

About Speed and Time

By Roald C. Maximo
rc-maximo@uol.com.br

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Abstract: In this short essay, I am starting from very basic concepts to try, step by step, to establish a valid physical relation between speed, energy and time. My intention, right from the start, is to by pass the Theory of Relativity and also avoid the application of the Lorentz transformation as a canned good. If it has to be part of the solution it shall arise spontaneously during the formulations as it ended up being the case.

In the derivations that follow, I am using the expression $MV^2 / 2$ to emphasize the fact that it refers to kinetic energy and not to relativistic mass energy, though, in the calculations, it doesn't make much of a difference apart from a conceptual one. All derivations bellow are as seen from the moving or a co-moving frame of reference.

So, starting with energy

$$E_h = \frac{M \cdot V^2}{2} \quad [1]$$

$$E_l = \frac{M \cdot v^2}{2} \quad [2]$$

we get

from [1]
$$V = \sqrt{\frac{2 \cdot E_h}{M}} \quad [3]$$

from [2]
$$v = \sqrt{\frac{2 \cdot E_l}{M}} \quad [4]$$

Where E_h = higher energy, E_l = lower energy, V = higher speed, v = lower speed and T representing a longer time and t a shorter time. D = an arbitrary distance

$$\frac{D}{t} = \sqrt{\frac{2 \cdot E_h}{M}} \quad [5] \quad \leq \text{ a shorter time for a higher speed}$$

$$\frac{D}{T} = \sqrt{\frac{2 \cdot E_l}{M}} \quad [6] \quad \leq \text{ a larger time for a lower speed}$$

from [6]
$$T = \frac{D}{\sqrt{\frac{2 \cdot E_l}{M}}} \quad [7]$$
 and from [5] =>
$$D = t \cdot \sqrt{\frac{2 \cdot E_h}{M}} \quad [8]$$

substituting [8] in [7]

$$T = \frac{t \cdot \sqrt{\frac{2 \cdot E_h}{M}}}{\sqrt{\frac{2 \cdot E_l}{M}}} = t \cdot \sqrt{\frac{E_h}{E_l}}$$

r. c. maximo

$$E_h = h \cdot F \quad E_l = h \cdot f \quad \text{where } h \text{ is the Plank's constant, } F \text{ a higher frequency and } f \text{ a lower frequency.}$$

We get

$$T = t \cdot \sqrt{\frac{E_h}{E_l}} = t \cdot \sqrt{\frac{h \cdot F}{h \cdot f}} = t \cdot \sqrt{\frac{F}{f}} \quad [9]$$

Here f is to be taken as a reference frequency for a body at rest. Possibly de Broglie frequency $f = \frac{M \cdot c^2}{h}$. Now let's see where F comes from. For that matter we resort to the Doppler equations for a moving electromagnetic source.

$$f_f = f \cdot \frac{c}{c - v} \quad [10] \quad \text{leading edge frequency}$$

$$f_b = f \cdot \frac{c}{c + v} \quad [11] \quad \text{trailing edge frequency}$$

$$\lambda_f = \frac{c - v}{f} \quad [12] \quad \text{leading edge wave length}$$

$$\lambda_b = \frac{c + v}{f} \quad [13] \quad \text{trailing edge wave length}$$

$$\lambda = \frac{c}{f} \quad [14] \quad \text{rest wave length}$$

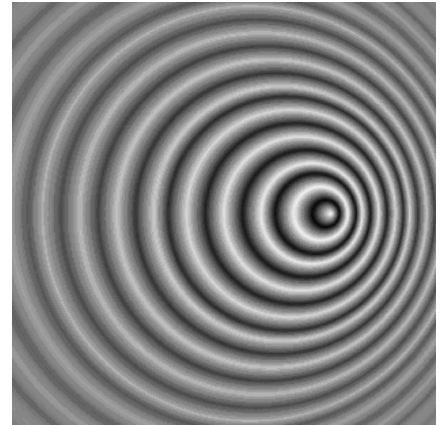


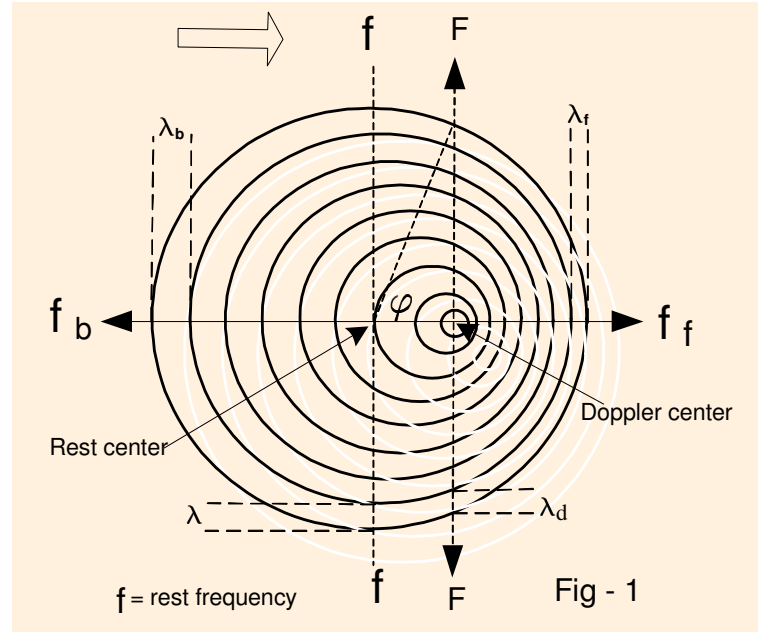
Fig [1] illustrates the situation. When at rest, the mean source frequency is f and will be centered on the diagram. For a moving source the rest frequency will be shifted in the trailing edge direction and the new center frequency location is here named **Doppler center**. It can easily be proven that the frequency F at the Doppler center is the arithmetic mean of the forward and backward frequencies f_f and f_b . As can be seen next, it has a shorter wave length and consequently a higher energy content than frequency f . This comes about, obviously, because of an existing asymmetry between the limits of f_f and f_b .

$$\lim_{v \rightarrow c} f_b = \frac{f}{2} \quad \lim_{v \rightarrow c} f_f = \infty$$

In what follows c , the speed of light, has been substituted for V , the higher speed .

$$F = \frac{f_f + f_b}{2} = \frac{f \cdot \left(\frac{c}{c-v}\right) + f \cdot \left(\frac{c}{c+v}\right)}{2} = f \cdot \frac{1}{1 - \frac{v^2}{c^2}}$$

$$F = f \cdot \frac{1}{1 - \frac{v^2}{c^2}} \quad [15]$$



From [9] we have

$$T = t \cdot \sqrt{\frac{F}{f}} \quad T = t \cdot \sqrt{\frac{f \cdot \frac{1}{1 - \frac{v^2}{c^2}}}{f}} \quad T = t \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad [16]$$

The relativistic time dilation equation [16] was obtained without resorting to SRT and, as can be seen, the Lorentz transformation emerged spontaneously, as expected, when we converted energy to frequency to time.

From fig [1] the angle φ is given by

$$\varphi = \arccos\left(\frac{v}{c}\right) = \arcsin\left(\sqrt{1 - \frac{v^2}{c^2}}\right) \quad [17]$$

which leads to the very simple relations

$$\lambda = \frac{c}{f} \quad [18] \quad \Leftarrow \text{Rest wave length}$$

$$T = \frac{t}{\sin(\varphi)} \quad [19] \quad \Leftarrow \text{Time delay}$$

$$F = \frac{f}{\sin(\varphi)^2} \quad [20] \quad \Leftarrow \text{Doppler center frequency}$$

$$\lambda_d = \lambda \cdot \sin(\varphi)^2 \quad [21] \quad \Leftarrow \text{Doppler center wave length}$$