Deformation of the Bodies Through Length Contraction: A new Approach to the Lorentz Contraction

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Abstract

It has been more than a century since Lorentz and later Einstein, explored relativistic events and there are still-important consequences of the theory that remain unclear to everyone. The present study focuses exclusively on the Lorentz length contraction phenomenon and takes a different approach to explain how the contraction of a body appears, if it indeed exists. We utilized the two postulates of Special Relativity that emphasize the constancy of the speed of light and common physical laws in all inertial frames. Finally, we propose the Lorentz contraction exists, but in a different form regarding the shape of the accelerated body. The deformation of its shape depends on the part that is measured.

I. Introduction

In his 'Dialogue Concerning the Two Chief World Systems', Galileo Galilei, in 1632, described Relativity with a ship model which travels at a constant speed with no lateral motion, on a smooth sea; any observer doing experiments below the deck would not be able to tell whether the ship was moving or stationary. This idea later became one of Einstein’s postulates in developing his new relativity theory about three centuries later. However, his explanation of the results didn’t guide him to a full understanding of the relativistic picture, because one aspect regarding speed of light was missing. That latter idea was discovered through the Michelson-Morley experiment in the mid 19th century. It contributed to the full understanding of relativity concepts.

Several new papers and articles were published that discussed the consequences of Einstein’s relativity but still, some aspects remained unclear to physicists, and more importantly, to physics educators. The study of length contraction and time dilation goes back to the 1950s with Terrell’s paper (1986) which discusses the relativistic consequences of the invisibility of the Lorentz contraction. The author showed that anybody could deduce the formulas and claimed that even though one can use methods of various measuring techniques (e.g. stereoscopic photography), length contraction cannot be pictured.

Boas (1961) strongly believed that the relativistic contraction could be seen as curved rather than contracting. Boas focused on the famous Gamow’s bicyclist example to show that the shape of the picture is curved rather than shortened. Based on the outcomes of previous research ideas, the current paper supports Boas’ ideas from different perspectives. We strongly believe that the shape of the moving body appears curved at both ends and we will demonstrate it by using time dilation ideas and the constancy of light.
Rindler (1961) discussed the deformation of an accelerated body with an example of a man walking very fast and trying to get over a deep and wide well which is certainly unsuccessful. He calls it a paradox but there is no paradox because the moving body is not in the same inertial frames as the grid. Grids are present in the observer’s frames of reference. However, I agree with other results which reveal that the warping of the shape of the moving body starts with the front of the body and pointed towards the front.

II. Claim

We strongly believe that the Lorentz contraction exists but we cannot suggest it can be measured or photographed in the same sense, because the apparatus currently used is not sufficient to reveal that it really occurs in a natural environment. We propose a new approach to the change of shape of the accelerated body. We make use of a train example. Consider that we have a wagon in a train moving with a relativistic velocity and we have two mirrors placed diagonally at the left lower and upper front of the wagon in figure 1. We generate a light signal from the rear end of the wagon and it is reflected back from the front mirror. A fixed observer next to the train station observes light signal (figure 2). We compare the measurement of time intervals by the train clock and the observer’s time.

Figure 1. Bouncing light rays from the diagonal mirrors as seen by a person in the wagon

\[
\ell = v \times t \quad (t \text{ is the proper time inside the train}) \quad (1)
\]

Let \( \tau \) be time according to the outside observer’s clock, so the related time differences between two persons, (time dilation equation)

\[
\tau = \gamma \times t \quad (2)
\]

Also, we can easily write down the height of the wagon by using the total travel times of light rays and the speed of light,
Equations 1-3 belong to figure one and are based on the observer inside the wagon. Now, we can look at the equations constructed according to the outside observer.

Figure 2. Bouncing light rays from the diagonal mirrors in the wagon as seen by outside observer

Total distance of the outgoing light ray according to the observer at the train station is:

\[ \ell' = c \times \tau \]  \hspace{1cm} (4)

Let us suppose the speed of the wagon is close to the speed of light so that the angle between light rays and the horizontal axis of the wagon is zero. We can easily construct a right angle triangle, and by using equations 1, 3, and 4 (Pythagoras theorem)

\[ (\ell)^2 + (\ell')^2 = (h)^2 \]

That gives us the famous length contraction formula,

\[ c^2 t^2 - c^2 \gamma^2 t^2 = \ell^2 \]

\[ \ell' = \ell \times \gamma (\ell': \text{length of the wagon measured by the observer at the station}) \]  \hspace{1cm} (5)

Now that we have proved the deformation and shrinking of the wagon with the usage of time dilation, we can look at the corners of the wagon. However, their shapes change during the motion.

We can construct a different right angle triangle, than the prior case. This time, the wagon travels at 0.5c (half the speed of light). Based on that, the light rays in both cases (inside observer and outside observer) do not create a right angle triangle, but if we include the length of the wagon, it constructs two triangles as in Figure 3,
Figure 3. Construction of two triangles at a speed of $\frac{1}{2} c$

The equations are constructed by two right angle triangles,

\[
(ct)^2 - (V\tau)^2 = (h)^2 \tag{6}
\]

\[
(ct)^2 - (V\tau)^2 = (h')^2 \tag{7}
\]

By eliminating $V$ terms first, and time terms next in equations 6 and 7 with $\tau = \gamma t$, we can easily deduce the following equations,

\[
(h')^2 + (h)^2 = (ct)^2 - (ct)^2
\]

\[
(h')^2 - (h)^2 = 2 (V\tau)^2 - (V\tau)^2
\]

Finally, we can deduce the deformation formulas as,

\[
(h')^2 = (ct)^2 - (V\tau)^2 \tag{8}
\]

\[
h = ct \tag{9}
\]

Equations 8 and 9 prove the shape-changing of the wagon and give an approximation of the rate of change, which depends on where it is measured on the wagon. Finally, at speeds close to the speed of light, the rectangular train wagon will look like a parallelogram and oval:

Figure 4. Formation shape of the wagon at the speeds close to the speed of light
III. Conclusion

In this article, only one of the SR postulates, the constancy of the speed of light, was used to implement a new perspective of the length contraction phenomenon according to the Lorentz formulas. The method used to explain the deformation of objects because of their high speed is a very simple example and can be found in several elementary SR textbooks which also derive the time dilation equation. However, we used and generalized it by looking at the light rays with various velocities as they reach the speed of light. This new and more generalized method can be used as an advanced approach in relativity books to illustrate the deformation degrees. We don’t claim that this deformation is visible or can be photographed. We will investigate that in a separate and later study.

In conclusion, we claim that the angle of the light rays (both incoming and reflected) will create smaller angles with the horizontal length of the wagon because of the first postulate of Einstein’s SR, the constancy of the speed of light, and it is obvious that this angle will get smaller as the wagon speeds up, reaching a cut-off value of 0 as the wagon travels at the speed of light. Hence, we expect major deformation from both ends of the wagon shaping it like two rockets connected at their rear ends.

Our assertion creates a new approach to length contraction phenomena but it shows similarities to the previous research findings by Rindler (1961). The only difference between our expectations and his is that we expect both ends of the wagon to shrink at some level but at the same rate, and also the length of the wagon will reduce in size.

4. Rindler, W., American Journal of Physics, 29, 8 (1961)