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a^2-b^2 =(a+b)(a-b) is Not Obeyed by Einstein's Derivation of E=mc^2

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Einstein's 27 Sep 1905 paper available at http://www.fourmilab.ch/etexts/einstein/E_mc2/www/

Abstract
a^2-b^2 =(a+b)(a-b) is the basic algebraic identity taught in middle schools. It is surprising to learn that this identity is not obeyed by the central equation on which Einstein's E=mc^2 is based, under some conditions. The derivation of E=mc^2 was given by Einstein in Sep 1905 and the central equation in the June 1905 paper which is known as the Special Theory of Relativity. This aspect is discussed here logically.

1.0 Transformation of energy of light rays and E=mc^2
Einstein's [1,2] argument: let there be a luminous body at rest in co-ordinate system (x, y, z) whose energy relative to this system is E_o. The system (**ξ,** η, **ζ**) is in uniform parallel translation w.r.t. system (x, y, z); and the origin of which moves along x-axis with velocity v. Let the energy of the body be H_o relative to the system (**ξ,** η, **ζ**). Let a system of plane light waves have an energy ℓ relative to system (x, y, z), the ray direction makes an angle φ with the x-axis of the system. The quantity of light measured in system [**ξ,** η, **ζ**] has the energy.

$\ell^* = \ell\{1 – v/c \cos φ \} /\sqrt{[1 – v^2 /c^2]}$ (1)

Eq.(1) was proposed by Einstein [2] in Section (8) entitled *Transformation of the Energy of Light Rays.* Eq.(1) was extensively used in the derivation of light energy (L) mass eqaution i.e.

L =mc^2 (2)

From this equation Einstein generalised the result for every energy (sound , heat , chemical , electric, energy in form of of invisible radiations etc.) without mathematical justification as

E=mc^2 (3)

Eq.(3) directly follows from Einstein's generalised deduction [1]

" The mass of a body is a measure of its energy-content; if the energy changes by L, the mass changes in the same sense by L/9 × 10^{20}, the energy being measured in ergs, and the mass in grammes"

However, eq.(1) is not justified for various forms of energy for which E =mc^2 is used. It implies the origin of E=mc^2 is not scientifically consistent.

Inconsistent deductions from eq.(1)

Eq.(1) is different from the other relativistic equations i.e from relativistic mass, length contraction and time dilation as its numerator contains a term, [1 – v/c cos φ] which involves the angle φ. Thus, in addition to velocity, the light energy also varies with angle φ. The mass, length and time are said to be relative as they vary with velocity when it is comparable to that of light. But in eq.(1) the light energy also depends upon the angle at which the light ray is emitted with respect to the moving frame of reference. If the body, while moving with the same velocity c/2 emits light energy at three different angles i.e., 89º, 90º and 91º , then the values of ℓ^* are 1.14465ℓ, 1.4142ℓ and 1.6477ℓ respectively. Thus even if the velocity remains the same then light energy varies with angle φ. This result is quite different from other relativistic variations and in that case, quantity only depends upon the velocity. Also it leads to inconsistent results as discussed below :

(i) Inconsistency of dimensional homogeneity when $v \to c$ in eq.(1).

If $\varphi = 0^o$, then eq. (1) becomes,

$\ell^* = \ell\,(1 - v/c) / \sqrt{(1 - v^2/c^2)}$ (4)

If the system [ξ, η, ζ] moves with velocity equal to that of light i.e. v = c which realistically means that velocity v tends to c i.e. $v \to c$. (some Quasars or other heavenly bodies may attain such high velocities) .Thus,

$\ell^* = 0/0$

which is undefined or ℓ^* tends to 0/0 which has the same meaning.

The dimensions of LHS are $M L^2 T^{-2}$ [energy] and that of RHS undefined. It is the inherent requirement that an equation must obey the principle of dimensional homogeneity [3-4] but it is not so in case of eq.(1) under this particular condition.

(ii) Non-compliance of identity $a^2 - b^2 = (a+b)(a-b)$ by eq.(1).

Further contradictory results are also self-evident if eq. (1) is solved and the same condition ($v = c$ or $v \to c$) is applied i.e. $\{1-v^2/c^2\} = (1-v/c)(1+v/c)$ is the simple algebraic result.

$\ell^* = \ell \sqrt{(1 - v/c)} / \sqrt{(1 + v/c)}$ (5)

Now again, if the velocity v tends to c, i.e. $v \to c$ in the above equation becomes,

$\ell^* = 0$

Under this condition, eq.(1) in unsolved form is $\ell^* = 0/0$

Thus under this condition light energy becomes zero when $v \to c$. This is contradictory to other predictions.

Thus the same equation (in solved and unsolved forms) under similar conditions ($v \to c$) gives different results ($\ell^* = 0/0$ and $\ell^* = 0$), which is purely arbitrary and illogical. Thus results from eq.(1) are contradictory to the basic identity of algebra; and in addition, the result is not consistent with dimensional homogeneity.

The basic principle of conservation of mass and energy should not be based upon an equation which is full of limitations e.g. it disobeys dimensional homogeneity and basic algebraic identities.

(iii) If angle $\varphi = 180^o$, then $\cos 180^o = -1$, thus under the condition when velocity tends to c ($v \to c$) or becomes equal to c then,

$\ell^* = \infty$

Thus, entirely different results are obtained, i.e. $\ell^* = \infty$ simply if the angle of the wave is exactly reversed ($\varphi = 180^o$) compared to the first case (when $\varphi = 0^o$, then $\ell^* = 0/0$ or $\ell = 0$)

Thus, the central equation which is used in the derivation of $E=mc^2$ should be free from the limitations and must lead to consistent results in all cases. Further, eq.(1) is relativistic in nature and it must give accurate results when v is comparable to c.

2.0 Consequences

In view of the discussion, the mass energy equation has been derived by a simple method as $\Delta E = Ac^2 \Delta m$, where A is the coefficient of proportionality like numerous others in existing physics. The new equation can be applied to explain the anomalous experimental data. $\Delta E = Ac^{2\Delta} M$ has been suggested, which implies that energy emitted on annihilation of mass (or vice versa) can be equal, less or more than predicted by $\Delta E = c^2 \Delta m$. The total kinetic energy of fission fragments of U^{235} or Pu^{239} is found experimentally 20-60 MeV less than Q-value predicted by Δmc^2 [5-8] , it is explainable with $\Delta E = Ac^2 \Delta M$ with value of A less than one. The mass of particle Ds (2317) discovered at SLAC, have mass lower than current estimates [9]; it can be explained with value of A more than one. $\Delta E = c^2 \Delta m$ is yet unconfirmed in chemical reactions, if energy emitted is found to be less, then it can be explained with value of A less than one. There are various perceptions [10-11] for variations in speed of light c, if materialised then it will automatically confirm $\Delta E = Ac^2 \Delta m$.

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