Inertial and Universal Velocities in the Scaling Theory

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Abstract
The concepts of the dimensionless universal and inertial velocities, which are intimate to the structure of the scaling theory, are discussed. The mechanics constructed on the bases of the new concepts, named universal mechanics, admits superluminal velocities, but yet coincides with the relativistic mechanics in its familiar basic dynamical components and their inter-relations. The current formalism accommodates consistently the results of CERN experiment with neutrinos.

1.The Galilean Form of the Scaling Transformations

Assume that a body b is moving in the universal frame S at velocity $\vec{u} = u\vec{\iota},$ and $B \in S$ is a body at rest in S with a geometric time distance $T\,sec$ from the observer $O \in S$. Suppose that at an instant $t = 0$ corresponding to b passing by $B \in S$, a spherical light wave emanates from observer $O \in S$. The wave arrives at B at an instant T and intercepts the source at a point $b' \in S$ an instant t. The sides' lengths of the triangle OBb' satisfy all triangle relations in Euclidean trigonometry, and in particular, $\overrightarrow{Ob'} = \overrightarrow{OB} + \overrightarrow{Bb'}.$ Since O and B are at rest in S, we have $\overrightarrow{OB} = cT(-\vec{e}),$ but since the body b is moving in S we set provisionally $\overrightarrow{Bb'} = a\vec{u}t$ and $\overrightarrow{Ob'} = act(-\vec{e}_L),$ where a is a factor that depends on the magnitude of the velocity of the body b and must go to 1 as u goes to zero. The sides vectors of the aforementioned triangle

(1.1) $$\overrightarrow{OB} = cT(-\vec{e}), \qquad \overrightarrow{Bb'} = at\vec{u}, \qquad \overrightarrow{Ob'} = act(-\vec{e}_L),$$

satisfy the vector relation

(1.2) $$at(-\vec{e}_L) = T(-\vec{e}) + at\vec{\beta},$$

where $\vec{\beta} = \vec{u}/c.$ Solving for t we obtain

(1.3) $$t = a^{-1}G(\beta, \pi - \theta)T,$$

where

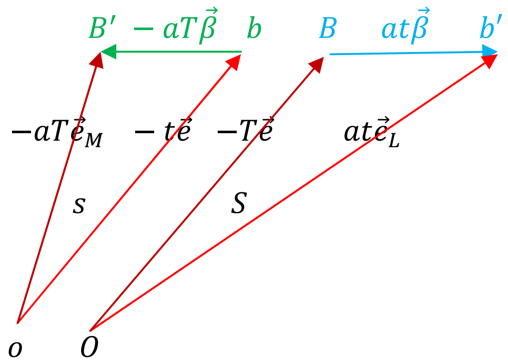
(1.4) $$G(\pi - \theta, \beta) = \frac{\beta cos\theta + \sqrt{1 - \beta^2 sin^2\theta}}{1 - \beta^2}$$

is the Euclidean factor [1,2].

The scaling theory preserves the universal nature of time. Our goal therefore, is to find the transformation from a universal frame to a moving frame such that the transformation and its inverse yield time durations as frame independent entities. The demand made on the transformation and its inverse is necessary so that either frame (but not both) can be started with as universal. Consider thus an inertial frame s that is moving in the universal frame S at velocity $\vec{u}$; the body b will be at rest in s. Let $o \in s$ be the s-observer that is contiguous to $O \in S$ at the instant $t = 0.$ i.e., when the spherical wave emanates from $O \in S$. While b remains at rest in the moving frame s, the body $B \in S$ moves at velocity $(-\vec{u})$. In the frame s the wave hits b at the instant t and intercepts B at the instant T at a position $B' \in s$. We seek thus the transformation that maps the triangle OBb' in S to the triangle $oB'b$ in s, with the time lengths of $OB\ and\ Ob'$ are equal to the time lengths of $oB'\, and\ ob$ respectively, and vice versa. Since the role of the ordered pair (t, T) in s must identical to the role of the pair (T, t) in S, the relation between t and T in s must results from that in S by interchanging t and T in (1.3) and replacing $\vec{u}$ by $(-\vec{u})$, or equivalently $\pi - \theta$ by θ. This yields

(1.5) $$T = a^{-1}G(\beta, \theta)t.$$

On the other hand, the quantities t and T already satisfy (1.3). Substituting one of the equations (1.3) or (1.5) in the other yields



(1.6) $$1 = a^{-2}G(\pi - \theta, \beta)\, G(\theta, \beta) = a^{-2}\,\gamma^2$$

or

(1.7) $$a = \gamma = \frac{1}{\sqrt{1 - \beta^2}}.$$

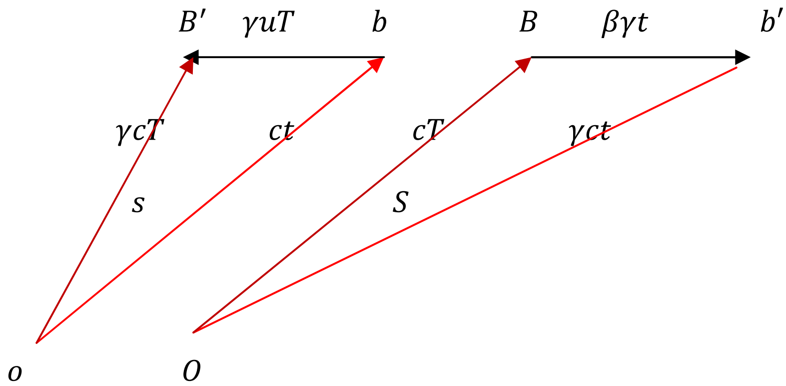
The required transformation is therefore

(1.8a) $$t = \Gamma(\theta, \beta)T \equiv \sqrt{1 - \beta^2} G(\pi - \theta, \beta)T,$$

or

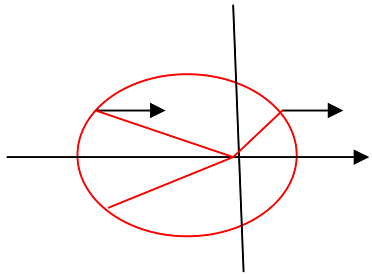
(1.8b) $$\frac{t}{\sqrt{1 - \beta^2}} = G(\pi - \theta, \beta)T.$$

We call the function $\Gamma(\theta, \beta)$ the scaling factor and the relation $t = \Gamma(\theta, \beta)T$ the scaling transformation. The relation (8b) is called the Galilean form of the scaling transformation.



The scaling transformation holds within the same frame whether it was the stationary or the moving frame, as well as between the universal and moving frame, and allows for each frame to be the stationary frame. Note that the geometric lengths of the corresponding trips in S and s are different.

It is not difficult to convince oneself that if a light wave emanates from b when at B then the period it takes to arrive at O is given by (1.8), .i.e. if a light wave emanates from b simultaneously with a wave emanating from O then when the former arrives at O the latter arrives at b'.



The above figure illustrates that all s-pulses that emanate at $t = 0$ from geometric distances $T = t_0 \,/\, \Gamma(\theta, \beta)$ seconds from $O \in S$, where t_0 is fixed, arrive simultaneously in t_0 seconds at O. In particular the pulse of the s-source which is heading (receding) directly towards (from) O and initially at the distance $\Gamma(0, \beta) = ((1 + \beta)/(1 - \beta))^{1/2}$ (at a distance $((1 - \beta)/(1 + \beta))^{1/2}$) seconds, arrive in one second at O. With respect to s-observer that is contiguous to O at the instant t_0, all sources that are at a geometric distance T seconds from O are at equal geometric distance t_0 from o.

2. On the Concept of Time in Scaling Theory

Time and geometric length in Newton's mechanics are independent absolute entities, and the velocity of an object in an inertial frame S is defined as the ratio $\beta_N = d/t$ of the travelled distance and the corresponding time interval. In measuring β_N light signals are redundant, for it is only necessary for point-wise observers to read the initial and final positions of the moving object, together with the corresponding instants of time employing their timers, or resorting to the universal timer (1.XII). Even if light signals are used to inform an observer O of such readings, these signals carry only information about the local measurements and can be sent at a later time.

In the scaling theory (ST), the frame independent entity, namely, "the time" is defined in terms of spatial displacements and can be measured by length units, which implies that velocity is a dimensionless quantity [2, section 3]. Indeed, global timing in a stationary frame S is set up by synchronization with an arbitrary observer $O \in S$ employing light's signals. The concept of time emerges through envisaging a 1-1 correspondence between each instant of time t read by the timer O and the compound event: (the wave front of the pulse that was emitted from O at $t = t_0$ occupies, at t, points at equal distances R from O). Through this correspondence, time duration Δt , is essentially measured by distance R, i.e. $\Delta t = (t - t_0) = a\,R$. The proportionality constant is the reciprocal of the velocity of light. Time durations defined in this way are independent of the master timer's position O. The homogeneity of time follows also from the homogeneity of space. Timers in S, have to reflect the facts stated in setting up the global time. By its way of construction, the global timing is unique up of course to an arbitrary choice of a unit and of zeroing, i.e., up to a transformation of the form $t' = at +$

t_0, where $a > 0$ and t_0 are arbitrary numbers. The global time in S prevails in any other inertial frame s, and every s-timer, if needed, must read the same instant of time read by the S-clock that is touching it.

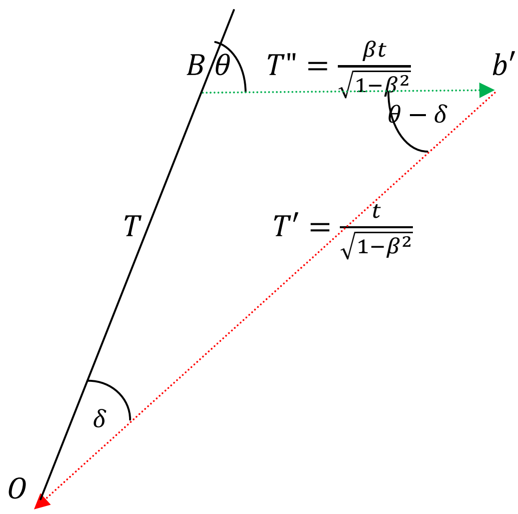
Instead of S, any other inertial frame s can be claimed universal, and a global time set up in s and prevailing in all other inertial frames must be the same as that constructed in S. The scaling transformations fulfill this quest by preserving time durations. In the frame S the ratio between the duration of a given light's trip and its geometric time length ($i.e.$, $t/T = \Gamma(\beta, \theta)$) is determined by the velocity of the source and its direction, and time therefore, is set up mathematically in terms of equally primitive concepts, the geometric length (through T) and inertial frames (through β).

In reality, time durations and instants of time, become measurable as soon as an inertial frame is endowed with a global time. The instants of time corresponding, for example, to a moving body passing by some given points in S are determined by the time reading of hypothetical timers at these points. Thus, the primitive observables, the true time and geometric distances, exist together and are measurable in a timed inertial frame.

3.The Euclidean Body-Observer Triangle

The initial and final positions of the moving body together with the observer's position form a Euclidean triangle with sides length's

(3.1) $$\mathrm{T},\ \ \mathrm{T}'' = \frac{\beta t}{\sqrt{1-\beta^2}},\ \ \mathrm{T}' = \frac{t}{\sqrt{1-\beta^2}}.$$



The given lengths satisfy all triangle relations in Euclidean geometry, and yield a value t for the time duration as it is prescribed by the scaling transformation STI. An elementary fact in Euclidean geometry asserts that when three numbers are legitimate to form a triangle, this triangle is unique (up of course to arbitrary rotations, translations, or reflections). Thus the latter values determine a unique Euclidean triangle; it is the *body-observer triangle*.

By (3.1) we have

(3.2) $$\frac{T''}{\beta}=\frac{T'}{1}=\frac{t}{\sqrt{1-\beta^2}},$$

which are equivalent to the relations

(3.3) $$T''=\beta T',\quad T''^2-T'^2=t^2.$$

In terms of the initial geometric distance T we have

(3.4) $$\frac{T''}{\beta}=\frac{T'}{1}=\frac{\Gamma(\beta,\theta)T}{\sqrt{1-\beta^2}}\equiv G(\beta,\pi-\theta)T.$$

By the sinuses law in trigonometry,

(3.5) $$\frac{\gamma\beta t}{\sin\delta}=\frac{\gamma t}{\sin\theta}=\frac{T}{\sin(\theta-\delta)}\ ,$$

we have

(3.6) $$\sin\delta=\beta sin\theta,\quad \sin(\theta-\delta)=sin\theta/G(\beta,\pi-\theta).$$

Since the pair of sides (T'',T') are in 1-1 correspondence with the pair (β,t), the body-observer triangle is determined by (β,t,T), which expresses the obvious fact that the direction of the vector $\vec{\beta}$ relative to the observer, i.e. θ, is determinable by the quantities (β,t,T) through the scaling transformations $\Gamma(\beta,\theta)=t/T$, by which we can determine one out of the quantities $\{T,t,\beta,\theta\}$ in terms of the remaining three. In other words, the triangle of body-observer is fully determined by three out of the four variables t,T,β,θ. This implies that in correspondence with each body-observer triangle there is one value of β, and hence the same value of β is obtained whether calculated from the expressions of $T'\, or\, T''$ or from the scaling transformations.

The Inertial Velocity

By (3.2), the displacement of the source, the distance travelled by the signal, and their duration *t* are in 1-1 correspondence. The quantity

(3.7) $$\beta=\frac{T''}{T'}\ ,$$

which is obtained from (3.2), will be called the *inertial velocity* of the body *b*. The definition (3.7) expresses the inertial speed in terms of geometric distances; *it is the quotient of the distance* (T'') *travelled by the body b to the distance* T' *travelled by the light emanating from the observer O when they intercept each other.* The initial time for both motions is the instant (*b at B*). By Galileetion [1, section 13], T' is also the distance travelled by the pulse emitted from *b* when at *B* till arriving at *O.* The inertial velocity of a body $B\in S$ is nil, because *B* is not displaced whatever was the distance travelled by a pulse intercepting it and emitted from an *S*-observer.

The concept of universal velocity which we introduce in the next section, differs from the seemingly identical concept, the Newtonian velocity, in being compatible with observation through light signal.

4. The Universal Mechanics

The universal velocity of a body (or just velocity) refers to its velocity in a universal space; its definition is identical to that of velocity in classical mechanics. Thus the universal velocity of the body *b,* considered before, is the ratio of the distance it travels to the corresponding time interval. By (3.2) we have

(4.1) $$\beta_U\equiv\frac{T''}{t}=\frac{\beta}{\sqrt{1-\beta^2}}.$$

The universal velocity $\vec{U} = c\vec{\beta}_U$ (or $\vec{\beta}_U$ if we take $c = 1$) is measured locally by reading the instants t_1 and t_2 at which the particle occupies the positions $B \in S\ and\ b' \in S$ respectively, and the length of the corresponding displacement vector $\overrightarrow{Bb'}$, which may of course be infinitesimal. It is noted that light signals do not enter in the measurement process, but yet, β_U local measurement is compatible with observation by light's signals.

According to its expression, the magnitude of the universal velocity can assume any non-negative value with no upper bound (i.e. $0 \leq \beta_U < \infty$) and that β_U goes to zero with β.

The ***momentum*** of the particle *b* is defined by the product of its mass *m* and universal velocity $\vec{U} = c\vec{\beta}_U$:

(4.2) $$\vec{p} = m\vec{U} = \frac{mc\vec{\beta}}{\sqrt{1-\beta^2}} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}}.$$

Noting the identity

(4.3) $$\frac{\beta^2}{1-\beta^2} = \frac{1}{1-\beta^2} - 1,$$

we obtain

(4.4) $$\frac{p^2}{c^2} = \frac{m^2}{1-\beta^2} - m^2.$$

In the reduced system of units RSUI or RSUII (see appendix) mass and energy have the same dimension, and the *right hand-side can be envisaged as a difference between the squares of two values of the mass or energy of the moving body corresponding to the states of motion and rest respectively.* Denoting these values by $E\ (\equiv M)$ and $E_0 (\equiv m)$ respectively, i.e,

(4.5) $$E(kg) \equiv M(kg) = \frac{m}{\sqrt{1-\beta^2}}\ (kg), \quad E_0 \equiv m\ (kg),$$

we write (4.4) in the form

(4.6) $$\frac{p^2}{c^2} = E^2 - E_0^2 = M^2 - m^2.$$

The latter relation reads: the state of motion of a body with rest mass *m* that is characterized by momentum of magnitude *p* is accompanied by a ***total kinetic energy, or kinetic mass***,

(4.7) $$E = \frac{m}{\sqrt{1-\beta^2}} = \sqrt{m^2 + (p/c)^2}.$$

When *p* goes to zero, the total kinetic energy (or kinetic mass) tends to the rest energy (or rest mass) $E_0 = m$. The Hamiltonian of the particle coincides with its total kinetic energy:

(4.8) $$H = \sqrt{m^2 + (p/c)^2}.$$

It is customary to measure mass in kg, energy in $(c^{-2}kg = Joule)$ and momentum in $(c^{-1}kg = m.sec^{-1}.kg)$ (see appendix), which corresponds to using the reduced system of units

$$RSUII \equiv \{LS = m, TS = sec = c.m, MS = kg\}.$$

In RSUII, the expressions (4.5) become

(4.9) $$E = \frac{mc^2}{\sqrt{1-\beta^2}}(c^{-2}kg), \qquad E_0 \equiv mc^2\ (c^{-2}kg),$$

where the mass *m* is measured in kilogram and the energy E in Joule $= c^{-2}kg$.
Equations (4.7) and (4.8) become

(4.10) $$E = \frac{mc^2}{\sqrt{1-\beta^2}} = \sqrt{m^2c^4 + p^2c^2},$$

(4.11) $$H = c\sqrt{m^2c^2 + p^2}.$$

In RSUI, $c = 1$, and the general relations (4.8) and (4.11) take the particular forms

(4.12) $$H = E = \sqrt{m^2 + p^2}.$$

The accommodation of the electromagnetic theory in the formalism of the scaling theory will be considered in a future work.

5. Measuring Inertial Velocity

A method of measuring the inertial velocity was discussed by the end of section 3. Another method for measuring the velocity of the s-frame is considered here. We know that time durations, and instants of time, become measurable as soon as an inertial frame is endowed with a global time. On measuring *T* and *t* the right hand-side of the scaling transformations $\Gamma(\beta,\theta) = t/T$ becomes known. The value of this ratio determines the inertial velocity β when the angle θ is known. Thus *the time of the arrival of a pulse emitted from the body at the observer O suffices to determine its inertial velocity provided its initial position and the direction of its velocity relative to the observer's position are known.*

Knowing *t* and *T* determines the component of the relative velocity $\vec{\beta}$ in the direction of the position vector, $\vec{R} = cT(-\vec{e})$, of the source relative to the observer. Indeed, from the expression of the scaling factor

(5.1) $$\Gamma(\beta,\theta) = \frac{t}{T} = \frac{\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}}{\sqrt{1-\beta^2}},$$

we obtain

(5.2) $$\frac{\Gamma^2 - 1}{2\Gamma} = \frac{\beta cos\theta}{\sqrt{1-\beta^2}} = \beta_U cos\theta.$$

For $\Gamma = 1$, which is equivalent to $t = T$, we have by the latter equation, $\beta cos\theta = 0$, which means that either the body is at rest ($\beta = 0$) in *S* or moving at a right angle with the radius vector $\vec{R}$ with unknown velocity. Assuming $\theta \neq \pi/2$, and dividing both sides in (5.2) by $cos\theta$, we obtain

$$\frac{\beta}{\sqrt{1-\beta^2}} \equiv \beta_U = \frac{\Gamma^2 - 1}{2\Gamma cos\theta}.$$

Contrary to what is conveyed by its expression, the value of the last term is independent of θ. In fact, the value $\Gamma = t/T$ changes with θ in a manner makes the last term, as it should be, independent of θ. This means that for all *s*-object the same value of β is obtained by the latter formula, which yields

(5.2*a*) $$\beta = \frac{|\Gamma^2 - 1|}{\sqrt{(\Gamma^2 - 1)^2 + 4\Gamma^2 cos^2\theta}}$$

$$(5.2b) \qquad = \frac{|\Gamma^2 - 1|}{\sqrt{(\Gamma^2+1)^2 - 4\Gamma^2 sin^2\theta}},$$

where

$$(5.3a) \qquad |\Gamma^2 - 1| = \Gamma^2 - 1 \text{ for } 0 \le \theta < \frac{\pi}{2}$$

$$(5.3b) \qquad |\Gamma^2 - 1| = 1 - \Gamma^2 \text{ for } \frac{\pi}{2} < \theta \le \pi$$

It is clear from (5.2a) that β is strictly less than 1.

In the expressions (5.2) the dependency of the measured value of Γ on θ is such that the right hand-side is independent of θ . To calculate the velocity β by formula (5.2) we need to read the time when the pulse arrives at O and to know the body's initial geometric vector position as well as its velocity's direction relative to this vector.

When θ is known the relation (5.2), supplemented by (5.3), sets up a 1-1 correspondence between β and $\Gamma = \mathrm{t/T}$. On the other hand, and for a fixed value of the ratio Γ, the value of β depends on the angle $\theta = \measuredangle(\vec{R}, \vec{v})$.

The inertial velocity of an s-frame is most simply measured by picking up an s-object that directly recedes from, or approaches, the observer $O \in S$.

(i) When the source approaches the observer radially, which corresponds to $\theta = \pi$,

$$\beta = \frac{1 - \Gamma^2}{\Gamma^2 + 1}.$$

The same result could have been obtained from the scaling transformation $\Gamma = \Gamma(\beta, \pi)$. In this case $\Gamma \le 1$ (because β is positive), or $0 < t \le T$. Since $t \to 0$ as $\beta \to 1^-$, objects approaching us with a great speed are detected within a limited period of time although they may be at the time of light emission at an huge distance T from us.

(ii)When the source recedes radially from the observer, which corresponds to $\theta = 0$,

$$\beta = \frac{\Gamma^2 - 1}{\Gamma^2 + 1}.$$

The same result could have been obtained from the scaling transformation $\Gamma = \Gamma(\beta, 0)$. In this case, $\Gamma = t/T \ge 1, or\ \ t \ge T,$ with no upper limit on the value of the time duration *t*. Moreover, $\beta \to 1^- \Rightarrow t \to \infty$. It follows that objects receding from us at enormous speed can take incredibly long time to be detected through light signals. Neither the objects nor their radiation is practically seen or felt; they are a sort of dark matter and dark energy. This applies also to objects that may be at the time of light emission quite close to us. Moreover, the great velocities at which these particles recede, confines the radiation we receive to quite long wavelengths.

Although a moving object can overtake the pulse emanating from its starting position when the object was there, the object can never overtake the pulse it emits in the direction of its motion. The latter fact will be considered in section 7.

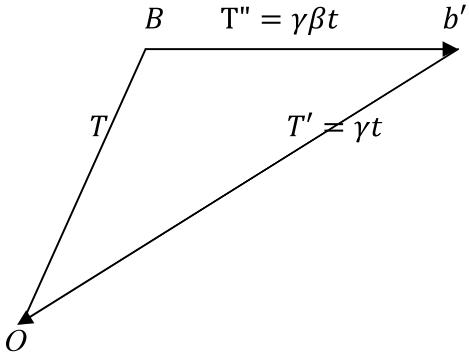
6. Point-wise Measurement of the Inertial Velocity

Suppose that the free source of light *b* is detected at a point $B \in S$ at $t = 0$ and at $b' \in S$ at an instant of time *t*. The pulse which was emitted from *b* when was at *B* arrives at the instant t at $O \in S$. i.e. the arrivals of b at b' and the pulse at O are simultaneous; they both occur at *t*. Let $T'' = |\overrightarrow{Bb'}|$ be the geometric time length of the displacement $\overrightarrow{Bb'}$. The source *b,* at the time *t*, is at geometric distance

(6.1) $$\frac{\beta t}{\sqrt{1-\beta^2}} \equiv T"$$

from $B \in S$. This yields

(6.2) $$\beta = \frac{1}{\sqrt{1+(t/T")^2}}$$



The latter formula determines the inertial velocity of a moving body b in S in terms of the time t read at its final position $b' \in S$ and the geometric distance $T"$ of b' from the body's initial position $B \in S$. The formula (6.2) digresses from the Newtonian (or universal) definition of the velocity as the ratio of distance travelled to the corresponding time interval, which takes in our units the form $\beta_U = T"/t$. In terms of the universal velocity, the inertial velocity takes the form

(6.3) $$\beta = \frac{\beta_U}{\sqrt{\beta_U^2+1}} = \frac{U}{\sqrt{U^2+c^2}}.$$

The following comments illustrate some facts concerning the inertial velocity which applies, of course, only to material bodies, but not to light signals.

(i) Because the right hand side of (6.2) is less than 1, $0 \leq \beta < 1$, and hence the inertial velocity β of any material object can not reach the value 1, i.e. can not reach the velocity of light.

(ii) If a body b is at rest at $B \in S$ which is distinct from $b' \in S$, then b will never be found at b' . Setting $t = \infty$ in (6.2) yields $\beta = 0$.

(iii) In spite of the fact that the inertial velocity of any object can not reach the velocity of light, the object itself can overtake the pulse emanating from its starting position $B \in S$. If the moving body b and the light signal emitted from $B \in S$ when b passed by, arrive simultaneously at b', then $t = T"$, and the inertial velocity of the body is $\beta = \frac{1}{\sqrt{2}} \approx 0.707$. However, the universal velocity of the same body is $\beta_U = T"/t = 1$. While inertial velocity can not reach the velocity of light, universal velocity is unbounded and can exceed that of light. These facts demonstrate that there is nothing mysterious about the result of the CERN experiment with Neutrinos.

(iv) For a fixed value of $T"$, β is a decreasing function of t; it tends to zero for t tending to infinity and to 1 for t tending to zero. This expresses the obvious fact, the faster the particle is the shorter time it takes to arrive at b'.

(v) For small velocities, $t/T" \gg 1$, and

(6.4) $$\beta = \frac{T''}{t}\frac{1}{\sqrt{1+(T''/t)^2}} \approx \frac{T''}{t} - \frac{1}{2}(\frac{T''}{t})^3 = \beta_U - \frac{1}{2}\beta_U^3$$

If the inertial velocity is sufficiently small we can neglect the third order term in comparison with the first order term and write

(6.5) $$\beta \approx \frac{T''}{t} = \beta_U.$$

We also obtain the same result simply by neglecting 1 in the dominator on the right hand-side of (6.2) in comparison with the much larger term $(t/T'')^2$. Therefore, for small values, the classical expression is an approximation of the inertial velocity formula (6.2).

For high velocities, $t/T'' \ll 1$, and the formula (6.2) can be approximated by

(6.6) $$\beta \approx 1 - \frac{1}{2}(\frac{t}{T''})^2 + \frac{3}{8}(\frac{t}{T''})^4 = 1 - \frac{1}{2}\beta_U^{-2} + \frac{3}{8}\beta_U^{-4}$$

The inertial velocity can also be deduced in terms of T'. Indeed, on solving the expression of T' for β we obtain the following equivalent expression of the inertial velocity,

(6.7) $$\beta = \sqrt{1-(t/T')^2}.$$

The μ −meson particles lifetime once more

The $\mu -$ meson particles whose mean lifetime is $t = 2 \times 10^{-6} sec$, are generated at $60km$ above the earth surface, that is, $T'' = 60km/(300{,}000km/\text{sec}) = 2 \times 10^{-4} sec$. Inserting $t/T'' = 10^{-2}$ in formula (6.2) yields the inertial velocity of the particles that just arrive at the earth surface as

(6.8) $$\beta = \frac{1}{\sqrt{1+10^{-4}}} \approx 1 - \tfrac{1}{2}10^{-4} = 0.99995$$

The particles with inertial velocities not less than the latter value can cover *60 km* in the earth's frame *S* in spite of the fact that a pulse of light emitted from an *S* observe can travel during the period of the $\mu - meson$ lifetime only

(6.9) $$t \times c = 2 \times 10^{-6} \times 3 \times 10^5 km = 0.6\ km$$

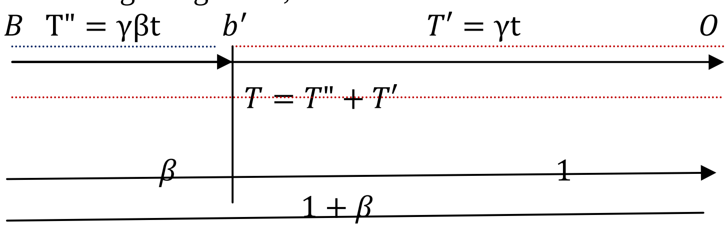
Relative to the Earth frame the universal velocity the mesons particles that reach the earth surface is

(6.10) $$\beta_U = \frac{T''}{t} = \frac{2 \times 10^{-4}}{2 \times 10^{-6}} = 100,$$

one hundred times of the velocity of light!.

7. The Simultaneous Positions of a Particle and its Emitted Signal.

Suppose that the body b heads towards O with inertial velocity β. A pulse emitted from b when at $B \in S$, say at $t = 0$, arrives at $O \in S$ simultaneously with the body arriving at $b' \in S$. The Galilean picture in which light is envisaged to emanate from the body's current position, corresponds to body-triangle (3.1), which reduces in the current head-on motion to a 3 straight segments, with



(7.1) $$\frac{T"}{\beta}=\frac{T'}{1}=\frac{\mathrm{T}}{1+\beta}=\gamma \mathrm{t}.$$

The third ratio is obtained from the properties of the proportion. Equivalently,

(7.2) $$T"=\gamma\beta\mathrm{t},\ \ \mathrm{T}'=\gamma \mathrm{t}\qquad T=T"+T'=\gamma(\beta+1)t$$

The first and third ratios show that when the body is at a distance $T"$ from *B*, the pulse it emits is at a distance

(7.3) $$T=\frac{1+\beta}{\beta}T"$$

from *B*. Thus, a particle always lacks behind the pulse it emits in its direction. As an example, a hypothetical light signal emitted from the meson particles when generated, travel by the time during which a meson particle arrives at the earth surface the distance

$$T=\frac{1+0.99995}{0.99995}\times 60\approx 2\times 60\ km$$

which (assuming not absorbed) is almost twice as much the distance travelled by the particle itself.

By (7.1), the particle and the light, arrive at b', at t_p and t_l respectively, where

(7.4) $$t_p=\sqrt{1-\beta^2}\frac{T"}{\beta}.$$

(7.5) $$t_l=\sqrt{\frac{1-\beta}{1+\beta}}T^{"}=\sqrt{1-\beta^2}\,\frac{T"}{1+\beta}=\frac{\beta}{1+\beta}t_p.$$

Thus light arrives first at b' advancing *b* by

(7.6) $$t_p-t_l=t_p\left(1-\frac{\beta}{1+\beta}\right)=\frac{t_p}{1+\beta}=\frac{t_l}{\beta}$$

Employing the STI to determine the position of the light front when *b* arrives at b'; i.e. at the instant of time t_p, we get

(7.7) $$T=\sqrt{\frac{1+\beta}{1-\beta}}t_p=\sqrt{\frac{1+\beta}{1-\beta}}\sqrt{1-\beta^2}\frac{T"}{\beta}=\frac{1+\beta}{\beta}T"$$

which coincides with (7.3). The same relation has been obtained by the Galilean picture, which on scaling $T^{"} and\ T'=T-T"$, yields the particle travelling the distance $\sqrt{1-\beta^2}T"$ with velocity β and light travelling the distance T' with velocity 1. i.e.

$$\sqrt{1-\beta^2}\left(T-T^{"}\right)=t_p$$

Substituting for t_p from (7.4) we obtain *T* as given by (7.3). On changing to distances instead of geometric time distances we write (7.3) in the form

$$\frac{X}{c+u}=\frac{X"}{u}$$

which is the same as the classical picture, apart from the fact that the quotients are γt, but not *t*. and hence *u* refers to the inertial velocity. For small velocities, $X"/u\approx t$, and $X\approx(c+u)t$.

Appendix: The Reduced System of Units

If the unit of time *TS* in *S* is defined as the duration required by light to cross the unit distance *LS* (a given rod stationary in *S*) from one end to another, say $LS=1meter$, we may designate the unit of time also by “meter”, to mean the time required by a light’s signal to cross this distance. In terms of a system of units of time, length, and mass $\{TS = LS = m, MS = kg\}$, the dimensions of some mechanical observables are listed in the table:

(3.4i) $[velocity] = LS.TS^{-1} = 1 \ (dimensionless)$

(3.4ii)

(3.4iii) $[momentum] = kg.LS.TS^{-1} = kg$

(3.4iv) $[force] = kg.LS.TS^{-2} = kg.m^{-1}$

(3.4v) $[energy] = [work] = kg.LS^2.TS^{-2} = kg = [mass]$

(3.4vi) $[angular\ momentum] = m.kg$

(3.4v) $[torque] = \ kg$

As seen from the first relation, the velocity $\vec{v} = \Delta\vec{R}/\Delta t$ in this system of units is a dimensionless 3-vector, and the speed of light in vacuum is 1 regardless of the chosen unit of length LS, provided we choose $TS = LS$. Mass and energy have the same unit, “kilogram”. In the reduced system of units (I) $\{TS = LS, MS = kg\}$, LS and MS are arbitrarily chosen, once and for all. In practical applications it is convenient to take $LS = 1\,meter \equiv m,$ and adopt a multiple of the unit $TS = 1\,meter,$ namely, “$second$”. The latter is defined by the period taken by light to travel a distance of $c\ meters = 3 \times 10^8 m$. Thus $1\ second = c\ meters,$ or $1\,meter = \frac{1}{c} seconds$. In the reduced system of units (II)

$$(3.5)\qquad \{LS = meter, second = c.meter, kg\} \equiv \left\{m = \frac{1}{c}sec, sec, kg\right\},$$

$$(3.6i)\qquad [velocity] = m.(c.meter)^{-1} = \frac{1}{c},$$

$$(3.6ii)\qquad [acceleration] = c^{-2}m^{-1} = (c\,sec)^{-1},$$

$$(3.6iii)\qquad [momentum] = \frac{1}{c}kg$$

$$(3.6iv)\qquad [force] = kg.c^{-2}m^{-1} = kg.(c\,\mathrm{sec})^{-1} = Newton$$

$$(3.6v)\qquad [energy] = [work] = kg.c^{-2} = Joule$$

$$(3.6vi)\qquad [angular\ momentum] = c^{-1}\,m.kg = c^{-2}sec.kg = Joule.sec = [action]$$

$$(3.4v)\qquad [torque] = c^{-2}\,kg$$

The reduced systems of units (RSU), I or II, suggest that observables which are measurable by the same unit are of the same nature, and merely appear as different facets of the same entity. Mass and energy for instance are both scalar quantities and both are measurable in RSUI by kg. This means that 1 kg of mass is equal to 1 kg of energy, and that under suitable circumstances either quantity can be transformed to the other. In the RSUII,

$$1kg = c^2(c^{-2}kg) = c^2 Joule,$$

and $m(kg) = mc^2 Joule$.

For a vector observable $\vec{A}$ that has the same dimension as a scalar observable B, the squares of these observables, A^2 and B^2 are of the same nature and in principle are transformable to each other.

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