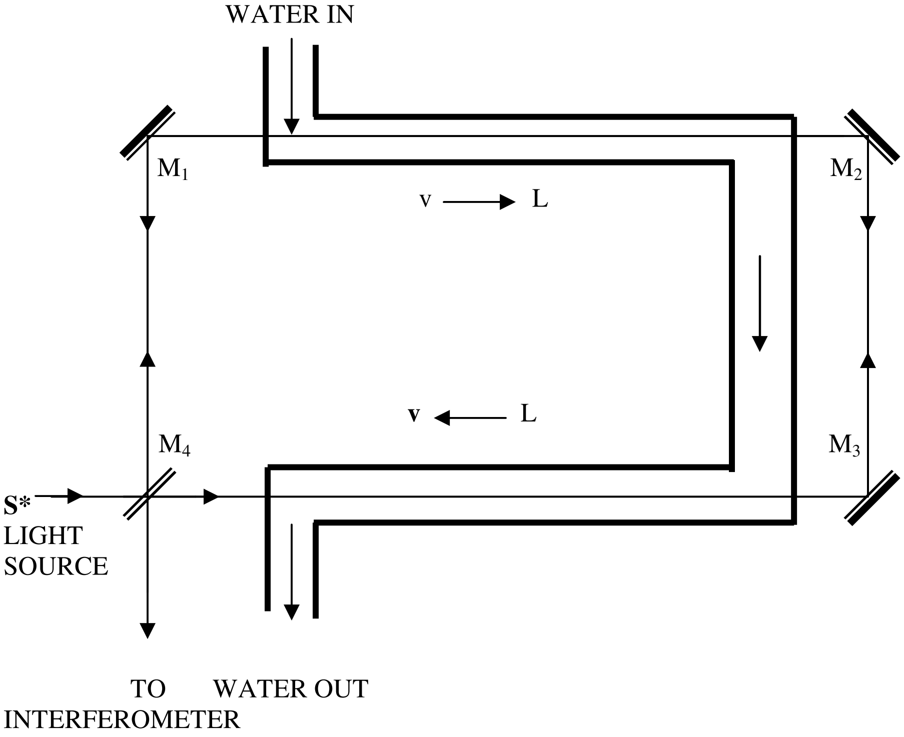

Figure 2. Schematic diagram of the apparatus of Fizeau's experiment

5 Non-relativistic explanation of the result of Fizeau's experiment

According to the Galilean-Newtonian relativity of classical mechanics, the speed of light in a medium that is moving in a vacuum with velocity $\boldsymbol{v}$, in the opposite direction of the normal, is given in terms of the refractive index μ of the medium, by equation (6). This gives the speed

w of transmission and propagation of light (downstream). If the medium is water of index of refraction μ moving with velocity v in a vacuum (Figure 2), the time t_1 taken for the beam that is going downstream to cover the distance $2L$ (where the magnitude v is very small compared with the speed of light c and, therefore, v^2/c^2 can be neglected), is obtained as given by:

$$t_1 = \frac{2L}{\frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right)} \approx \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} \tag{7}$$

For the beam going upstream, with velocity $-\boldsymbol{v}$, the longer transit time t_2 is:

$$t_2 = \frac{2L}{\frac{c}{\mu} - v\left(1 - \frac{1}{\mu}\right)} \approx \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} \tag{8}$$

The time difference $(\Delta t) = (t_2 - t_1)$ between the two beams in moving water, is:

$$t_2 - t_1 = \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} - \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\}$$

$$\Delta t = \frac{4Lv\mu^2}{c^2}\left(1 - \frac{1}{\mu}\right) \tag{9}$$

The fringe shift δ_x, for light of wavelength λ, is obtained as:

$$\delta_x = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c}\left(1 - \frac{1}{\mu}\right) \tag{10}$$

In the experiments performed by Fizeau [1], $L = 3\ m$, $v = 7\ m/sec.$, $\lambda = 6 \times 10^{-7}\ m$ (yellow light), $c = 3 \times 10^8\ m/sec$ and $\mu = 4/3$ (for water). The fringe shift δ_x is obtained as ≈ 0.2, which was easily observable and measurable in the interferometer.

6 Relativistic explanation of the result of Fizeau's experiment

According to Einstein's *relativistic velocity addition rule,* if you move with velocity $\boldsymbol{u}$ relative to a medium moving with velocity $\boldsymbol{v}$ (as light propagated in water moving with velocity $\mathbf{v}$), the magnitude of your velocity s, relative to an observer, is:

$$s = \frac{u+v}{1+\frac{uv}{c^2}} \tag{11}$$

where c is the speed of light in a vacuum. For speeds much less than *c*, or if *c* is infinitely large, Einstein's relativistic formula (equation 11) reduces to the Galilean (classical) relativity.

According to the theory of special relativity, the velocity of light, relative to a medium moving in a vacuum with velocity ***v***, remains as a constant ***c***. The velocity of light within the medium of refractive index μ is ***c/μ***. Einstein's *velocity addition rule*, with $u = c/\mu$, gives the magnitude of velocity ***w*** of light in the moving medium (with respect to an observer) as:

$$w = \frac{\frac{c}{\mu}+v}{1+\frac{cv}{\mu c^2}} = \frac{\frac{c}{\mu}+v}{1+\frac{v}{\mu c}} \approx \frac{c}{\mu}+v\left(1-\frac{1}{\mu^2}\right)$$

$$w \approx \frac{c}{\mu}+v\left(1-\frac{1}{\mu^2}\right) \tag{12}$$

Equation (12), Fresnel's law, compared with equation (6), is used to obtain the transit time difference between the two beams in Fizeau's experiment, and thereby explain the result from the relativistic point of view.

Transit time of beam going downstream, with speed *v* very small compared with *c*, is:

$$t_1 = \frac{2L}{\frac{c}{\mu}+v\left(1-\frac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c}\left\{1-\frac{\mu v}{c}\left(1-\frac{1}{\mu^2}\right)\right\}$$

$$t_1 \approx \frac{2L\mu}{c}\left(1+\frac{v}{\mu c}-\frac{\mu v}{c}\right)$$

The transit time of the beam going upstream (with speed $-v$) is:

$$t_2 = \frac{2L}{\frac{c}{\mu} - v\left(1 - \frac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu^2}\right)\right\}$$

$$t_2 \approx \frac{2L\mu}{c}\left(1 - \frac{v}{\mu c} + \frac{\mu v}{c}\right)$$

The time difference between the beam going downstream and the other beam going upstream is obtained as:

$$\Delta t = t_2 - t_1 \approx \frac{4Lv\mu^2}{c^2}\left(1 - \frac{1}{\mu^2}\right) \tag{13}$$

The fringe shift δ_y, for light of wavelength λ, is obtained as:

$$\delta_y = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c}\left(1 - \frac{1}{\mu^2}\right) \tag{14}$$

This δ_y in equation (14) is larger than the fringe shift δ_x as given by equation (10).

7 Conclusion

Michelson and Morley in 1886 and later P. Zeeman and associates in 1915 repeated Fizeau's experiment with greater precision in which the interferometer could measure a fringe shift as low as *0.01*. The influence of the motion of the medium on the propagation of light has thus been verified. As to which is the correct explanation, equations (6) and (10) for δ_x in accordance with the Galilean relativity of classical mechanics or equations (12) and (14) for δ_y according to the theory of special relativity, remains to be seen.

The treatment of refraction of light in this paper, clearly demonstrate the relativity of the velocity of light with respect to a moving medium, in accordance with Galileo's relativity of classical mechanics. The result of Fizeau's experiment [1] is not a direct consequence of the relativistic *velocity addition rule* but due to the effect of motion of the transmission medium on the speed of light.

In equations (6) and (12) the speed v, of the moving medium (Figure 1 with light at normal incidence), can take any value between 0 and $\pm c$. For $v = 0$, both equations give the speed of light in the medium $w = c/\mu$, as expected. For $\mu = 1$ both equations give $w = c$, as expected. For $v = c$, equation (6) gives $w = c$, also as expected but equation (12) gives $w = c(1+1/\mu - 1/\mu^2)$ which may be greater than c as $1 < \mu < 2$. For $v = -c$, equation (6) gives $w = c(2/\mu - 1)$, which is reasonable, but the relativistic equation (12) gives $w = c(1/\mu^2 + 1/\mu - 1)$, which may be negative.

Equation (14) is another good example of Beckmann's *correspondence theory* [4], whereby the desired result is obtained mathematically but based on the wrong underlying principles. Fizeau's experiment might as well have verified the fringe shift in equation (10), rather than the relativistic equation (14), for the transmission of light in a moving medium. Curt Renshaw [5] analysed the results of several experiments conducted to measure the effect of the speed of a medium on the speed of transmission of light in the medium and he concluded that the results could be explained without invoking special relativity.

8 References

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