Mathematical Invalidity of the Lorentz Transformation in Relativity Theory

Aleksandar Vukelja
aleksandar@masstheory.org

April 2005.

Abstract

MATHEMATICAL PROCEDURE by which Albert Einstein derived Lorentz transformation is incorrect. The transformation is an imaginary "solution" to a set of equations which evaluate to zero throughout the derivation process.

Author derives Lorentz transformation the way Einstein did, and shows the places where errors were made.

Critical Analysis of Einstein's paper
"Simple Derivation of the Lorentz Transformation"

Readers will find in the appendix B unmodified copy of Einstein's derivation, as published in "Relativity: The Special and General Theory", 1920.

We have two frames of reference, K which is still, and K' which is moving along the positive side of x axes of K with speed v.

A light signal, which is proceeding along the positive axis of x, is transmitted according to the equation

$$x = ct \quad \text{or} \quad x - ct = 0 \tag{1}$$

where c is the speed of light.

(Postulate 1) Assuming that the speed of light is independent of the speed of observer v, in K' for the same front of the light signal we write

$$x' = ct' \quad \text{or} \quad x' - ct' = 0 \tag{2}$$

To find out how to transform coordinates between K and K',
(Error 1) (Postulate 2) We assume that there is a proportionality quotient λ,

$$x' - ct' = \lambda\,(x - ct) \tag{3}$$

(Error 2) If we apply quite similar considerations to light rays which are being transmitted along the negative x axis,
(Postulate 3) then we have another quotient μ

$$x' + ct' = \mu\,(x + ct) \tag{4}$$

By adding (3) and (4) we have

$$x' = a\,x - b\,ct \tag{5a}$$
$$ct' = act - bx \tag{5b}$$

where for convenience we introduced $a = \dfrac{\lambda + \mu}{2}$ and $b = \dfrac{\lambda - \mu}{2}$.

If we would know quotients a and b, our derivation would be complete. For the base of K' we always have x' = 0,

(Error 3) then from (5a) we have

$$x = \frac{bc}{a}t \quad \text{or} \quad v = \frac{x}{t} = \frac{bc}{a} \tag{6}$$

where v is the speed of K' system relative to K.

Observed from *K*, relation of x' and x recorded when t = 0, from (5a) is

$$x' = ax \qquad (7)$$

Two points of the x' axis which are separated by the distance $x' = 1$ measured in K' are thus separated at the same moment in *K* by the distance

$$\Delta x = \frac{1}{a} \qquad (8)$$

But if the snapshot be taken from K' at the moment t' = 0,

(Error 4) Observed from K', relation between x' and x recorded when t' = 0, from (5b) we have

$$act = bx \quad \text{or} \quad t = \frac{bx}{ac} \qquad (9)$$

(Error 5) and by using (9) in the same equation (5a) from which it was derived, we get

$$x' = a\left(1 - \frac{b^2}{a^2}\right)x \qquad (10)$$

Using expression (6) for speed *v*, expression (10) becomes:

$$x' = a\left(1 - \frac{v^2}{c^2}\right)x \qquad (11)$$

From this we conclude that two points on the x axis and separated by the distance 1 (relative to *K*) will be represented on our snapshot by the distance:

$$\Delta x' = a\left(1 - \frac{v^2}{c^2}\right) \qquad (12)$$

But from what has been said, the two snapshots must be identical; hence Δx in (8) must be equal to $\Delta x'$ in (12), so we obtain:

$$a^2 = \frac{1}{1 - \frac{v^2}{c^2}} \qquad (13)$$

The equations (6) and (13) determine the constants *a* and *b*. By inserting the values of these constants in (5a), we obtain the Lorentz Transformation:

$$x' = \frac{x - vt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}, \quad t' = \frac{t - \frac{v}{c^2}x}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \qquad (14)$$

Explanations of Errors

Error 1 Expression (3) is useless. We have

$$x - ct = 0 \quad (1)$$
$$x' - ct' = 0 \quad (2)$$

In (3) Einstein writes

$$x' - ct' = \lambda\,(x - ct) \quad (3)$$

Because of (1) and (2) we can write (3) as

$$0 = \lambda\ 0$$

One can postulate that meaningful values, which are at least sometimes both different from zero, are somehow related. Introducing proportionality quotient between nothing and nothing has no meaning.

Error 2 Explanation is inapplicable. If rays of light are traveling in both positive and negative directions of the x axes, then one cannot combine their positive and negative x coordinates as they represent different events (addition of apples and oranges).

Equation (4) can be used in a valid context, and that is postulation of new relationship between the same coordinates on the positive side, besides existing postulate (3). However this is not what he wrote.

Error 3 Expression (6) is a cardinal error. Derivation begins with (1) $x = ct$ for the front of the beam of light.

Few lines below, in (6) Einstein now presents relative speed of two coordinate systems as

$v = \frac{x}{t}$ or $x = vt$ in the same context.

This can only be valid so long as $vt = ct$ or $v = c$. As that was the point of inclusion of relative speed v into equations, all that follows is meaningless.

Error 4 Expression (9) is valid only when x' = 0, which according to (2) implies also that t' = 0. This can happen only when also x = 0 and t = 0, rendering (9) useless. Otherwise, because (5a) and (5b) are the same equation, when x' = 0 then t' = 0 and we have

$$\frac{x}{t} = \frac{bc}{a} \quad \text{from (5a)}$$
$$\frac{x}{t} = \frac{ac}{b} \quad \text{from (5b)}$$

Which would it be if $x \neq 0$?

Error 5 Expression (11) is yet another cardinal error. It is accumulation of all previous errors, and adds additional nonsense:

One cannot assume x' = 0 in (5b) and then include (9) which was derived from that assumption, back into the same equation (5a) and then pretend that x' is now different from zero. This is how (11) was derived.

It takes exceptionally strong illusions and lack of math skills to make five such errors on a single sheet of paper.

In all, entire logic of Einstein is inherently erroneous.

$$t = 0,\ t' = 0,\ x = 0,\ x' = 0 \qquad \text{(a)}$$

None of assumptions made in derivation at various places, such as (a) is permitted. This is erroneous because the same requirements remain inherently present in equations derived through such simplifications.

In other words, if we have an equation set (A), and obtain some new equation set (B) from it by using special cases such as (a), and combine (A) and (B) into (C), then (C) inherently contains (a) and is valid so long as (a) is satisfied. Lorentz transformation derived this way is a case of (C), and is valid only for values t = 0, x = 0, t' = 0, x' = 0.

By making such errors, one can derive nearly anything, Lorentz transformation included. Unfortunately for science, this is one thing that his many followers learned very well from him, and happily applied.

References

Albert Einstein
Relativity: The Special and General Theory. (a copy of examined text is included in appendix)

This article was inspired by work of

Milan Pavlović
“Ajnštajnova teorija relativnosti – naučna teorija ili OBMANA”, downloaded from http://users.net.yu/~mrp

Appendix – copy of Einstein's original derivation

Albert Einstein (1879–1955). Relativity: The Special and General Theory. 1920.

Simple Derivation of the Lorentz Transformation

[SUPPLEMENTARY TO SECTION XI]

FOR the relative orientation of the co-ordinate systems indicated in Fig. 2, the x-axes of both systems permanently coincide. In the present case we can divide the problem into parts by considering first only events which are localized on the x-axis. Any such event is represented with respect to the co-ordinate system K by the abscissa x and the time t, and with respect to the system k' by the abscissa x' and the time t'. when x and t are given.

A light-signal, which is proceeding along the positive axis of x, is transmitted according to the equation

$$x = ct \quad \text{or} \quad x - ct = 0 \qquad (1)$$

Since the same light-signal has to be transmitted relative to k' with the velocity c, the propagation relative to the system k' will be represented by the analogous formula

$$x' - ct' = 0 \qquad (2)$$

Those space-time points (events) which satisfy (1) must also satisfy (2). Obviously this will be the case when the relation

$$(x' - ct') = \lambda\,(x - ct) \qquad (3)$$

is fulfilled in general, where l indicates a constant; for, according to (3), the disappearance of (x – ct) involves the disappearance of (x' – ct').

If we apply quite similar considerations to light rays which are being transmitted along the negative x-axis, we obtain the condition

$$(x' + ct') = \mu\,(x + ct) \qquad (4)$$

By adding (or subtracting) equations (3) and (4), and introducing for convenience the constants a and b in place of the constants l and m where

$$a = \frac{\lambda + \mu}{2}$$

and

$$b = \frac{\lambda - \mu}{2}$$

we obtain the equations

$$\begin{aligned} x' &= ax - bct \\ ct' &= act - bx \end{aligned} \qquad (5)$$

We should thus have the solution of our problem, if the constants a and b were known. These result from the following discussion.

For the origin of k' we have permanently x' = 0, and hence according to the first of the equations (5)

$$x = \frac{bc}{a}t$$

If we call v the velocity with which the origin of k' is moving relative to K, we then have

$$v = \frac{bc}{a} \qquad (6)$$

The same value v can be obtained from equation (5), if we calculate the velocity of another point of k' relative to K, or the velocity (directed towards the negative x-axis) of a point of K with respect to K'. In short, we can designate v as the relative velocity of the two systems.

Furthermore, the principle of relativity teaches us that, as judged from K, the length of a unit measuring-rod which is at rest with reference to k' must be exactly the same as the length, as judged from K', of a unit measuring-rod which is at rest relative to K. In order to see how the points of the x'-axis appear as viewed from K, we only require to take a “snapshot” of k' from K; this means that we have to insert a particular value of t (time of K), e.g. t = 0. For this value of t we then obtain from the first of the equations (5)

$$x' = ax$$

Two points of the x'-axis which are separated by the distance x'=1 when measured in the k' system are thus separated in our instantaneous photograph by the distance

$$\Delta x = \frac{1}{a} \tag{7}$$

But if the snapshot be taken from K'(t' = 0), and if we eliminate t from the equations (5), taking into account the expression (6), we obtain

$$x' = a\left(1 - \frac{v^2}{c^2}\right)x$$

From this we conclude that two points on the x-axis and separated by the distance 1 (relative to K) will be represented on our snapshot by the distance

$$\Delta x' = a\left(1 - \frac{v^2}{c^2}\right) \tag{7a}$$

But from what has been said, the two snapshots must be identical; hence Δx in (7) must be equal to Δx' in (7a), so that we obtain

$$a^2 = \frac{1}{1 - \frac{v^2}{c^2}} \tag{7b}$$

The equations (6) and (7b) determine the constants a and b. By inserting the values of these constants in (5), we obtain the first and the fourth of the equations given in Section XI.

$$\begin{aligned} x' &= \frac{x - vt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \\ t' &= \frac{t - \frac{v}{c^2}x}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \end{aligned} \tag{8}$$

Thus we have obtained the Lorentz transformation...