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The Moving Mirror Thought Experiment Reviewed

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Abstract

Reinterpreting the moving mirror thought experiment Special Relativity it will be shown that for inertial frames approaching the speed of light the anticipated duration of an event decreases and the distance travelled per unit time increases according to the gamma factor correction. This approach offers resolution to the paradoxes of Relativity by employing the principle of the constancy of the velocity of light and absolute time as basis to arrive at accepted relativity formula for transformations in time and length. The transmission of light by optical fibre is employed as an example of the mirror thought experiment.

1. Introduction

Although the Special theory of relativity has displaced earlier classical formulae the Galilean and Newtonian conceptual framework of absolute time and space deserves further consideration. Einstein's kinematics for relativity asserts two hypotheses. Firstly, *the principle of relativity* asserts that there are an infinite number of inertial systems moving with respect to one another in which all physical laws assume the simplest form. Secondly, *the principle of the constancy of the velocity of light* asserts that the velocity of light has the same value in all inertial frames. As a consequence, relativity theory imposes varying time and length rates as inertial frames approach the speed of light.

Einstein introduced *thought experiments* to test, via the imagination, physical phenomena travelling at speeds approaching the speed of light; approximately 300,000 kilometres per second. Thought experiments do not represent empirical verification, however, they have been widely employed as educational aids to elucidate relativity theory. The moving mirror thought experiment is, perhaps, the most well known consisting of an inertial frame travelling with a speed approaching the speed of light while firing a light beam at a moving mirror and timing the return; see figures 1 and 2. The constancy of the speed of light is employed as the reference for time measurement. The timing of the return light beam is compared to that of a stationary frame so that the difference can be calculated. The difference in timing, termed the gamma (or beta) factor, is employed as an adjustment to the time for events and the length of distances in moving inertial frames.

Reinterpreting the moving mirror thought experiment it will be shown that for inertial frames approaching the speed of light the anticipated duration of an event decreases and the distance travelled per unit time increases according to the gamma factor correction. Employing the principle of the constancy of the velocity of light and absolute time as basis to arrive at the accepted relativity formula for transformations in time and length. Following the terminology of relativity, the duration of an event is termed 'proper time'. An increase in distance travelled per unit time in the direction of motion is termed 'proper velocity'.

To quote from the founder of gravitation, Sir Isaac Newton, from "The Principia" 1729 [1]:

"*Absolute, True, and Mathematical Time, of itself, and from its own nature flows equably without regard to any thing external, and by another name is called Duration: Relative, Apparent and common time is some sensible and external (whether accurate or unequable) measure of Duration by the means of motion, which is commonly used instead of True time, such as an hour, a day, a month..*

All Motions may be accelerated and retarded, but the True, or equable progress, of Absolute time is liable to no change. The duration or perseverance of things remains the same, whether the motions are swift or slow, or none at all..."

Newton espoused the view that absolute time is independent of the motion of inertial frames and relegated relative time as a secondary sensory phenomenon. This view of absolute time is eloquently expressed in the fourth Galilean velocity transformation asserting that in a moving or stationary inertial frame the duration of an event is equal. The fourth Galilean transformation can be written in terms of a moving inertial frame having a primed superscript, t', and the time, t, unprimed for an event in a stationary frame as shown:

$t' = t$

Alternatively, absolute time can be written in terms of the constancy of the rate of change of time within inertial frames to mathematically describe the Newtonian view that the duration remains constant and is independent of motion:

$dt' / dt = 1$

The optical fibre and waveguide employed internationally for high speed, wide-bandwidth communications have been given as a practical example of the moving mirror thought experiment. The optical fibre waveguides exhibit total internal reflection of the light signal and employ a stationary light source thereby representing a specific application of theory. Moreover, to reinterpret special relativity in terms of absolute time provides a method to resolve the 'Paradoxes of Relativity'.

2. Proper Time

The aim of the moving mirror experiment is to employ light and its constancy as the reference to establish the time for events in two inertial frames, one moving and the other stationary [1]. A light beam is fired from a moving inertial frame towards a moving mirror and the return of the beam is timed. The inertial frame with primed notation moves with a velocity, V' as is shown in figure 1. In comparison to a stationary frame a light beam is fired at the mirror and the return is timed. Given that the speed of light remains constant a difference in time will be recorded for both cases because the path length of the inertial frame in motion is longer. Figures 1 and 2 demonstrate the two cases.

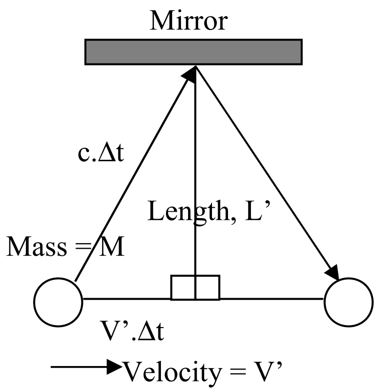

Figure 1. Moving Inertial Frame

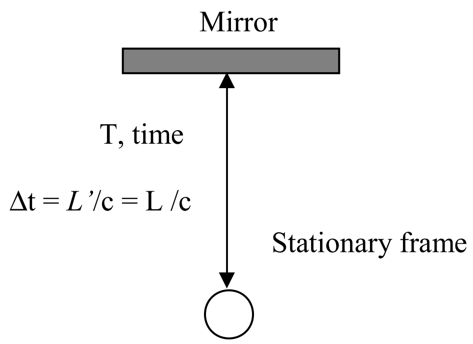

Figure 2. Stationary Inertial Frame

The terms are defined below:
c is the speed of light.
The elapsed time Δt' is measured in the moving inertial frame.
Δt is the proper time measured in the stationary frame.
L is the perpendicular length in the moving frame.
L' is the perpendicular length in the stationary frame, that is, equivalent to L.

As a preliminary the times taken to measure a fixed length, *L',* in both the stationary and moving frames will be found to be different. The fixed length in this case are the perpendicular lengths L and L' between the mirror and the source. The proper time, T_O, is defined here as the measured time for an event in the stationary frame, differing from Relativistic notation.

Using the Pythagorean triangle in figure 1 it is found that:

$$(c.\Delta t)^2 = (L')^2 + (V'.\Delta t)^2$$

$$(\Delta t)^2 = L'^2 / (c^2 - V'^2)$$

$$(\Delta t)^2 = L'^2 / c^2 .1 / (1 - V'^2 / c^2)$$

$$\Delta t = (L' / c) .1 / (1 - V'^2 / c^2)$$

and it can be seen that the fixed length, *L'* = L is equivalent in both frames,

$$\Delta t = (L / c) .1 / \sqrt{(1 - V'^2 / c^2)}$$

and the proper time for an event, Δt = L / c where L is the distance to the mirror.

$$\Delta t' = \Delta t . (1 / \sqrt{(1 - V'^2 / c^2)}$$

Let, T_O, be the proper time for an event and T be the time measured from the moving inertial frame.

$$T = T_O / \sqrt{(1 - V'^2 / c^2)}$$

Let $\beta = 1/\sqrt{(1 - (V'^2 / c^2))}$ be treated as a scalar constant because the velocity of the inertial frame is constant and it is noted that $\beta > 1$.

$$\therefore T = T_O . \beta$$

The time, T, is the anticipated Coordinate time for an event and is dilated for measurement of a fixed length, according to Special Relativity. However, the coordinate time does not actually occur. Rather, the proper time for an event is contracted against the anticipated coordinate time when measured against a fixed length according to the formula:

$$\mathbf{T_O = T / \beta} \qquad \mathbf{1.}$$

The proper time for an event, T_O, can be understood as the time for the event as measured in both the moving and stationary inertial frames. The proper time in formula 1 has been interpreted as the total reduced time for an event as compared to the anticipated time. For instance, when the muon particle moves at near light speeds of 0.995c the coordinate time corresponds to the coordinate velocity of the particle. While, the anticipated coordinate time for travel at approximately the speed of light is 6.3 microseconds, when the measured time was in fact 0.7 microseconds:

$$\delta = \beta = 1/\sqrt{(1 - (V'^2 / c^2))} = 9.0$$

The anticipated coordinate time, T is:

$$T = \delta . T_O = 9.0 \times 0.7 \times 10^{-6} = 6.3 \times 10^{-6} \text{ secs}$$

In this view time dilation or contraction do not occur within the stationary or moving inertial frame for the reduced anticipated total time of an event. Furthermore, experiments in relativity confirm that the measured duration time for events is equivalent to the proper time [11-16].

3. Proper Length

As a preliminary, the measured lengths will be taken from the stationary and the moving frame for a fixed time interval and will be found to be different. The proper length is defined as the product of the proper velocity is considered to be time and the length, L', is simply a product of velocity and time. While the perpendicular length remains constant the length measurement in the direction of motion is the variable, following the convention for Special Relativity. The length of a rod, L', is measured from the primed frame to indicate that it is parallel measurement. The time interval, T, is fixed in both frames and is measured as the rod passes by a fixed point.

The length, L', of the rod as measured in the moving frame and the magnitude, V', of its velocity as observed in the moving frame is given by the equation below:

$L' = V'. T$

In the stationary frame the proper length of the rod, L_O, measured for a fixed time, T, is given by:

$L_O = V_O. T$

and substituting Lorentz transformation for proper velocity, $V_O = V'. \beta$:

$$L'/L_O = (V'. T) / (V_O. T)$$
$$= V' / (V'. \beta) = 1 / \beta$$

$L' = L_O / \beta$

The parallel length is decreased in the direction of motion for a moving inertial frame when measured for a fixed duration of time. Conversely the proper length is increased for a fixed time interval of time and length parallel to motion as shown below:

$$\mathbf{L_O = L'. \beta} \qquad \mathbf{2.}$$

Length values do not change, as in the theory of relativity, however an apparent increase in length will be measured per unit time as a anticipated distance of travel, L, will be exceeded by the beta correction factor correction for length. Thus, the proper length per unit time can be viewed in terms of velocity.

4. Proper Velocity

An inertial frame travelling with Coordinate velocity, V', is transformed with respect to the proper velocity. The proper velocity can be derived from the proper distance divided by the time taken in the stationary frame. For the moving mirror experiment the velocity transformation of a moving inertial frame can be derived for single dimensional motion. Please refer to figure 1 and 2. The following points are of note:

- The distance travelled by the primed inertial frame is $V'.\Delta t$.
- The time for light travel reflection of a stationary frame is t.

The velocity of the inertial frame with respect to the stationary frame is:

$$V_O = V'. \Delta t/ t$$
$$= V'. (1/\sqrt{(1 - v^2 /c^2)})$$

$$\mathbf{V_O = V'. \beta} \qquad \mathbf{3.}$$

Following from (3), proper velocity is greater than or equal to the coordinate velocity as beta, $\beta \geq 1$. As beta is unlimited, proper velocity may exceed the speed of light. Empirical evidence has found that coordinate velocity is less than the speed of light, c, while proper velocity diverges from the coordinate as the speed of light is approached.

The proper velocity, V_O, can also be defined in terms of proper length and coordinate time, T, measured in the stationary frame. As absolute time is assumed, the duration of events will be identical in all inertial frames. A point to consider is that a transformation of length and time cannot be applied simultaneously; that is, either length or time must be applied.

$$\mathbf{V_O = L_O / T = V'.\beta} \qquad \mathbf{3a.}$$

The proper velocity, V_O, can also be defined in terms of coordinate length, L, and proper time, T_O. The proper velocity is then the distance, L, measured in the observer's inertial frame per the proper time. Therefore the proper velocity of astronomical bodies [17] could be found using formula 3b below.

$$\mathbf{V_O = L / T_O = V'.\ \beta} \qquad \mathbf{3b.}$$

Points of Note:

i) The moving and stationary inertial frames in figures 1 and 2 represent two separate events with differing durations.
ii) Absolute time and fixed length form the basis for the moving mirror thought experiment.
iii) The speed of light is measured as constant in the stationary and the moving inertial frames. Similarly, the perpendicular length is constant in both inertial frames.
iv) The velocity of the moving inertial frame is the only variable.
v) The velocity of the inertial frame may be adjusted in order to maintain the speed of light constancy of all inertial frame measurements.
vi) The beta adjustment factor can be applied to the time for events in both the stationary and the moving inertial frame.
vii) In order to maintain speed of light constancy the time for events in the moving inertial frame is reduced to that of the stationary frame.

5. The Paradoxes of Relativity

According to Relativity theory clocks change time rates within the inertial frames, and time is dilated or contracted according to the velocity of the inertial frame. If there is an increase in proper velocity and a subsequent decrease in proper time, however, a time rate change is not required within inertial frames. Although event time reduction can be measured, time dilation or contraction must be inferred and may not actually occur in the stationary frame or the inertial frame in motion.

To reconsider the 'Twin paradox': for twins, one is an astronaut who travels at relativistic speeds to another planet that is 5 light years distant with a return journey of 10 light years and the other remains on earth. On returning, the astronaut had aged only several months while his twin had aged by 10 years. Relativity theory holds that the clock on the astronaut's space ship actually moved slower as a result of time dilation and the change at the cellular level was reflected in the astronaut aging less than his twin.

The 'Twin Paradox' could be resolved if the time rate in the spaceship and on earth remained the same and total time for the journey was reduced. The coordinate velocity of 99.9% of the speed of light transformed the proper velocity of the spaceship to superluminal proper speeds at 22.3 times the speed of light. The coordinate time for the return journey would be anticipated to be 10 years as the spaceship would be travelling at approximately the speed of light. The twin brothers would age at the same rate in the stationary and moving frame. The velocity transformation for a body with near light speed would reduce the return journey by a factor of 22. In this case the time was reduced from 10 years to 5 months and 13 days.

The paradox of the fast walker involves a walker who walks across a grid that is smaller than his foot length. However, as the walker approaches the speed of light his feet reduce in length until he falls into the grid. The length of physical objects remains fixed in all inertial frames for any velocity including near light speed travel. The increase in length is only apparent as the increased proper velocity dilates the distance travelled per unit time. This solution also resolves the paradox of Relativity including the "Pole and Barn" paradox [6].

6. Optical Fibre and the Vanishing LED

Acting as a waveguide, the optical fibre is a practical version of the moving mirror thought experiment. The light source is a LED with a modulated laser light beam and a velocity of zero. The total internal reflection of the light beam occurs at incident angles greater than the critical angle for a light beam while the interface between the core and cladding performs the role of a mirror. This is shown in figure 3 below.

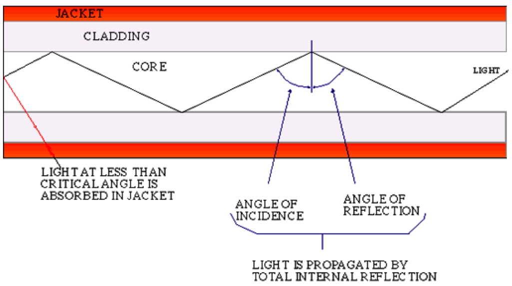

Figure 3. Optical waveguide showing internally reflected light

Proper Time, $T_O = T / \beta$
where $\beta = 1/\sqrt{(1 - v^2/c^2)}$

The velocity of the LED source is zero, $v = 0$.
$v = 0$ and $\beta = 1 \therefore T_O = T / \beta = T$
Proper Length, $L_O = L.\ \beta$
$v = 0$ and $\beta = 1 \therefore L_O = L.\ \beta = L$

It has been firmly established by engineering theory and application that there is no change in the proper length or time for the transmission of optical signals within the optical fibre.

However, practical examples of the moving mirror thought experiment would be furnished if the LED source were able to move in relation to the optical fibre. Consider, a thought experiment in which the LED source itself became a relativistic inertial frame moving at speeds approaching the speed of light. Special relativity theory asserts that physical dimensions of the length of the LED, the length, L', would contract according to the equation:

$L' = L_O / \beta$

As the velocity of the LED approaches the speed of light beta, β, approaches infinity:

As $V \rightarrow c$ then $\beta \rightarrow \infty$

And the length of the LED in its relativistic inertial frame falls away to zero:

$L' \rightarrow 0$

According to Relativity as the velocity of the LED approaches the speed of light its length would diminish to zero. However, Relativity provides no clue as to why a physical object would cease to be when it reaches a certain threshold of velocity.

7. Conclusion

The review of the moving mirror thought experiment using the principle of the constancy of the velocity of light and absolute time provides the following formula for proper time, length and velocity for constant translational velocity:

(i) Time, $\mathbf{T_O = T / \beta}$

where T_O is the measured proper time for an event and T is apparent coordinate time per unit length and $\beta = 1/\sqrt{(1 - (V'^2/c^2))}$.

(ii) Length, $\mathbf{L_O = L.\ \beta}$

where L_O is proper length and L is apparent length measured parallel to motion in the moving frame per unit time.

(iii) Velocity, $\mathbf{V_O = V'.\ \beta = L_O/T = L/T_O}$

where V_O is proper velocity, V' is Coordinate velocity in moving frame, L_O is proper length and T_O is duration time for the event.

When reinterpreted, the moving mirror thought experiment provides the established formula for special relativity without the need to assume relativistic transformations in time and length both between and within the moving and stationary inertial frames. However, the time for an event will be contracted relative to the expectation of the time for events, while, the time for events would be identical in the moving and stationary inertial frames. Similarly, the distance travelled per unit time in the direction of motion would increase per unit time and this is termed the proper velocity. The overall effect of near light speed motion is to produce an increase in the proper velocity relative to the coordinate velocity.

The concept of absolute time as described by Newton in the following "*The duration or perseverance of things remains the same, whether the motions are swift or slow, or none at all...*" is mathematically defined in the following:

$dt' / dt = 1$.

The concept of absolute time combined with principle of constancy of the speed of light matches known physical measurements for inertial frames approaching the speed of light [13].

In section 4 of his 1905 paper Einstein notes a 'peculiar consequence' [10] of relativity theory for the time dilation of a moving clock relative to its stationary twin. Academic discussion of this point became known as the 'Twin Paradox'. Proper time and length as transformed against for the expectation of coordinate event time and distance, for near light speed inertial motion, can provide a solution for this paradox and the other paradoxes associated with Relativity.

8. References

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