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Indeterminate Form in the Equations of Archimedes, Newton and Einstein.

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Abstract

A very peculiar similarity is observed between equations of Archimedes, Newton and Einstein. As under some condition the equations based upon the established principles/laws lead to indeterminate form i.e. $\frac{0}{0}$. The reason is that in the case of Archimedes principle, equations became feasible in 1935 after enunciation of the principle in 1685, when Newton defined g in *The Principia* . Such specific studies do not mean any comment or conclusion of the established status of the well-known laws.

1.0 Indeterminate form of volume leads to a generalization of Archimedes principle in this particular case.

1.0 Completely submerged floating balloons lead to indeterminate volume of fluid in them

Consider a balloon filled with a medium of density D_m floating in water of density D_w . The volume of the sheath of tje balloon /vessel is v (mass =m) and the volume of the medium filled inside the balloon is V (say wood, metal and gases). According to Archimedes principle the upthrust experienced by the balloon is equal to the weight of fluid displaced [1-2]. The body displaces fluid equal to its own volume.

$$V D_m g + mg = (V+v) D_w g \qquad (1)$$

$$\text{Or } m = (V+v) D_w - V D_m \qquad (1)$$

From Eq.(1) different values of V, D_w , D_m and v can be written as

$$V = \frac{(m - vD_w)}{(D_w - d_m)} \qquad (2)$$

$$D_m = \frac{(m + VD_w)}{(V + v)} \quad (3)$$

and

$$D_w = \left\{ \frac{D_m (V + v) - m}{V} \right\} \quad (4)$$

$$v = \left\{ \frac{m - V(D_m - D_w)}{D_w} \right\} \quad (5)$$

Now we can try to calculate the volume (V) of a fluid filled in a balloon (wood, metal and gases), the sheath of the balloon has a volume v and mass m, such that the density of fluid filled inside is equal to that of water ($D_m = D_w$). The obvious volume should turn out equal to V, which is the actual value. Hence substituting, mass from eq.(1) in eq.(2), we get

$$V = \frac{(vD_w - vD_w)}{(D_w - D_w)} = \frac{0}{0} \quad (6)$$

which is the indeterminate form i.e. volume of medium filled in the balloon (wood, metal and gases) becomes undefined but in the actual experimental setup, the volume is V consisting of metal, wood and gases. Thus RHS of Eq.(6) becomes devoid of units and dimensions which are not defined. Although division by zero is not permitted, yet it smoothly follows from equations based upon Archimedes' principle as per its definition. Also in this case, the numerator of the equation also becomes zero. Also v, D_m, D_w cannot be calculated from Eqs.(3-5) due to eq.(6) .

Thus Archimedes' principles in its original form; in this particular case is not applicable.
L Hospital Rule is not applicable here, as it is an exact equation, NOT a function or limit or differentiation of the function. No such method can give the exact value of V. Thus Archimedes' principles in its original form in this particular case, is not applicable. It can be solved by generalizing the 2265 years old Archimedes principle.

It may be understood in view of the fact that mathematical equations based upon Archimedes principle became feasible after 1935 years of enunciation of the principle when Newton published *The Principia* and defined acceleration due to gravity in 1685.

This limitation can be easily explained if Archimdes principle is generalized for this particular case i.e.

' upthrust experienced by balloon is proportional to the weight of fluid displaced'

$$U_{gen} \propto (V+v)\, D_w\, g \quad \text{or} \quad U_{gen} = f\,(V+v)\, D_w\, g \quad (7)$$

Now eqs.(1-2) become

$m= f\,(V+v)\,D_w - VD_m$ (8)

$$V = \frac{\left(m - fvD_w\right)}{\left(fD_w - d_m\right)} \quad (9)$$

Under the similar condition ($D_m = D_w$)

$$V = \frac{\left(f - 1\right)V}{f - 1} = V \quad (10)$$

Now, consistent results are obtained. Also now, correct values of v, D_m and D_w are obtained. Unlike eq.(6), in this case, no division by zero is involved, also the numerator is non-zero, hence a consistent and logical result is obtained. Correctly in this case, the internal volume of the balloon is V which is filled with wood, metal and gases. The physical significance of the co-efficient of proportionality, f can be understood as it may be regarded as accounting for the shape of body. According to original form of the principle, the body floats when its density is equal to that of the medium ($D_b = D_m$) and other factors, including shape, have no role to play.

This analysis does not mean any comment or judgement on the established status of the Archimedes principle, as this analysis is valid under certain conditions only.

2.0 Indeterminate form of mass in Newton's Second of Motion when it reduces to first law of motion

According to *The Principia* [2] the second Law of motion is

The alteration of motion is equal to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

Mathematically force,

F =ma = mass x acceleration

This law also gives the inertial mass of the body.

$$M = \frac{F}{a} = \quad (11)$$

The second law of motion is the real law of motion as the first and third laws of motion can be derived from it. If no external force F acts (a=0), then the body either remains at rest, v=0 or moves with uniform velocity v =u (which is the first law of motion). So the second law of motion reduces to the first law if F=0. Under this condition (when the second law of motion reduces to the first law, F=0, v=u) the inertial mass is given by

$$M = \frac{F}{a} = \frac{0}{0} \qquad (12)$$

which is an inconsistent and unphysical result in this particular situation only . It is similar to eq.(6). L Hospital rule is not applicable here.

3.0 Indeterminate form of frequency in Einstein's equation of Doppler principle when v=c

In the paper which is widely known as the special theory of relativity, Einstein [3] put forth a equation of relativistic variation of frequency of light

$$\nu^* = \nu \frac{\left[1 - \frac{v}{c}\cos\phi\right]}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (13)$$

where ν^* is the frequency measured by an observer in a frame moving with velocity v, and ν is the frequency in the rest frame. ϕ is the angle between the source and observer. If the value of ϕ is 0 , and the observer moves with the speed of light i.e. v=c then

$$\nu^* = \frac{0}{0} \qquad (14)$$

which is again a similar result, and frequency becomes undefined. If v $\rightarrow$ c then $\nu^* \rightarrow \frac{0}{0}$. Hence the similar results.

4.0 Conclusions

Thus equations of the scientific legends i.e. Archimedes, Newton and Einstein lead to indeterminate form. In the case of Archimedes principle, the indeterminate form arises from the applications in completely submerged balloons, whereas in the case of Newton's and Einstein's equations it follows directly. The reason is that at the time of formation of the equations, all possibilities were not taken in account, hence there are limitations in the equations in some cases. Anyhow there is some similarity in the equations of Einstein, Newton and Archimedes.

This analysis does not imply any comment or judgement on the established status of the doctrines, as this analysis is valid under certain conditions only. The laws of the scientific legends are not applicable under some conditions as discussed, otherwise the laws hold good. If Archimedes principle is generalized, it leads to logical and consistent results.

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